On the Possibility of Instant Displacements in the Space-Time of General Relativity

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Employing the mathematical apparatus of chronometric invariants (physical observable quantities), this study finds a theoretical possibility for the instant displacement of particles in the space-time of the General Theory of Relativity. This is to date the sole theoretical explanation of the well-known phenomenon of photon teleportation, given by the purely geometrical methods of Einstein's theory.

As it is known, the basic space-time of the General Theory of Relativity is a four-dimensional pseudo-Riemannian space, which is, in general, inhomogeneous, curved, rotating, and deformed. There the square of the space-time interval $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$, being expressed in the terms of physical observable quantities — chronometric invariants [1, 2], takes the form

$$ds^2 = c^2 d\tau^2 - d\sigma^2.$$

Here the quantity

$$d\tau = \left(1 - \frac{\mathrm{w}}{c^2}\right) dt - \frac{1}{c^2}\, v_i dx^i$$

is an interval of physical observable time, $\mathrm{w}=c^2(1-\sqrt{g_{00}})$ is the gravitational potential, $v_i = -c\frac{g_{0i}}{\sqrt{g_{00}}}$ is the linear velocity of the space rotation, $d\sigma^2 = h_{ik}\, dx^i dx^k$ is the square of a spatial observable interval, $h_{ik} = -g_{ik} + \frac{1}{c^2}\, v_i v_k$ is the metric observable tensor, g_{ik} are spatial components of the fundamental metric tensor $g_{\alpha\beta}$ (space-time indices are Greek $\alpha, \beta = 0, 1, 2, 3$, while spatial indices — Roman $i, k = 1, 2, 3$).

Following this form we consider a particle displaced by ds in the space-time. We write ds^2 as follows

$$ds^2 = c^2 d\tau^2 \left(1 - \frac{\mathrm{v}^2}{c^2}\right),$$

where $\mathrm{v}^2 = h_{ik}\, \mathrm{v}^i \mathrm{v}^k$, and $\mathrm{v}^i = \frac{dx^i}{d\tau}$ is the three-dimensional observable velocity of the particle. So ds is: (1) a substantial quantity under $\mathrm{v} < c$; (2) a zero quantity under $\mathrm{v} = c$; (3) an imaginary quantity under $\mathrm{v} > c$.

Particles of non-zero rest-masses $m_0 \neq 0$ (substance) can be moved: (1) along real world-trajectories $cd\tau > d\sigma$, having real relativistic masses $m = \frac{m_0}{\sqrt{1 - \mathrm{v}^2/c^2}}$; (2) along imaginary world-trajectories $cd\tau < d\sigma$, having imaginary relativistic masses $m = \frac{im_0}{\sqrt{\mathrm{v}^2/c^2 - 1}}$ (tachyons). World-lines of both kinds are known as *non-isotropic trajectories*.

Particles of zero rest-masses $m_0 = 0$ (massless particles), having non-zero relativistic masses $m \neq 0$, move along world-trajectories of zero four-dimensional lengths $cd\tau = d\sigma$ at the velocity of light. They are known as *isotropic trajectories*. Massless particles are related to light-like particles — quanta of electromagnetic fields (photons).

A condition under which a particle may realize an instant displacement (*teleportation*) is equality to zero of the observable time interval $d\tau = 0$ so that the *teleportation condition* is

$$\mathrm{w} + v_i u^i = c^2,$$

where $u^i = \frac{dx^i}{dt}$ is its three-dimensional coordinate velocity. From this the square of that space-time interval by which this particle is instantly displaced takes the form

$$ds^2 = -d\sigma^2 = -\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik}\, dx^i dx^k,$$

where $1 - \frac{\mathrm{w}}{c^2} = \frac{v_i u^i}{c^2}$ in this case, because $d\tau = 0$.

Actually, the signature $(+---)$ in the space-time area of a regular observer becomes $(-+++)$ in that space-time area where particles may be teleported. So the terms "time" and "three-dimensional space" are interchanged in that area. "Time" of teleporting particles is "space" of the regular observer, and vice versa "space" of teleporting particles is "time" of the regular observer.

Let us first consider substantial particles. As it easy to see, instant displacements (teleportation) of such particles manifests along world-trajectories in which $ds^2 = -d\sigma^2 \neq 0$ is true. So the trajectories represented in the terms of observable quantities are purely spatial lines of imaginary three-dimensional lengths $d\sigma$, although being taken in ideal world-coordinates t and x^i the trajectories are four-dimensional. In a particular case, where the space is free of rotation ($v_i = 0$) or its rotation velocity v_i is orthogonal to the particle's coordinate velocity u^i (so that $v_i u^i = |v_i||u^i| \cos(v_i; u^i) = 0$), substantial particles may be teleported only if gravitational collapse occurs ($\mathrm{w} = c^2$). In this case world-trajectories of teleportation taken in ideal world-coordinates become also purely spatial $ds^2 = g_{ik}\, dx^i dx^k$.

Second, massless light-like particles (photons) may be teleported along world-trajectories located in a space of the metric

$$ds^2 = -d\sigma^2 = -\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik}\, dx^i dx^k = 0,$$

because for photons $ds^2 = 0$ by definition. So the space of photon teleportation characterizes itself by the conditions $ds^2 = 0$ and $d\sigma^2 = c^2 d\tau^2 = 0$.

The obtained equation is like the "light cone" equation $c^2 d\tau^2 - d\sigma^2 = 0$ ($d\sigma \neq 0$, $d\tau \neq 0$), elements of which are world-trajectories of light-like particles. But, in contrast to the light cone equation, the obtained equation is built by ideal world-coordinates t and x^i — not this equation in the terms of observable quantities. So teleporting photons move along trajectories which are elements of the world-cone (like the light cone) in that space-time area where substantial particles may be teleported (the metric inside that area has been obtained above).

Considering the photon teleportation cone equation from the viewpoint of a regular observer, we can see that the spatial observable metric $d\sigma^2 = h_{ik}\, dx^i dx^k$ becomes degenerate, $h = \det ||h_{ik}|| = 0$, in the space-time area of that cone. Taking the relationship $g = -h\, g_{00}$ [1, 2] into account, we conclude that the four-dimensional metric $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ degenerates there as well $g = \det ||g_{\alpha\beta}|| = 0$. The last fact implies that signature conditions defining pseudo-Riemannian spaces are broken, so that photon teleportation manifests outside the basic space-time of the General Theory of Relativity. Such a fully degenerate space was considered in [3, 4], where it was referred to as a *zero-space* because, from viewpoint of a regular observer, all spatial intervals and time intervals are zero there.

When $d\tau = 0$ and $d\sigma = 0$ observable relativistic mass m and the frequency ω become zero. Thus, from the viewpoint of a regular observer, all particles located in zero-space (in particular, teleporting photons) having zero rest-masses $m_0 = 0$ appear as zero relativistic masses $m = 0$ and the frequencies $\omega = 0$. Therefore particles of this kind may be assumed to be the ultimate case of massless light-like particles.

We will refer to all particles located in zero-space as *zero-particles*.

In the frames of the particle-wave concept each particle is given by its own wave world-vector $K_\alpha = \frac{\partial \psi}{\partial x^\alpha}$, where ψ is the wave phase (eikonal). The eikonal equation $K_\alpha K^\alpha = 0$ [5], setting forth that the length of the wave vector K^α remains unchanged*, for regular massless light-like particles (regular photons), becomes a travelling wave equation

$$\frac{1}{c^2}\left(\frac{^*\partial \psi}{\partial t}\right)^2 - h^{ik}\frac{^*\partial \psi}{\partial x^i}\frac{^*\partial \psi}{\partial x^k} = 0\,,$$

that may be obtained after taking $K_\alpha K^\alpha = g^{\alpha\beta}\frac{\partial \psi}{\partial x^\alpha}\frac{\partial \psi}{\partial x^\beta} = 0$ in the terms of physical observable quantities [1, 2], where we formulate regular derivatives through chronometrically invariant (physical observable) derivatives $\frac{^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$ and $\frac{^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2} v_i \frac{^*\partial}{\partial t}$ and we use $g^{00} = \frac{1}{g_{00}}\left(1 - \frac{1}{c^2} v_i v^i\right)$, $v_k = h_{ik} v^i$, $v^i = -c g^{0i}\sqrt{g_{00}}$, $g^{ik} = -h^{ik}$.

The eikonal equation in zero-space takes the form

$$h^{ik}\frac{^*\partial \psi}{\partial x^i}\frac{^*\partial \psi}{\partial x^k} = 0$$

because there $\omega = \frac{^*\partial \psi}{\partial t} = 0$, putting the equation's time term into zero. It is a standing wave equation. So, from the viewpoint of a regular observer, in the frames of the particle-wave concept, all particles located in zero-space manifest as *standing light-like waves*, so that all zero-space appears filled with a system of light-like standing waves — a light-like hologram. This implies that an experiment for discovering non-quantum teleportation of photons should be linked to stationary light.

There is no problem in photon teleportation being realised along fully degenerate world-trajectories ($g = 0$) outside the basic pseudo-Riemannian space ($g < 0$), while teleportation trajectories of substantial particles are strictly non-degenerate ($g < 0$) so the lines are located in the pseudo-Riemannian space†. It presents no problem because at any point of the pseudo-Riemannian space we can place a tangential space of $g \leqslant 0$ consisting of the regular pseudo-Riemannian space ($g < 0$) and the zero-space ($g = 0$) as two different areas of the same manifold. A space of $g \leqslant 0$ is a natural generalization of the basic space-time of the General Theory of Relativity, permitting non-quantum ways for teleportation of both photons and substantial particles (previously achieved only in quantum fashion — quantum teleportation of photons in 1998 [6] and of atoms in 2004 [7, 8]).

Until now teleportation has had an explanation given only by Quantum Mechanics [9]. Now the situation changes: with our theory we can find physical conditions for the realisation of teleportation of both photons and substantial particles in a non-quantum way.

The only difference is that from the viewpoint of a regular observer the square of any parallely transported vector remains unchanged. It is also an "observable truth" for vectors in zero-space, because the observer reasons standards of his pseudo-Riemannian space anyway. The eikonal equation in zero-space, expressed in his observable world-coordinates, is $K_\alpha K^\alpha = 0$. But in ideal world-coordinates t and x^i the metric inside zero-space, $ds^2 = -\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik}\, dx^i dx^k = 0$, degenerates into a three-dimensional $d\mu^2$ which, depending

*According to Levi-Civita's rule, in a Riemannian space of n dimensions the length of any n-dimensional vector Q^α remains unchanged in parallel transport, so $Q_\alpha Q^\alpha = \text{const}$. So it is also true for the four-dimensional wave vector K^α in a four-dimensional pseudo-Riemannian space — the basic space-time of the General Theory of Relativity. Since $ds = 0$ is true along isotropic trajectories (because $cd\tau = d\sigma$), the length of any isotropic vector is zero, so that we have $K_\alpha K^\alpha = 0$.

†Any space of Riemannian geometry has the strictly non-degenerate metric feature $g < 0$ by definition. Pseudo-Riemannian spaces are a particular case of Riemannian spaces, where the metric is sign-alternating. So the four-dimensional pseudo-Riemannian space of the signature $(+---)$ or $(-+++)$ on which Einstein based the General Theory of Relativity is also a strictly non-degenerated metric ($g < 0$).

on gravitational potential w uncompensated by something else, is not invariant, $d\mu^2 = g_{ik}\,dx^i dx^k = \left(1-\frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 \neq$ $\neq$ inv. As a result, within zero-space, the square of a transported vector, a four-dimensional coordinate velocity vector U^α for instance, being degenerated into a spatial U^i, does not remain unchanged

$$U_i U^i = g_{ik}\, U^i U^k = \left(1-\frac{\mathrm{w}}{c^2}\right)^2 c^2 \neq \text{const},$$

so that although the geometry is Riemannian for a regular observer, the real geometry of zero-space within the space itself is non-Riemannian.

We conclude from this brief study that instant displacements of particles are naturally permitted in the space-time of the General Theory of Relativity. As it was shown, teleportation of substantial particles and photons realizes itself in different space-time areas. But it would be a mistake to think that teleportation requires the acceleration of a substantial particle to super-light speeds (the tachyons area), while a photon needs to be accelerated to infinite speed. No — as it is easy to see from the teleportation condition $\mathrm{w} + v_i u^i = c^2$, if gravitational potential is essential and the space rotates at a speed close to the velocity of light, substantial particles may be teleported at regular sub-light speeds. Photons can reach the teleportation condition easier, because they move at the velocity of light. From the viewpoint of a regular observer, as soon as the teleportation condition is realised in the neighbourhood of a moving particle, such particle "disappears" although it continues its motion at a sub-light coordinate velocity u^i (or at the velocity of light) in another space-time area invisible to us. Then, having its velocity reduced, or by the breaking of the teleportation condition by something else (lowering gravitational potential or the space rotation speed), it "appears" at the same observable moment at another point of our observable space at that distance and in the direction which it obtained by u^i there.

In connection with the results, it is important to remember the "Infinity Relativity Principle", introduced by Abraham Zelmanov (1913–1987), a prominent cosmologist. Proceeding from his cosmological studies [1], he concluded that "... in homogeneous isotropic cosmological models spatial infinity of the Universe depends on our choice of that reference frame from which we observe the Universe (the observer's reference frame). If the three-dimensional space of the Universe, being observed in one reference frame, is infinite, it may be finite in another reference frame. The same is as well true for the time during which the Universe evolves."

We have come to the "Finite Relativity Principle" here. As it was shown, because of a difference between physical observable world-coordinates and ideal ones, the same space-time areas may be very different, being defined in each of the frames. Thus, in observable world-coordinates, zero-space is a point ($d\tau = 0$, $d\sigma = 0$), while $d\tau = 0$ and $d\sigma = 0$ taken in ideal world-coordinates become $-\left(1-\frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 +$ $+\, g_{ik}\,dx^i dx^k = 0$, which is a four-dimensional cone equation like the light cone. Actually here is the "Finite Relativity Principle" for observed objects — an observed point is the whole space taken in ideal coordinates.

Conclusions

This research currently is the sole explanation of virtual particles and virtual interaction given by the purely geometrical methods of Einstein's theory. It is possible that this method will establish a link between Quantum Electrodynamics and the General Theory of Relativity.

Moreover, this research is currently the sole theoretical explanation of the observed phenomenon of teleportation [6, 7, 8] given by the General Theory of Relativity.

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References

1. Zelmanov A. L. Chronometric invariants. Dissertation, 1944. First published: CERN, EXT-2004-117, 236 pages.
2. Zelmanov A. L. Chronometric invariants and co-moving coordinates in the general relativity theory. *Doklady Acad. Nauk USSR*, 1956, v. 107 (6), 815–818.
3. Rabounski D. D. and Borissova L. B. Particles here and beyond the Mirror. Editorial URSS, Moscow, 2001, 84 pages.
4. Borissova L. B. and Rabounski D. D. Fields, vacuum, and the mirror Universe. Editorial URSS, Moscow, 2001, 272 pages (the 2nd revised ed.: CERN, EXT-2003-025).
5. Landau L. D. and Lifshitz E. M. The classical theory of fields. GITTL, Moscow, 1939 (ref. with the 4th final exp. edition, Butterworth–Heinemann, 1980).
6. Boschi D., Branca S., De Martini F., Hardy L., and Popescu S. Experimental realization of teleporting an unknown pure quantum state via dual classical and Einstein-Podolsky-Rosen Channels. *Phys. Rev. Lett.*, 1998, v. 80, 1121–1125.
7. Riebe M., Häffner H., Roos C. F., Hänsel W., Benhelm J., Lancaster G. P. T., Korber T. W., Becher C., Schmidt-Kaler F., James D. F. V., and Blatt R. Deterministic quantum teleportation with atoms. *Nature*, 2004, v. 429, 734–736.
8. Barrett M. D., Chiaverini J., Schaetz T., Britton J., Itano W. M., Jost J. D., Knill E., Langer C., Leibfried D., Ozeri R., Wineland D. J. Deterministic quantum teleportation of atomic qubits. *Nature*, 2004, v. 429, 737–739.
9. Bennett C. H., Brassard G., Crepeau C., Jozsa R., Peres A., and Wootters W. K. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Phys. Rev. Lett.*, 1993, v. 70, 1895–1899.