Notes on Pioneer Anomaly Explanation by Sattellite-Shift Formula of Quaternion Relativity: Remarks on "Less Mundane Explanation of Pioneer Anomaly from Q-Relativity"

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Use of satellite shift formula emerging in Quaternion (Q-) model of relativity theory for explanation of Pioneer anomaly [1] is critically discussed. A cinematic scheme more suitable for the case is constructed with the help of Q-model methods. An appropriate formula for apparent deceleration resulting from existence of observer-object relative velocity is derived. Preliminary quantitative assessments made on the base of Pioneer 10/11 data demonstrate closure of the assumed "relativistic deceleration" and observed "Doppler deceleration" values.

1 Introduction. Limits of satellite-shift formula

Recently [1] there was an attempt to give an explanation of Pioneer anomaly essentially using formula for relativistic shift of planet's fast satellites observed from the Earth. This formula was derived within framework of Q-method developed to calculate relativistic effects using $SO(1,2)$ form-invariant quaternion square root from space-time interval rather than the interval itself [2]; in particular this advantageously permits to describe relativistic motions of any non-inertial frames. The last option was used to find mentioned formula that describes cinematic situation comprising three Solar System objects: the Earth (with observer on it), a planet, and its satellite revolving with comparatively large angular velocity. Due to existence of Earth-planet relative velocity, not great though and variable but permanent, the cycle frequency of satellite rotation (observed from the Earth) is apparently less that in realty, i.e. the "planet's clock" is slowing down, and calculation shows that the gap is growing linearly with time. Visually it looks that the satellite position on its orbit is apparently behind an expected place. For very fast satellites (like Jupiter's Metis and Adrastea) and for sufficiently long period of time the effect can probably be experimentally detected. Same effect exists of course for Mars's satellites and it is computed that monthly apparent shift on its orbit of e.g. Phobos is about 50 meters (that is by the way can be important and taken into account when planning expedition of spacecraft closely approaching the moon).

In paper of F. Smarandache and V. Christianto [1] the discussed formula was used to describe famous Pioneer effect, implying that the last great acceleration the space probe received when approached very close to Jupiter; in particular data concerning Adrastea, whose location was as close to Jupiter as the space probe, were cited in [1]. Combined with ether drift effect the formula gives good coincidence (up to 0.26%) with value of emission angle shift required to explain observation data of Pioneer's signal Doppler residuals [3].

This surprisingly exact result nevertheless should not lead to understanding that obtained by Q-method mathematical description of a specific mechanical model can bear universal character and fit to arbitrary relativistic situation. One needs to recognize that Pioneer cinematic scheme essentially differs from that of the Earth-planet-satellite model; but if one tries to explain the Pioneer effect using the same relativistic idea as for satellite shift then an adequate cinematic scheme should be elaborated. Happily the Q-method readily offers compact and clear algorithm for construction and description of any relativistic models. In Section 2 a model referring observed frequency shift of Pioneer spacecraft signals to purely relativistic reasons is regarded; some quantitative assessments are made as well as conclusions on ability of the model to explain the anomaly. In Section 3 a short discussion is offered.

2 Earth-Pioneer Q-model and signal frequency shift

Paper [3] enumerates a number of factors attracted to analyze radio data received from Pioneer 10/11 spacecraft, among them gravitational planetary perturbations, radiation pressure, interplanetary media, General Relativity*, the Earth's precession and nutation. It is worth noting here that one significant factor, time delay caused by relative probe-observer motion, is not distinguished in [3]. The fact is understandable: relative motion of spacecraft and observer on the Earth is utterly non-inertial one; Special Relativity is not at all able to cope with the case while General Relativity methods involving specific metric and geodesic lines construction

*Unfortunately paper [3] does not indicate to what depth General Relativity is taken into account: whether only Newtonian gravity is modified by Schwarzschild, Kerr (or other) metrics, or cinematic effects are regarded too.

(with all curvature tensor components zero) or additional vector transport postulates are mathematically difficult. Contrary to this the Q-relativity method easily allows building of any non-inertial relativistic scheme; an example describing a spacecraft (probe) and an Earth's observer is given below.

Assume that Pioneer anomaly is a purely relativistic effect caused by existence of Earth-Pioneer relative velocity, variable but permanent. Construct respective model using the Q-method algorithm. Choose Q-frames. Let $\Sigma = (\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3)$ be the Earth's frame whose Cartesian directing vectors are given by quaternion "imaginary" units $\mathbf{q}_k$ obeying the multiplication rule*

$$1\,\mathbf{q}_k = \mathbf{q}_k\,1 = \mathbf{q}_k\,, \qquad \mathbf{q}_k\,\mathbf{q}_l = -\delta_{kl} + \varepsilon_{klj}\,\mathbf{q}_j\,. \quad (1)$$

Let Q-frame $\Sigma' = \{\mathbf{q}_{k'}\}$ belong to a probe. Suppose for simplicity that vectors $\mathbf{q}_2$, $\mathbf{q}_3$ are in the ecliptic plane as well as (approximately) the probe's trajectory. Assume that vector $\mathbf{q}_2$ of Σ is always parallel to Earth-probe relative velocity V. Now one is able to write rotational equation, main relation of Q-relativity, which ties two frames

$$\Sigma' = O_1^{-i\psi}\,\Sigma\,, \quad (2)$$

here $O_1^{-i\psi}$ is 3×3 orthogonal matrix of rotation about axis No. 1 at imaginary angle $-i\psi$

$$O_1^{-i\psi} = \begin{pmatrix} \cos(i\psi) & -\sin(i\psi) & 0 \\ \sin(-i\psi) & \cos(i\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cosh\psi & -i\sinh\psi & 0 \\ i\sinh\psi & \cosh\psi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

thus "converting" frame Σ into Σ'. The first row in the matrix equation (2)

$$\mathbf{q}_{1'} = \mathbf{q}_1 \cosh\psi - \mathbf{q}_2\, i \sinh\psi$$

after straightforward algebra

$$\mathbf{q}_{1'} = \cosh\psi\,(\mathbf{q}_1 - \mathbf{q}_2\, i \tanh\psi) \;\Rightarrow\; \mathbf{q}_{1'} = \frac{dt}{dt'}(\mathbf{q}_1 - \mathbf{q}_2\, i V\psi)$$

with usual relativistic relations

$$V = \tanh\psi, \qquad dt = dt' \cosh\psi \quad (3)$$

acquires the form of basic cinematic space-time object of Q-relativity

$$i dt'\,\mathbf{q}_{1'} = i dt\,\mathbf{q}_1 + dr\,\mathbf{q}_2\,,$$

a specific quaternion square root from space-time interval of Special Relativity

$$(i dt'\mathbf{q}_{1'})(i dt'\mathbf{q}_{1'}) = (i dt\,\mathbf{q}_1 + dr\,\mathbf{q}_2)(i dt\,\mathbf{q}_1 + dr\,\mathbf{q}_2) \;\Rightarrow\; \Rightarrow\; dt'^2 = dt^2 - dr^2,$$

dt' being proper time segment of the probe. Eq. (3) yields ratio for probe-Earth signal period (small compared to time of observation) $T = T' \cosh\psi$, i.e. observed from Earth the period is apparently longer than it really is. Vice versa, observed frequency $f = 1/T$ is smaller than the real one f'

$$f = \frac{1}{T} = \frac{1}{T\cosh\psi} = \frac{f'}{\cosh\psi} = f'\sqrt{1-(V/c)^2}, \quad (4)$$

or for small relative velocity

$$f \cong f'\left(1 - \frac{V^2}{2c^2}\right).$$

This means that there exists certain purely apparent relativistic shift of the probe's signal detected by the Earth observer

$$\Delta f = f' - f = f'\frac{V^2}{2c^2}, \quad \text{or} \quad \frac{\Delta f}{f'} = \frac{V^2}{2c^2} = \frac{\varepsilon}{c^2}, \quad (5)$$

ε being the probe's kinetic energy per unit mass computed in a chosen frame. Contrary to pure Doppler effect the shift given by Eq. (5) does not depend on the direction of relative velocity of involved objects since in fact it is just another manifestation of relativistic delay of time. Light coming to observer from any relatively (and arbitrary) moving body is universally "more red" than originally emitted signal; as well all other frequencies attributed to observed moving bodies are smaller then original ones, and namely this idea was explored for derivation of satellite shift formula.

Experimental observation of the frequency change (5) must lead to conclusion that there exists respective "Doppler velocity" V_D entering formula well known from Special Relativity

$$f = \frac{f'}{\sqrt{1-(V_D/c)^2}}\left(1 - \frac{V_D}{c}\cos\beta\right), \quad (6)$$

β being angle between velocity vector and wave vector of emitted signal. If $\beta = 0$ and smaller relativistic correction are neglected then Eq. (6) can be rewritten in the form similar to Eq. (5)

$$\frac{\Delta f}{f'} \cong \frac{V_D}{c^2}\,; \quad (7)$$

comparison of Eqs. (7) and (5) yields very simple formula for calculated (and allegedly existent) "Doppler velocity" corresponding to observed relativistic frequency change

$$V_D \cong \frac{\varepsilon}{c}\,. \quad (8)$$

Estimation of the value of V_D can be done using picture of Pioneer 10/11 trajectories (Fig.1) projected upon ecliptic plane (provided in NASA report [4]); other spacecraft traces are also shown, the Earth's orbit radius too small to be indicated.

Schematically the cinematic situation for Pioneer 10 is shown at Fig. 2 where the trajectory looks as a straight line inclined at constant angle λ to axis $\mathbf{q}_2$, while the Earth's position on its orbit is determined by angle $\alpha = \Omega t$, $\Omega = 3.98\times10^{-7}\,s^{-1}$ being the Earth's orbital angular velocity. Vectors of the probe's and Earth's velocities in Solar Ecliptic

*Latin indices are 3-dimensional (3D), δ_{kl} is 3D Kroneker symbol, ε_{jkl} is 3D Levi-Civita symbol; summation convention is assumed.

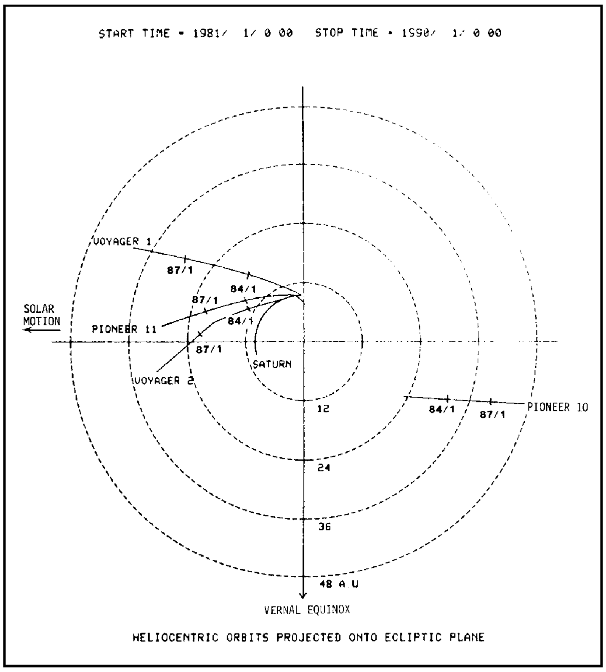

Fig. 1: Spacecraft trajectories on the ecliptic plane. (After NASA original data [4]. Used by permission.)

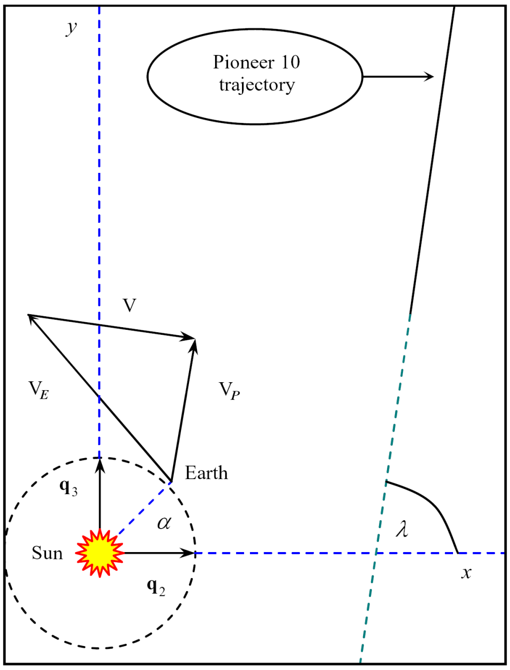

Fig. 2: Earth-Pioneer 10 cinematic scheme, where the trajectory looks as a straight line inclined at constant angle λ to axis $\mathbf{q}_2$.

(SE) coordinate system* are respectively denoted as $\mathbf{V}_P$ and $\mathbf{V}_E$; their vector subtraction gives relative Earth-probe velocity $\mathbf{V} = \mathbf{V}_P - \mathbf{V}_E$ so that

$$V_D(t) = \frac{V^2}{2c} = \frac{V_P^2 + V_E^2 - 2V_P V_E \cos(\Omega t - \lambda)}{2c}, \qquad (9)$$

and respective "Doppler acceleration" is

$$a_D = \dot{V}_D(t) = \\ = \frac{V_P \dot{V}_P - \dot{V}_P V_E \cos(\Omega t - \lambda) + \Omega V_P V_E \sin(\Omega t - \lambda)}{c}. \qquad (10)$$

In Eq. (10) the first term in the numerator claims existence of secular deceleration, since escaping from the Sun's and Jupiter's gravity the probe is permanently decelerated, $\dot{V}_p < 0$; the result is that the frequency gap shrinks giving rise to pure relativistic blue shift. Other sign-changing terms in right-hand-side of Eq. (10) are periodic (annual) ones; they may cause blue shift as well as red shift. Thus Eq. (10) shows that, although relative probe-Earth velocity incorporates into difference between real and observed frequency, nevertheless secular change of the difference is to be related only to relative probe-Sun velocity. Distinguish this term temporary ignoring the annual modulations; then the secular deceleration formula is reduced as

$$a_{SD} \cong \frac{\dot{V}_P V_P}{c}. \qquad (11)$$

*The SE is a heliocentric coordinate system with the z-axis normal to and northward from the ecliptic plane. The x-axis extends toward the first point of Aries (Vernal Equinox, i.e. to the Sun from Earth in the first day of Spring). The y-axis completes the right handed set.

Below only radial components of the probe's velocity and acceleration in Newtonian gravity are taken into account in Eq. (11); it is quite a rough assessment but it allows to conceive order of values. The probe's acceleration caused by the Sun's Newtonian gravity is

$$\dot{V}_P = -\frac{GM_\odot}{R^2}, \qquad (12)$$

$G = 6.67 \times 10^{-11}$ m^3/kg$\times$s^2, $M_\odot = 1.99 \times 10^{30}$ kg are respectively gravitational constant and mass of the Sun. NASA data [5] show that in the very middle part (1983–1990) of the whole observational period of Pioneer 10 its radial distance from the Sun changes from $R \cong 28.8$ AU $= 4.31 \times 10^{12}$ m to $R \cong 48.1$ AU $= 7.2 \times 10^{12}$ m, while year-mean radial velocity varies from $V_P = 15.18 \times 10^3$ m/s to $V_P = 12.81 \times 10^3$ m/s. Respective values of the secular "relativistic deceleration" values for this period computed with the help of Eqs. (11), (12) vary from $a_{SD} = -3.63 \times 10^{-10}$ m/s^2 to $a_{SD} = -1.23 \times 10^{-10}$ m/s^2. It is interesting (and surprising as well) that these results are very close in order to anomalous "Doppler deceleration" of the probe $a_P = -(8 \pm 3) \times 10^{-10}$ m/s^2 cited in [3].

Analogous computations for Pioneer 11, as checking point, show the following. Full time of observation of Pioneer 11 is shorter so observational period is taken from 1984 to 1989, with observational data from the same source [5]. Radial distances for beginning and end of the period are $R \cong 15.1$ AU $= 2.26 \times 10^{12}$ m, $R \cong 25.2$ AU $= 3.77 \times 10^{12}$ m; respective year-mean radial velocities are $V_P = 11.86 \times 10^3$ m/s, $V_P = 12.80 \times 10^3$ m/s. Computed "relativistic deceleration" values for this period are then $a_{SD} = -10.03 \times 10^{-10}$ m/s^2,

$a_{SD} = -5.02 \times 10^{-10}$ m/s^2: this is even in much better correlation (within limits of the cited error) with experimental value of a_P.

3 Discussion

Quantitative estimations presented above allow to conclude: additional blue shift, experimentally registered in Pioneer 10 and 11 signals, and interpreted as Sun-directed acceleration of the spacecraft to some extent, support the assumption of pure relativistic nature of the anomaly. Of course one notes that while Pioneer 11 case shows good coincidence of observed and calculated values of deceleration, values of a_{SD} for Pioneer 10 constitute only (45–15)% of observed Doppler residual; moreover generally in this approach "relativistic deceleration" is a steadily decreasing function, while experimentally (though not directly) detected deceleration a_P is claimed nearly constant. These defects could find explanation first of all in the fact that a primitive "Newtonian radial model" was used for assessments. Preliminary but more attentive reference to NASA data allows noticing that observed angular acceleration of the probes too could significantly incorporate to values of "relativistic deceleration". This problem remains to be regarded elsewhere together with analysis of the angular acceleration itself.

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