Fermions as Topological Objects

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A preon-based composite model of the fundamental fermions is discussed, in which the fermions are bound states of smaller entities — primitive charges (preons). The preon is regarded as a dislocation in a dual 3-dimensional manifold — a topological object with no properties, save its unit mass and unit charge. It is shown that the dualism of this manifold gives rise to a hierarchy of complex structures resembling by their properties three families of the fundamental fermions. Although just a scheme for building a model of elementary particles, this description yields a quantitative explanation of many observable particle properties, including their masses. PACS numbers: 12.60.Rc, 12.15.Ff, 12.10.Dm

1 Introduction

The hierarchical pattern observed in the properties of the fundamental fermions (quarks and leptons) points to their composite nature [1], which goes beyond the scope of the Standard Model of particle physics. The particles are grouped into three generations (families), each containing two quarks and two leptons with their electric charges, spins and other properties repeating from generation to generation: the electron and its neutrino, e^- , ν_e , the muon and its neutrino, μ^-, ν_μ , the tau and its neutrino, τ^-, ν_τ , the up and down quarks, $u^{+2/3}$, $d^{-1/3}$, charm and strange, $c^{+2/3}$, $s^{-1/3}$, top and bottom, $t^{+2/3}$, $b^{-1/3}$ (here the charges of quarks are indicated by superscripts). The composite models of quarks and leptons [2] are based on fewer fundamental particles than the Standard Model (usually two or three) and are able to reproduce the above pattern as to the electric and colour charges, spins and, in some cases, the variety of species. However, the masses of the fundamental fermions are distributed in a rather odd way [3]. They cannot be predicted from any application of first principles of the Standard Model; nor has any analysis of the observed data [4] or development of new mathematical ideas [5] yielded an explanation as to why they should have strictly the observed values instead of any others. Even there exist claims of randomness of this pattern [6]. However, the history of science shows that, whenever a regular pattern was observed in the properties of matter (e.g., the periodical table of elements or eight-fold pattern of mesons and baryons), this pattern could be explained by invoking some underlying structures. In this paper we shall follow this lead by assuming that quarks and leptons are bound states of smaller particles, which are usually called "pre-quarks" or "preons" [7]. Firstly, we shall guess at the basic symmetries of space, suggesting that space, as any other physical entity, is dual. We propose that it is this property that is responsible for the emergence

of different types of interactions from a unique fundamental interaction. To be absolutely clear, we have to emphasise that our approach will be based on classical (deterministic) fields, which is opposed to the commonly-held view that quarks and leptons are quantum objects. But we shall see that by using classical fields on small scales we can avoid the problems related to the short-range energy divergences and anomalies, which is the main problem of all quantum field theories.

2 The universe

Let us begin from a few conjectures (postulates) about the basic properties of space:

- P1 Matter is structured, and the number of its structural levels is finite;
- P2 The simplest (and, at the same time, the most complex) structure in the universe is the universe itself;
- P3 The universe is self-contained (by definition);
- P4 All objects in the universe spin (including the universe itself).

The postulate P1 is based on the above mentioned historical experience with the patterns and structures behind them. These patterns are known to be simpler on lower structural levels, which suggests that matter could be structured down to the simplest possible entity with almost no properties. We shall relate this entity to the structure of the entire universe (postulate P2). This is not, of course, a novelty, since considering the universe as a simple uniform object lies in the heart of modern cosmology. The shape (topology) of this object is not derivable from Einstein's equations, but for simplicity it is usually considered as a hyper-sphere (S^3) of positive, negative or zero curvature. However, taking into account the definition of the universe as a self-contained object (postulate P3), the spherical shape becomes inappropriate, because any sphere has at least two unrelated

hyper-surfaces, which is incompatible with the definition of the uniqueness and self-containedness of the universe. More convenient would be a manifold with a unique hyper-surface, such as the Klein-bottle, K^3 [8]. Similarly to S^3 , it can be of positive, negative or zero curvature. An important feature of K^3 is the unification of its inner and outer surfaces (Fig. 1). In the case of the universe, the unification might well occur on the sub-quark level, giving rise to the structures of elementary particles and, supposedly, resulting in the identification of the global cosmological scale with the local microscopic scale of elementary particles. In Fig. 1b the unification region is marked as Π (primitive particle).



Fig. 1: (a) Klein-bottle and (b) its one-dimensional representation; the "inner" (I) and "outer" (II) hyper-surfaces are unified through the region Π (primitive particle); R and ρ are, respectively, the global and local radii of curvature.

3 The primitive particle

Let as assume that space is smooth and continuous, i.e., that its local curvature cannot exceed some finite value ε : $|\rho|^{-1} < \varepsilon$. Then, within the region Π (Fig. 1b) space will be locally curved "inside-out". In these terms, the primitive particle can be seen as a dislocation (topological defect) of the medium and, thus, cannot exist independently of this medium. Then, the postulate P4 about the spinning universe gives us an insight into the possible origin of the idea of spinning universe was proposed many years ago by A. Zelmanov [9] and K. Gödel [10]. It comes from the common fact that so far non-rotating objects have never been observed.

The universe spinning with its angular velocity ω (of course, if considered from the embedding space) would result in the linear velocity $\pm \omega R$ of the medium in the vicinity of the primitive particle, where R is the global radius of curvature of the universe; and the sign depends on the choice of the referent direction (either inflow or outflow from the inversion region).

Due to the local curvature, ρ^{-1} , in the vicinity of the primitive particle, the spinning universe must give rise to a local acceleration, a_g , of the medium moving through the region Π , which is equivalent to the acceleration of the particle itself. According to Newton's second law, this acceleration can be described in terms of a force, $F_g =$

 $= m_{\rm g} a_{\rm g}$, proportional to this acceleration. The coefficient of proportionality between the acceleration and the force corresponds to the inertial mass of the particle. However, for an observer in the coordinate frame of the primitive particle this mass will be perceived as gravitational (m_{g}) because the primitive particle is at rest in this coordinate frame. Thus, the spinning universe implies the accelerated motion of the primitive particle along its world line (timeaxis). If now the particle is forced to move along the spatial coordinates with an additional acceleration a_i , it will resist this acceleration in exactly the same way as it does when accelerating along the time-axis. A force $F_i = m_i a_i$, which is required in order to accelerate the particle, is proportional to a_i with the coefficient of proportionality m_i (inertial mass). But, actually, we can see that within our framework the inertial, $m_{\rm i}$, and gravitational, $m_{\rm g}$, masses are generated by the same mechanism of acceleration. That is, mass in this framework is a purely inertial phenomenon ($m_i \equiv m_g$).

It is seen that changing the sign of ωR does not change the sign of the second derivative $a_g = \frac{\partial^2(ict)}{\partial t^2}$, i.e., of the "gravitational" force $F_g = m_g a_g$. This is obvious, because the local curvature, ρ^{-1} , is the property of the manifold and does not depend on the direction of motion. By contrast, the first derivative $\frac{\partial(ict)}{\partial t}$ can be either positive or negative, depending on the choice of the referent direction. It would be natural here to identify the corresponding force as electrostatic. For simplicity, in this paper we shall use unit values for the mass and electric charge of the primitive particle, denoting them as m_o and q_o .

In fact, the above mass acquisition scheme has to be modified because, besides the local curvature, one must account for torsion of the manifold (corresponding to the Weyl tensor). In the three-dimensional case, torsion has three degrees of freedom, and the corresponding field can be resolved into three components (six — when both manifestations of space, I and II, are taken into account). It is reasonable to relate these three components to three polarities (colours) of the strong interaction.

Given two manifestations of space, we can resolve the field of the particle into two components, ϕ_s and ϕ_e . To avoid singularities we shall assume that infinite energies are not accessible in nature. Then, since it is an experimental fact that energy usually increases as distance decreases, we can hypothesise that the energy of both ϕ_e and ϕ_s , after reaching a maximum, decays to zero at the origin. The simplest form for the split field that incorporates the requirements above is the following:

$$F = \phi_{\rm s} + \phi_{\rm e} , \phi_{\rm s} = s \exp(-\rho^{-1}) , \quad \phi_{\rm e} = -\phi_{\rm s}'(\rho) .$$
(1)

Here the signature $s = \pm 1$ indicates the sense of the interaction (attraction or repulsion); the derivative of ϕ_s is taken with respect to the radial coordinate ρ . Far from the

 $V(\rho)$

0.8

0.6

0.40.2

0.0

 ρ/ρ_0

source, the second component of the split field F mimics the Coulomb gauge, whereas the first component extends to infinity being almost constant (similarly to the strong field).

In order to formalise the use of tripolar fields we have to introduce a set of auxiliary 3×3 singular matrices Π^i with the following elements:

$${}^{\pm}\pi^{i}_{jk} = \pm \delta^{i}_{j} (-1)^{\delta^{k}_{j}},$$
 (2)

where δ_i^i is the Kronecker delta-function; the (±)-signs correspond to the sign of the charge; and the index i stands for the colour (i = 1, 2, 3 or red, green and blue). The diverging components of the field can be represented by reciprocal elements: $\tilde{\pi}_{jk} = \pi_{jk}^{-1}$. Then we can define the (unit) charges and masses of the primitive particles by summation of these matrix elements:

$$\begin{array}{ll} q_{\Pi} = \mathbf{u}^{\mathsf{T}} \Pi \mathbf{u}, & \tilde{q}_{\Pi} = \mathbf{u}^{\mathsf{T}} \tilde{\Pi} \mathbf{u} \\ m_{\Pi} = \mid \mathbf{u}^{\mathsf{T}} \Pi \mathbf{u} \mid, & \tilde{m}_{\Pi} = \mid \mathbf{u}^{\mathsf{T}} \tilde{\Pi} \mathbf{u} \mid \end{array}$$
(3)

(**u** is the diagonal of a unit matrix; \tilde{q}_{Π} and \tilde{m}_{Π} diverge). Assuming that the strong and electric interactions are manifestations of the same entity and taking into account the known pattern [11] of the colour-interaction (two like-charged but unlike-coloured particles are attracted, otherwise they repel), we can write the signature s_{ij} of the chromoelectric interaction between two primitive particles, say of the colours iand j, as:

$$s_{ij} = -\mathbf{u}^{\mathsf{T}} \Pi^{i} \Pi^{j} \mathbf{u} \,. \tag{4}$$

Colour dipoles 4

Obviously, the simplest structures allowed by the tripolar field are the monopoles, dipoles and tripoles, unlike the conventional bipolar (electric) field, which allows only the monopoles and dipoles. Let us first consider the colour-dipole configuration. It follows from (4) that two like-charged particles with unlike-colours will combine and form a charged colour-dipole, g^{\pm} . Similarly, a neutral colour-dipole, g^{0} , can also be formed - when the constituents of the dipole have unlike-charges.

The dipoles g^{\pm} and g^{0} are classical oscillators with the double-well potential $V(\rho)$, Fig. 2, derived from the split field (1). The oscillations take place within the region $\rho \in (0, \rho_{\max})$, with the maximal distance between the components $\rho_{\rm max} \approx 1.894 \rho_{\circ}$ (assuming the initial condition $E_0 =$ = V(0) and setting this energy to zero).

Let us assume that the field $F(\rho)$ does not act instantaneously at a distance. Then, we can define the mass of a system with, say, N primitive particles as proportional to the number of these particles, wherever the field flow rate is not cancelled. For this purpose we shall regard the total field flow rate, v_N , of such a system as a superposition of the individual volume flow rates of its N constituents. Then



$$m_N = \mid N \mid v_N. \tag{5}$$

Here v_N is calculated recursively from

$$v_i = \frac{q_i + v_{i-1}}{1 + |q|_i v_{i-1}},\tag{6}$$

with i = 2, ..., N and putting $v_1 = q_1$. Then, when two unlike-charged particles combine (say red and antigreen), the magnitudes of their oppositely directed flow rates cancel each other (resulting in a neutral system). The corresponding, acceleration also vanishes, which is implicit in (5), formalising the fact that the mass of a neutral system is nullified. This formula implies the complete cancellation of masses in the systems with vanishing electric fields, but this is only an approximation because in our case the primitive particles are separated by the average distance ρ_{\circ} , whereas the complete cancellation of flows is possible only when the flow source centres coincide.

In the matrix notation, the positively charged dipole, g_{12}^+ , is represented as a sum of two matrices, Π^1 and Π^2 :

$$g_{12}^{+} = \Pi^{1} + \Pi^{2} = \begin{pmatrix} -1 & +1 & +1 \\ +1 & -1 & +1 \\ 0 & 0 & 0 \end{pmatrix},$$
(7)

with the charge $q_{g_{12}^+} = +2$ and mass $m_{g_{12}^+} \approx 2$ and $\widetilde{m}_{g_{12}^+} = \infty$, according to (3). If two components of the dipole are oppositely charged, say, $g_{12}^0 = \Pi^1 + \overline{\Pi}^2$ (of whatever colour combination), then their electric fields and masses are nullified: $q_{q^0} = 0, \, m_{q^0} \approx 0$ (but still $\widetilde{m}_{q^0} = \infty$ due to the null-elements in the matrix g^0). The infinities in the expressions for the reciprocal masses of the dipoles imply that neither g^{\pm} nor q^0 can exist in free states (because of their infinite energies). However, in a large ensemble of neutral colour-dipoles q^0 . not only electric but all the chromatic components of the field can be cancelled (statistically). Then, the mass of the neutral

dipole g_{ik}^0 with an extra charged particle Π^l belonging this ensemble but coupled to the dipole, will be derived from the unit mass of Π^l :

$$m(\Pi^{i},\Pi^{i},\Pi^{l}) = 1,$$

but still $\widetilde{m}(\Pi^{i},\overline{\Pi}^{k},\Pi^{l}) = \infty.$ (8)

The charge of this system will also be derived from the charge of the extra charged particle Π^l .

5 Colour tripoles

Three primitive particles with complementary colour-charges will tend to cohere and form a Y-shaped structure (tripole). For instance, by completing the set of colour-charges in the charged dipole [adding the blue-charged component to the system (7)] one would obtain a colour-neutral but electrically charged tripole:

$$\mathsf{Y} = \Pi^1 + \Pi^2 + \Pi^3 = \left(egin{array}{cccc} -1 & +1 & +1\ +1 & -1 & +1\ +1 & +1 & -1\ +1 & +1 & -1 \end{array}
ight),$$

which is colour-neutral at infinity but colour-polarised nearby (because the centres of its constituents do not coincide). Both m and \tilde{m} of the tripole are finite, $m_{\rm Y} = \tilde{m}_{\rm Y} = 3 \ [m_{\circ}]$, since all the diverging components of its chromofield are mutually cancelled (converted into the binding energy of the tripole).

6 Doublets of tripoles



Fig. 3: The tripoles (Y-particles) can combine pairwisely, rotated by 180° (a) or 120° (b) with respect to each other.

One can show [12] that two like-charged Y-tripoles can combine pole-to-pole with each other and form a charged doublet $\delta^+ = \mathbf{Y} \mathbf{\lambda}$ (Fig. 3a). Here the rotated symbol $\mathbf{\lambda}$ is used to indicate the rotation of the tripoles through 180° with respect to each other, which corresponds to their equilibrium position angle. The marked arm of the symbol Y indicates one of the colours, say, red, in order to visualise mutual orientations of colour-charges in the neighbouring tripoles. The charge of the doublet, $q_{\delta} = +6 [q_{\circ}]$, is derived from the charges of its two constituent tripoles; the same is applied to its mass: $m_{\delta} = \tilde{m}_{\delta} = 6 \ [m_{\circ}]$. Similarly, if two unlikecharged Y-particles are combined, they will form a neutral doublet, $\gamma = \mathbf{Y} \mathbf{\lambda}$ (Fig. 3b) with $q_{\gamma} = 0$ and $m_{\gamma} = \tilde{m}_{\gamma} = 0$. The shape of the potential well in the vicinity of the doublet allows a certain degree of freedom for its components to rotate oscillating within $\pm 120^{\circ}$ with respect to their equilibrium position angle (see [12] for details). We shall use the symbols \circlearrowright and \circlearrowright to denote the clockwise and anticlockwise rotations.

7 Triplets of tripoles

The $\frac{2}{3}\pi$ -symmetry of the tripole allows up to three of them to combine if they are like-charged. Necessarily, they will combine into a loop, denoted hereafter with the symbol *e*. It is seen that this loop can be found in one of two possible configurations corresponding to two possible directions of rotation of the neighbouring tripoles: clockwise, $e_{\bigcirc}^+ = \mathbf{Y}\mathbf{Y}\mathbf{Y}$, and anticlockwise, $e_{\bigcirc}^+ = \mathbf{Y}\mathbf{Y}\mathbf{Y}$. The vertices of the tripoles can be directed towards the centre of the structure (Fig. 4a) or outwards (Fig. 4b), but it is seen that these two orientations



Fig. 4: Three like-charged tripoles joined with their vertices directed towards (a) and outwards (b) of the centre of the structure; (c): trajectories of colour charges in this structure.

correspond to different phases of the same structure, with its colour charges spinning around its ring-closed axis. These spinning charges will generate a toroidal (ring-closed) magnetic field which will force them to move along the torus. Their circular motion will generate a secondary (poloidal) magnetic field, contributing to their spin around the ring-axis, and so forth. The corresponding trajectories of colour-charges (currents) are shown in Fig. 4c. This mechanism, known as dynamo, is responsible for generating a self-consistent magnetic field of the triplet *e*.

To a first order of approximation, we shall derive the mass of the triplet from its nine constituents, suggesting that this mass is proportional to the density of the currents, neglecting the contribution to the mass of the binding and oscillatory energies of the tripoles. That is, we put $m_e = 9 [m_o]$ (bearing in mind that the diverging components, \tilde{m}_o , are almost nullified). The charge of the triplet is also derived from the number of its constituents: $q_e = \pm 9 [q_o]$.

8 Hexaplets

Unlike-charged tripoles, combined pairwisely, can form chains with the following patterns:

$$\nu_{e\circlearrowright} = \mathbf{Y}\overline{\mathbf{Y}} + \overline{\lambda}\lambda + \mathbf{Y}\overline{\mathbf{Y}} + \overline{\lambda}\lambda + \mathbf{Y}\overline{\mathbf{Y}} + \lambda\lambda + \dots$$

$$\nu_{e\circlearrowright} = \mathbf{Y}\overline{\mathbf{Y}} + \overline{\lambda}\lambda + \mathbf{Y}\overline{\mathbf{Y}} + \overline{\lambda}\lambda + \mathbf{Y}\overline{\mathbf{Y}} + \overline{\lambda}\lambda + \dots$$
(9)

corresponding to two possible directions of rotation of the neighbouring tripoles with respect to each other. The cycle of rotations repeats after each six consecutive links, making the orientation of the sixth link compatible with (attractive to) the first link by the configuration of their colour-charges.

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This allows the closure of the chain in a loop (which we shall call hexaplet and denote as ν_e). The pattern (9) is visualised in Fig. 5a where the antipreons are coded with lighter colours. The corresponding trajectories of charges (currents) are shown in Fig. 5b. They are clockwise or anticlockwise helices, similar to those of the triplet e^- . The hexaplet consists of $n_{\nu_e} = 36$ preons (twelve tripoles); it is electrically neutral and, therefore, almost massless, according to Eq. (3).

Some properties of the simple preon-based structures are summarised in Table 1.



Fig. 5: (a) Structure of the hexaplet $\nu_e = 6Y\overline{Y}$ and (b) the corresponding helical trajectories (currents) formed by the motions of the hexaplet's colour-charges.

9 Combinations of triplets and hexaplets

The looped structures e = 3Y and $\nu_e = 6Y\overline{Y}$ can combine with each other, as well as with the simple tripole Y, because of their $\frac{2}{3}\pi$ -symmetry and residual chromaticism. That is, separated from other particles, the structure ν_e will behave like a neutral particle. But, if two such particles approach one another, they will be either attracted or repulsed from each other because of van der Waals forces caused by their residual chromaticism and polarisation. The sign of this interaction depends on the twisting directions of the particles' currents. One can show [12] that the configuration of colour charges in the hexaplet ν_e matches (is attractive to) that of the triplet *e* if both particles have like-helicities (topological charges). On the contrary, the force between the particles of the same kind is attractive for the opposite helicities $(2e_{\bigcirc}^+)$ or $e_{\bigcirc}^+e_{\bigcirc}^-)$ and repulsive for like-helicities $(2e^+_{\bigcirc \bigcirc} \text{ or } e^+_{\bigcirc}e^-_{\bigcirc})$. So, the combined effective potential of the system 2e with unlikehelicities, will have an attractive inner and repulsive outer region, allowing an equilibrium configuration of the two particles. In the case of like-helicities, both inner and outer regions of the potential are repulsive and the particles e with like-helicities will never combine. This coheres with (and probably explains) the Pauli exclusion principle, suggesting that the helicity (topological charge) of a particle can straightforwardly be related to the quantum notion of spin. This conjecture is also supported by the fact that quantum spin is measured in units of angular momentum (\hbar), and so too - the topological charge in question, which is derived from the rotational motion of the tripoles Y around the ring-closed axis of the triplet *e* or hexaplet ν_e .

Relying upon the geometrical resemblance between the tripoles Y, triplets e, and hexaplets ν_e and following the pattern replicated on different complexity levels we can deduce how these structures will combine with each other. Obviously, the hexaplet ν_e , formed of twelve tripoles, is geometrically larger than a single tripole. Thus, these two structures can combine only when the former enfolds the latter. The combined structure, which we shall denote as $Y_1 = \nu_e + Y$, will have a mass derived from its 39 constituents: $m_{Y_1} = n_{\nu_e} + m_Y = 36 + 3 = 39 \ [m_o]$. Its charge will be derived from the charge of its central tripole: $q_{Y_1} = \pm 3 \ [q_o]$. By their properties, the tripole, Y, and the "helical tripole", Y₁, are alike, except for the helicity property of the latter derived from the helicity of its constituent hexaplet.

When considering the combination of the hexaplet, ν_e , with the triplet, e, we can observe that the hexaplet must be stiffer than the triplet because of stronger bonds between the unlike-charged components of the former, while the repulsion between the like-charged components of the latter makes the bonds between them weaker. Then, the amplitude of the fluctuations of the triplet's radius will be larger than that of the hexaplet. Thus, in the combined structure, which we shall denote as $W = 6Y\overline{Y}3Y$ (or $\nu_e e$), it is the triplet that would enfold the hexaplet. The charge of this structure will correspond to the charge of its charged component, $e: q_W =$ $= \pm 9 [q_o]$; its mass can also be derived from the masses of its constituents if oscillations are dampened:

$$m_W = m_e + n_{\nu_e} = 9 + 36 = 45 \ [m_\circ]$$

Like the simple Y-tripoles, the "helical" ones, Y_1 , can form bound states with each other (doublets, strings, loops, etc.). Two hexaplets, if both enfold like-charged tripoles, will *always* have like-topological charges (helicities), which means that the force between them due to their topological charges will be repulsive (in addition to the usual repulsive force between like-charges). Thus, two like-charged helical tripoles Y_1 will never combine, unless there exists an intermediate hexaplet (ν_e) between them, with the topological charge opposite to that of the components of the pair. This would neutralise the repulsive force between these components and allow the formation of the following positively charged bound state ("helical" doublet):

$$u^+ = \mathsf{Y}_{1\circlearrowright} \nu_{e\circlearrowright} \mathsf{Y}_{1\circlearrowright} \quad \text{or} \quad \mathsf{Y}_1 \circlearrowright \mathsf{Y}_1 \ .$$
 (10)

For brevity we have denoted the intermediate hexaplet with the symbol $\check{0}$, implying that it creates a bond force between the otherwise repulsive components on its sides. By its properties, the helical doublet can be identified with the *u*-quark. Its net charge, $q_u = +6 [q_o]$, is derived from the charges of its two charged components (Y₁-tripoles). Its mass is also derived from the number of particles that constitute these charged components: $m_u = 2 \times 39 = 78 [m_o]$.

Structure	Constituents of the structure	Number of colour charges in the structure	Charge (q_{\circ} units)	Mass (m_\circ -units)					
The primitive particle (preon Π)									
П	1П	1	+1	1					
First-order structures (combinations of preons)									
ρ	2П	2	+2	2					
g^0	$1\overline{\Pi} + 1\Pi$	2	-1 + 1 = 0	~ 0					
Y	3П	3	+3	3					
Second-order structures (combinations of tripoles Y)									
δ	2Y	6	+6	6					
γ	$1\overline{\mathbf{Y}} + 1\mathbf{Y}$	6	-3 + 3 = 0	~ 0					
<i>e</i> ⁻	3 Y	9	-9	9					
Third-order structures									
$2e^{-}$	$3\overline{Y} + 3\overline{Y}$	9 + 9 = 18	-18	18					
e^-e^+	$3\overline{Y} + 3Y$	9 + 9 = 18	-9 + 9 = 0	$\sim 16^{\dagger}$					
$ u_e$	6ŸY	$6 \times (3+3) = 36$	$6 \times (-3 + 3) = 0$	7.9×10^{-8} [†]					
Y_1	$ u_e + Y$	36 + 3 = 39	0 - 3 = -3	36 + 3 = 39					
W^-	$ u_e ~+~ e^-$	36 + 9 = 45	0 - 9 = -9	36 + 9 = 45					
u	$Y_1 \ Q \ Y_1$	39 + 36 + 39 = 114	+3 + 0 + 3 = +6	39 + 39 = 78					
$ u_{\mu}$	$Y_1 \stackrel{\cdot}{\cdot} \overline{Y}_1$	39 + 36 + 39 = 114	-3 + 0 + 3 = 0	1.4×10^{-7} [†]					
d	$u + W^-$	114 + 45 = 159	+6 - 9 = -3	78 + 45 = 123					
μ	$ u_{\mu} + W^{-}$	114 + 45 = 159	0 - 9 = -9	$\overline{48+39}^{\ddagger}$					
and so on									

Table	1:	Simple	preon-based	structures

[†]quantities estimated in [13]

[‡]system with two oscillating components (see further)

The positively charged *u*-quark can combine with the negatively charged structure $W^- = \overline{\nu}_e e^-$ (of 45-units mass), forming the *d*-quark:

$$d^- = u^+ + \overline{\nu}_e e^- \tag{11}$$

of a 123-units mass ($m_d = m_u + m_W = 78 + 45$). The charge of this structure will correspond to the charge of a single triplet: $q_d = q_u + q_e = +6 - 9 = -3 [q_o]$ (see Fig. 6 that below).



Fig. 6: Scheme of the *d*-quark. The symbol \diamond is used for the triplet (*e*), the symbols \langle and \rangle denote the tripoles (Y-particles), and the symbols \bigcap denote the hexaplets (ν_e).

10 The second and third generations of the fundamental fermions

When two unlike-charged helical tripoles combine, their polarisation modes and helicity signs will *always* be opposite (simply because their central tripoles have opposite charges). This would cause an attractive force between these two particles, in addition to the usual attractive force corresponding to the opposite electric charges of Y_1 and \overline{Y}_1 . Since all the forces here are attractive, the components of this system will coalesce and then disintegrate into neutral doublets γ . However, this coalescence can be prevented by an additional hexaplet ν_e with oscillating polarisation, which would create a repulsive stabilising force (barrier) between the combining particles:

$$\nu_{\mu} = \mathsf{Y}_{1\circlearrowright} \nu_{e\circlearrowright\circlearrowright} \mathsf{Y}_{1\circlearrowright}. \tag{12}$$

It is natural to identify this structure with the muonneutrino – a neutral lepton belonging to the second family of the fundamental fermions. The intermediate hexaplet oscillates between the tripoles $Y_{1\odot}$ and $\overline{Y}_{1\odot}$, changing synchronously its polarisation state: $\nu_{e\odot} \leftrightarrow \nu_{e\odot}$. For brevity, we shall use vertical dots separating the components of ν_{μ} to

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denote this barrier-hexaplet:

$$\nu_{\mu} = \mathsf{Y}_1 \vdots \,\overline{\mathsf{Y}}_1 \,. \tag{13}$$

By analogy, we can derive the tau-neutrino structure:

$$\nu_{\tau} = \mathbf{Y}_1 \vdots \, \overline{\mathbf{Y}}_1 \vdots \, \mathbf{Y}_1 \vdots \, \overline{\mathbf{Y}}_1 \,, \tag{14}$$

as well as the structures of the muon (Fig. 7):

$$\mu^- = \nu_\mu \overline{\nu}_e e^- \tag{15}$$

and tau-lepton (Fig. 8):

$$\tau^- = \nu_\tau \overline{\nu}_\mu \mu^-. \tag{16}$$

Drawing also an analogy with molecular equilibrium configurations, where the rigidness of a system depends on the number of local minima of its combined effective potential [14], we can consider the second and third generation fermions as non-rigid structures with oscillating components (clusters) rather than stiff entities with dampened oscillations. In Fig. 7 and Fig. 8 we mark the supposedly clustered components of the μ - and τ -leptons with braces. Obtaining the ground-state energies (masses) of these complex structures is not a straightforward task because they may have a great variety of oscillatory modes contributing to the mass. However, in principle, these masses are computable, as can be shown by using the following empirical formula:

$$m_{\text{clust}} = \overline{m_1 + m_2 + \dots + m_N} = m\widetilde{m},$$
 (17)

where N is the number of oscillating clusters, each with the mass m_i (i = 1, ..., N); m is the sum of these masses:

$$m=m_1+m_2+\cdots+m_N,$$

and \tilde{m} is the reduced mass based on the components (3):

$$\widetilde{m}^{-1} = \widetilde{m}_1^{-1} + \widetilde{m}_2^{-1} + \dots + \widetilde{m}_N^{-1}.$$

For simplicity, we assume that unit conversion coefficients in this formula are set to unity. Each substructure here contains a well-defined number of constituents (preons) corresponding to the configuration with the lowest energy. Therefore, the number of these constituents is fixed by the basic symmetry of the potential, implying that the input quantities in (17) are not free parameters. The fermion masses computed with the use of this formula are summarised in Table 2.

As an example, let us compute the muon's mass. The masses of the muon's substructures, according to Fig. 7, are: $m_1 = \tilde{m}_1 = 48$, $m_2 = \tilde{m}_2 = 39$ (in units of m_\circ). And the muon's mass will be: $m_\mu = \overline{48+39} = \frac{48+39}{1/48+1/39} = 1872[m_\circ]$. For the τ -lepton, the constituent masses are $m_1 = \tilde{m}_1 = 201$, $m_2 = \tilde{m}_2 = 156$ (Fig. 8), and its mass is $m_\tau = \overline{201+156} = 160$





Fig. 8: Scheme of the tau-lepton.

= 31356 $[m_{\circ}]$. For the proton, the positively charged fermion consisting of two up $(N_u = 2)$, one down $(N_d = 1)$ quarks and submerged into a cloud of gluons g^0 , the masses of its components are $m_u = \tilde{m}_u = 78$, $m_d = \tilde{m}_d = 123$. The total number of primitive charges comprising the proton's structure is $N_p = 2m_u + m_d = 2 \times 78 + 123 = 279$, which would correspond to the number of gluons (N_g) interacting with each of these charges $(N_g = N_p = 279)$. The masses of these gluons, according to (8), are $m_{g^0} = 1$, $\tilde{m}_{g^0} = \infty$, and the resulting proton mass is

$$m_p = \overline{N_u m_u + N_d m_d + N_g m_g} = 16523 \ [m_\circ], \quad (18)$$

which also reproduces the well-known but not yet explained proton-to-electron mass ratio, since $\frac{m_p}{m_e} = \frac{16523}{9} \approx 1836$.

With the value (18) one can convert m_e , m_{μ} , m_{τ} , and the masses of all other particles from units m_{\circ} into proton mass units, m_p , thus enabling these masses to be compared with the experimental data. The computed fermion masses are listed in Table 2 where the symbols Y_1 , Y_2 and Y_3 denote complex "helical" tripoles that replicate the properties of the simple tripole Y on higher levels of the hierarchy. These helical tripoles can be regarded as the combinations of "heavy neutrinos" with simple triplets. Like ν_e , the heavy neutrino consists of six pairs of helical triplets: $\nu_h = 6Y_1\overline{Y}_1$. They can further combine and form "ultra-heavy" neutrinos $\nu_{uh} = 3(\overline{Y}_1\nu_h u)e^-$ and so on. The components Y_2 and Y_3 of the *c* and *t* quarks have the following structures: $Y_2 =$ $= u\nu_e u\nu_e e^-$, consisting of 165 primitive particles, and $Y_3 =$ $= \nu_{uh}Y$, consisting of 1767 primitive particles.

Particle and its structure (components)		Number of charges in the non- cancelled mass components	Computed masses in units of $[m_p]$	Masses converted into m_p	Experimental masses [3] in units of $[m_p]$			
First family								
ν _e	6YY	≈ 0	7.864×10^{-8}	4.759×10^{-12}	< 3×10 ⁻⁹			
e ⁻	3 Y	9	9	0.0005447	0.0005446170232			
u	$Y_1 \ Q \ Y_1$	78	78	0.004720	0.0021 to 0.0058			
d	$u + \overline{\nu}_e e^-$	123	123	0.007443	0.0058 to 0.0115			
Second family								
$ u_{\mu} $	$Y_1 \stackrel{.}{\cdot} \overline{Y}_1$	≈ 0	1.4×10^{-7}	8.5×10^{-12}	$< 2 \times 10^{-4}$			
μ^{-}	$ u_{\mu}$ + $\overline{ u}_{e}e^{-}$	$\overline{48+39}$	1872	0.1133	0.1126095173			
с	$Y_2 \ \ Y_2$	$\overline{165 + 165}$	27225	1.6477	1.57 to 1.95			
s	$c + e^-$	$\overline{165 + 165 + 9}$	2751	0.1665	0.11 to 0.19			
Third family								
$\nu_{ au}$	$Y_1 \stackrel{\cdot}{\vdots} \overline{Y}_1 \stackrel{\cdot}{\vdots} Y_1 \stackrel{\cdot}{\vdots} \overline{Y}_1$	≈ 0	1.5896×10^{-7}	9.6192×10 ⁻¹²	$< 2 \times 10^{-2}$			
$ \tau^-$	$ u_{ au} + \overline{ u}_{\mu}\mu^{-}$	$\overline{156 + 201}$	31356	1.8977	1.8939 ± 0.0003			
t	Y ₃ ≬ Y ₃	$\overline{1767 + 1767}$	3122289	188.94	189.7 ± 4.5			
b	t + μ^-	$\overline{1767 + 1767 + 48 + 39}$	76061.5	4.603	4.3 to 4.7			

Table 2: Computed masses of quarks and leptons. The values in the 4th column taken in units of m_{\circ} are converted into proton mass units (5th column) m_p =16523, Eq.(18). The overlined ones are shorthands for Eq. (17). The masses of ν_e , ν_{μ} and ν_{τ} are estimated in [13].

11 Conclusions

The results presented in Table 2 show that our model agree with experiment to an accuracy better then 0.5%. The discrepancies should be attributed to the simplifications we have assumed here (e.g., neglecting the binding and oscillatory energies, as well as the neutrino residual masses, which contribute to the masses of many structures in our model).

By matching the pattern of properties of the fundamental particles our results confirm that our conjecture about the dualism of space and the symmetry of the basic field corresponds, by a grand degree of confidence, to the actual situation. Thus, our model seems to unravel a new layer of physical reality, which bears the causal mechanisms underlying quantum phenomena. This sets a foundation from which one can explain many otherwise inexplicable observational facts that plague modern physics.

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