A New Conformal Theory of Semi-Classical Quantum General Relativity

Indranu Suhendro

Department of Physics, Karlstad University, Karlstad 651 88, Sweden

E-mail: spherical_symmetry@yahoo.com

We consider a new four-dimensional formulation of semi-classical quantum general relativity in which the classical space-time manifold, whose intrinsic geometric properties give rise to the effects of gravitation, is allowed to evolve microscopically by means of a conformal function which is assumed to depend on some quantum mechanical wave function. As a result, the theory presented here produces a unified field theory of gravitation and (microscopic) electromagnetism in a somewhat simple, effective manner. In the process, it is seen that electromagnetism is actually an emergent quantum field originating in some kind of stochastic smooth extension (evolution) of the gravitational field in the general theory of relativity.

1 Introduction

We shall show that the introduction of an external parameter, the Planck displacement vector field, that deforms ("maps") the standard general relativistic space-time $\mathbb{S}_1$ into an evolved space-time $\mathbb{S}_2$ yields a theory of general relativity whose space-time structure obeys the semi-classical quantum mechanical law of evolution. In addition, an "already quantized" electromagnetic field arises from our schematic evolution process and automatically appears as an intrinsic geometric object in the space-time $\mathbb{S}_2$. In the process of evolution, it is seen that from the point of view of the classical space-time $\mathbb{S}_1$ alone, an external deformation takes place, since, by definition, the Planck constant does not belong to its structure. In other words, relative to $\mathbb{S}_1$, the Planck constant is an external parameter. However from the global point of view of the universal (enveloping) evolution space $\mathbb{M}_4$, the Planck constant is intrinsic to itself and therefore defines the dynamical evolution of $\mathbb{S}_1$ into $\mathbb{S}_2$. In this sense, a point in $\mathbb{M}_4$ is not strictly single-valued. Rather, a point in $\mathbb{M}_4$ has a "dimension" depending on the Planck length. Therefore, it belongs to both the space-time $\mathbb{S}_1$ and the space-time $\mathbb{S}_2$.

2 Construction of a four-dimensional metric-compatible evolution manifold $\mathbb{M}_4$

We first consider the notion of a four-dimensional, universal enveloping manifold $\mathbb{M}_4$ with coordinates x^μ endowed with a *microscopic* deformation structure represented by an exterior vector field $\phi\,(x^\mu)$ which maps the enveloped space-time manifold $\mathbb{S}_1 \in \mathbb{M}_4$ at a certain initial point P_0 onto a new enveloped space-time manifold $\mathbb{S}_2 \in \mathbb{M}_4$ at a certain point P_1 through the diffeomorphism

$$x^\mu(P_1) = x^\mu(P_0) + l\,\xi^\mu,$$

where $l = \sqrt{\frac{G\hbar}{c^3}} \approx 10^{-33}$ cm is the Planck length expressed in terms of the Newtonian gravitational constant G, the Dirac-Planck constant $\hbar$, and the speed of light in vacuum c, in such a way that

$$\phi^\mu = l\,\xi^\mu$$
$$\lim_{\hbar\to 0}\phi^\mu = 0\,.$$

From its diffeomorphic structure, we therefore see that $\mathbb{M}_4$ is a kind of *strain space*. In general, the space-time $\mathbb{S}_2$ evolves from the space-time $\mathbb{S}_1$ through the non-linear mapping

$$P(\phi) : \mathbb{S}_1 \to \mathbb{S}_2\,.$$

Note that the exterior vector field ϕ can be expressed as $\phi = \phi^\mu h_\mu = \bar{\phi}^\mu g_\mu$ (the Einstein summation convention is employed throughout this work) where h_μ and g_μ are the sets of basis vectors of the space-times $\mathbb{S}_1$ and $\mathbb{S}_2$, respectively (likewise for ξ). We remark that $\mathbb{S}_1$ and $\mathbb{S}_2$ are both endowed with metricity through their immersion in $\mathbb{M}_4$, which we shall now call the *evolution manifold*. Then, the two sets of basis vectors are related by

$$g_\mu = \left(\delta^\nu_\mu + l\,\nabla_\mu\,\xi^\nu\right) h_\nu$$

or, alternatively, by

$$g_\mu = h_\mu + l\left(\bar{\nabla}_\mu\,\bar{\xi}^\nu\right) g_\nu$$

where δ^ν_μ are the components of the Kronecker delta.

At this point, we have defined the two covariant derivatives with respect to the connections ω of $\mathbb{S}_1$ and Γ of $\mathbb{S}_2$ as follows:

$$\nabla_\lambda A^{\alpha\beta\ldots}_{\mu\nu\ldots} = \partial_\lambda A^{\alpha\beta\ldots}_{\mu\nu\ldots} + \omega^\alpha_{\sigma\lambda} A^{\sigma\beta\ldots}_{\mu\nu\ldots} + \omega^\beta_{\sigma\lambda} A^{\alpha\sigma\ldots}_{\mu\nu\ldots} + \ldots$$
$$- \omega^\sigma_{\mu\lambda} A^{\alpha\beta\ldots}_{\sigma\nu\ldots} - \omega^\sigma_{\nu\lambda} A^{\alpha\beta\ldots}_{\mu\sigma\ldots} - \ldots$$

and

$$\bar{\nabla}_\lambda B^{\alpha\beta\ldots}_{\mu\nu\ldots} = \partial_\lambda B^{\alpha\beta\ldots}_{\mu\nu\ldots} + \Gamma^\alpha_{\sigma\lambda} B^{\sigma\beta\ldots}_{\mu\nu\ldots} + \Gamma^\beta_{\sigma\lambda} B^{\alpha\sigma}_{\mu\nu\ldots} + \ldots$$
$$- \Gamma^\sigma_{\mu\lambda} B^{\alpha\beta\ldots}_{\sigma\nu\ldots} - \Gamma^\sigma_{\nu\lambda} B^{\alpha\beta\ldots}_{\mu\sigma\ldots} - \ldots$$

for arbitrary tensor fields A and B, respectively. Here $\partial_\mu = \partial/\partial x^\mu$, as usual. The two covariant derivatives above are equal only in the limit $\hbar \to 0$.

Furthermore, we assume that the connections ω and Γ are generally asymmetric, and can be decomposed into their symmetric and anti-symmetric parts, respectively, as

$$\omega^{\lambda}_{\mu\nu} = \left(h^{\lambda}, \partial_{\upsilon} h_{\mu}\right) = \omega^{\lambda}_{(\mu\nu)} + \omega^{\lambda}_{[\mu\nu]}$$

and

$$\Gamma^{\lambda}_{\mu\nu} = \left(g^{\lambda}, \partial_{\upsilon} g_{\mu}\right) = \Gamma^{\lambda}_{(\mu\nu)} + \Gamma^{\lambda}_{[\mu\nu]}.$$

Here, by (a, b) we shall mean the inner product between the arbitrary vector fields a and b.

Furthermore, by direct calculation we obtain the relation

$$\partial_{\nu} g_{\mu} = \left(\omega^{\lambda}_{\mu\nu} + l\left(\nabla^{\sigma}_{\mu\xi}\right)\omega^{\lambda}_{\sigma\nu} + l\,\partial_{\nu}\left(\nabla_{\mu}\xi^{\lambda}\right)\right) h_{\lambda}\,.$$

Hence, setting

$$F^{\lambda}_{\mu\nu} = \omega^{\lambda}_{\mu\nu} + l\left(\left(\nabla_{\mu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\nu} + \partial_{\nu}\left(\nabla_{\mu}\xi^{\lambda}\right)\right) =$$
$$= \omega^{\lambda}_{\mu\nu} + l\left(\left(\nabla_{\mu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\nu} + \partial_{\nu}\partial^{\lambda}_{\mu\xi} + \xi^{\sigma}\partial_{\nu}\omega^{\lambda}_{\sigma\mu} + \left(\partial_{\nu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\mu}\right)$$

we may simply write

$$\partial_{\nu} g_{\mu} = F^{\lambda}_{\mu\nu} h_{\lambda}\,.$$

Meanwhile, we also have the following inverse relation:

$$h_{\mu} = \left(\delta^{\nu}_{\mu} - l\,\bar{\nabla}_{\mu}\bar{\xi}^{\nu}\right) g_{\nu}\,.$$

Hence we obtain

$$\partial_{\nu} g_{\mu} = \left(\omega^{\lambda}_{\mu\nu} + l\left(\nabla_{\mu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\nu} + l\,\partial_{\nu}\partial^{\lambda}_{\mu\xi} + \right.$$
$$+ l\,\xi^{\sigma}\partial_{\nu}\omega^{\lambda}_{\sigma\mu} + l\left(\partial_{\nu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\mu} - l\,\omega^{\sigma}_{\mu\nu}\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} -$$
$$- l\left(\nabla_{\mu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\nu}\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} - l\left(\partial_{\upsilon}\partial_{\mu}\xi^{\sigma}\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} -$$
$$\left. - l\,\xi^{\rho}\left(\partial_{\nu}\omega^{\sigma}_{\rho\mu}\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} - l\left(\partial_{\nu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\mu}\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda}\right) g_{\lambda}\,.$$

Using the relation $\partial_{\nu} g_{\mu} = \Gamma^{\lambda}_{\mu\nu} g_{\lambda}$ (similarly, $\partial_{\nu} h_{\mu} = \omega^{\lambda}_{\mu\nu} h_{\lambda}$), we obtain the relation between the two connections Γ and ω as follows:

$$\Gamma^{\lambda}_{\mu\nu} = \omega^{\lambda}_{\mu\nu} + l\left(\left(\nabla_{\mu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\nu} + \partial_{\nu}\partial_{\mu}\xi^{\lambda} + \right.$$
$$+ \xi^{\sigma}\partial_{\nu}\omega^{\lambda}_{\sigma\mu}\left(\partial_{\nu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\mu} - \omega^{\sigma}_{\mu\nu}\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} - \left(\nabla_{\mu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\nu}\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} -$$
$$\left. - \left(\partial_{\nu}\partial_{\mu}\xi^{\sigma}\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} - \xi^{\rho}\left(\partial_{\nu}\omega^{\sigma}_{\rho\mu}\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} - \left(\partial_{\nu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\mu}\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda}\right)$$

which is a general non-linear relation in the components of the exterior displacement field ξ. We may now write

$$\Gamma^{\lambda}_{\mu\nu} = F^{\lambda}_{\mu\nu} + G^{\lambda}_{\mu\nu}$$

where, recalling the previous definition of $F^{\lambda}_{\mu\nu}$, it can be rewritten as

$$F^{\lambda}_{\mu\nu} = \omega^{\lambda}_{\mu\nu} + l\left(\left(\partial_{\nu}\omega^{\lambda}_{\sigma\mu} + \omega^{\rho}_{\mu\sigma}\omega^{\lambda}_{\rho\nu}\right)\xi^{\sigma} + \right.$$
$$\left. + \partial_{\nu}\partial_{\mu}\xi^{\lambda} + \left(\partial_{\mu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\nu} + \left(\partial_{\nu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\mu}\right)$$

and where

$$G^{\lambda}_{\mu\nu} = -l\left(\omega^{\sigma}_{\mu\nu} + l\left(\left(\nabla_{\mu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\nu}\right.\right.$$
$$\left.\left. + \partial_{\nu}\partial_{\mu}\xi^{\sigma} + \xi^{\rho}\partial_{\nu}\omega^{\sigma}_{\rho\mu} + \left(\partial_{\nu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\mu}\right)\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda}\,.$$

At this point, the intrinsic curvature tensors of the space-times $\mathbb{S}_1$ and $\mathbb{S}_2$ are respectively given by

$$K^{\sigma}{}_{\rho\mu\nu} = 2\left(h^{\sigma}, \partial_{[\mu}\partial_{\nu]}h_{\rho}\right) =$$
$$= \partial_{\mu}\omega^{\sigma}_{\rho\nu} - \partial_{\nu}\omega^{\sigma}_{\rho\mu} + \omega^{\lambda}_{\rho\nu}\omega^{\sigma}_{\lambda\mu} - \omega^{\lambda}_{\rho\mu}\omega^{\sigma}_{\lambda\nu}$$

and

$$R^{\sigma}{}_{\rho\mu\nu} = 2\left(g^{\sigma}, \partial_{[\mu}\partial_{\nu]}g_{\rho}\right) =$$
$$= \partial_{\mu}\Gamma^{\sigma}_{\rho\nu} - \partial_{\nu}\Gamma^{\sigma}_{\rho\mu} + \Gamma^{\lambda}_{\rho\nu}\Gamma^{\sigma}_{\lambda\mu} - \Gamma^{\lambda}_{\rho\mu}\Gamma^{\sigma}_{\lambda\nu}\,.$$

We may also define the following quantities built from the connections $\omega^{\lambda}_{\mu\nu}$ and $\Gamma^{\lambda}_{\mu\nu}$:

$$D^{\sigma}{}_{\rho\mu\nu} = \partial_{\mu}\omega^{\sigma}_{\rho\nu} + \partial_{\nu}\omega^{\sigma}_{\rho\mu} + \omega^{\lambda}_{\rho\nu}\omega^{\sigma}_{\lambda\mu} + \omega^{\lambda}_{\rho\mu}\omega^{\sigma}_{\lambda\nu}$$

and

$$E^{\sigma}{}_{\rho\mu\nu} = \partial_{\mu}\Gamma^{\sigma}_{\rho\nu} + \partial_{\nu}\Gamma^{\sigma}_{\rho\mu} + \Gamma^{\lambda}_{\rho\nu}\Gamma^{\sigma}_{\lambda\mu} + \Gamma^{\lambda}_{\rho\mu}\Gamma^{\sigma}_{\lambda\nu}$$

from which we may define two additional "curvatures" X and P by

$$X^{\sigma}{}_{\rho\mu\nu} = \left(h^{\sigma}, \partial_{\mu}\partial_{\nu}h_{\rho}\right) = \frac{1}{2}\left(K^{\sigma}{}_{\rho\mu\nu} + D^{\sigma}{}_{\rho\mu\nu}\right) =$$
$$= \partial_{\mu}\omega^{\sigma}_{\rho\nu} + \omega^{\lambda}_{\rho\nu}\omega^{\sigma}_{\lambda\mu}$$

and

$$P^{\sigma}{}_{\rho\mu\nu} = \left(g^{\sigma}, \partial_{\mu}\partial_{\nu}g_{\rho}\right) = \frac{1}{2}\left(R^{\sigma}{}_{\rho\mu\nu} + E^{\sigma}{}_{\rho\mu\nu}\right) =$$
$$= \partial_{\mu}\Gamma^{\sigma}_{\rho\nu} + \Gamma^{\lambda}_{\rho\nu}\Gamma^{\sigma}_{\lambda\mu}$$

such that $K^{\sigma}{}_{\rho\mu\nu} = 2\,X^{\sigma}{}_{\rho[\mu\nu]}$ and $R^{\sigma}{}_{\rho\mu\nu} = 2\,P^{\sigma}{}_{\rho[\mu\nu]}$.

Now, we see that

$$F^{\lambda}_{(\mu\nu)} = \omega^{\lambda}_{(\mu\nu)} + l\left(\frac{1}{2}D^{\lambda}{}_{\sigma\mu\nu}\xi^{\sigma} + \partial_{\nu}\partial_{\mu}\xi^{\lambda}\right) +$$
$$+ l\left(\left(\partial_{\mu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\nu} + \left(\partial_{\nu}\xi^{\sigma}\right)\omega^{\lambda}_{\sigma\mu}\right)$$

and

$$F^{\lambda}_{[\mu\nu]} = \omega^{\lambda}_{[\mu\nu]} + \frac{1}{2}\,l\,K^{\lambda}{}_{\sigma\mu\nu}\,\xi^{\sigma}\,.$$

In addition, we also have

$$G^{\lambda}_{(\mu\nu)} = l\left(\omega^{\sigma}_{(\mu\nu)} + l\left(\frac{1}{2}D^{\sigma}{}_{\rho\mu\nu}\xi^{\rho} + \partial_{\nu}\partial_{\mu}\xi^{\sigma}\right)\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda} +$$
$$+ l\left(l\left(\left(\partial_{\mu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\nu} + \left(\partial_{\nu}\xi^{\rho}\right)\omega^{\sigma}_{\rho\mu}\right)\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda}$$

and

$$G^{\lambda}_{[\mu\nu]} = l\left(\omega^{\sigma}_{[\mu\nu]} - \frac{1}{2}\,l\,K^{\sigma}{}_{\rho\mu\nu}\,\xi^{\rho}\right)\bar{\nabla}_{\sigma}\bar{\xi}^{\lambda}.$$

Now, the metric tensor g of the space-time $\mathbb{S}_1$ and the metric tensor h of the space-time $\mathbb{S}_2$ are respectively given by

$$h_{\mu\nu} = \left(h_{\mu}, h_{\nu}\right)$$

and

$$g_{\mu\nu} = \left(g_{\mu}, g_{\nu}\right)$$

where the following relations hold:

$$h_{\mu\sigma} h^{\nu\sigma} = \delta^\nu_\mu$$

$$g_{\mu\sigma} g^{\nu\sigma} = \delta^\nu_\mu$$

In general, the two conditions $h_{\mu\sigma}\, g^{\nu\sigma} \neq \delta^\nu_\mu$ and $g_{\mu\sigma}\, h^{\nu\sigma} \neq \delta^\nu_\mu$ must be fulfilled unless $l = 0$ (in the limit $\hbar \to 0$). Furthermore, we have the metricity conditions

$$\nabla_\lambda\, h_{\mu\nu} = 0\,,$$

and

$$\bar{\nabla}_\lambda\, g_{\mu\nu} = 0\,.$$

However, note that in general, $\bar{\nabla}_\lambda\, h_{\mu\nu} \neq 0$ and $\nabla_\lambda g_{\mu\nu} \neq 0$.

Hence, it is straightforward to see that in general, the metric tensor g is related to the metric tensor h by

$$g_{\mu\nu} = h_{\mu\nu} \;+\; 2\,l\, \nabla_{(\mu}\xi_{\nu)} + l^2\, \nabla_\mu \xi^\lambda\, \nabla_\nu \xi_\lambda$$

which in the linear approximation reads

$$g_{\mu\nu} = h_{\mu\nu} + 2l\, \nabla_{(\mu}\xi_{\nu)}\,.$$

The formal structure of our underlying geometric framework clearly implies that the same structure holds in n dimensions as well.

3 The conformal theory

We are now in the position to extract a physical theory of quantum gravity from the geometric framework in the preceding section by considering the following linear conformal mapping:

$$g_\mu = e^\varphi h_\mu$$

where the continuously differentiable scalar function $\varphi\,(x^\mu)$ is the generator of the quantum displacement field in the evolution space $\mathbb{M}_4$ and therefore connects the two space-times $\mathbb{S}_1$ and $\mathbb{S}_2$.

Now, for reasons that will be apparent soon, we shall define the generator φ in terms of the canonical quantum mechanical wave function $\psi\,(x^\mu)$ as

$$\varphi = \ln\,(1 + M\psi)^{\frac{1}{2}}$$

where

$$M = \pm\frac{1}{2}\, l \left(i\, \frac{m_0 c}{\hbar}\right)^2.$$

Here m_0 is the rest mass of the electron. Note that the sign $\pm$ signifies the signature of the space-time used.

Now, we also have the following relations:

$$g^\mu = e^{-\varphi} h^\mu,$$

$$h_\mu = e^{-\varphi} g_\mu,$$

$$h^\mu = e^\varphi g^\mu,$$

$$(g_\mu, g^\nu) = (h_\mu, h^\nu) = \delta^\nu_\mu\,,$$

$$(g_\mu, h^\nu) = e^{2\,\varphi}\delta^\nu_\mu\,,$$

$$(h_\mu, g^\nu) = e^{-2\,\varphi}\delta^\nu_\mu\,,$$

as well as the conformal transformation

$$g_{\mu\nu} = e^{2\varphi} h_{\mu\nu}\,.$$

Hence

$$g^{\mu\nu} = e^{-2\,\varphi}\, h^{\mu\nu}.$$

We immediately see that

$$g_{\mu\sigma}\, h^{\nu\sigma} = e^{2\varphi}\, \delta^\nu_\mu\,,$$

$$h_{\mu\sigma}\, g^{\nu\sigma} = e^{-2\varphi}\, \delta^\nu_\mu\,.$$

At this point, we see that the world-line of the space-time $\mathbb{S}_2$, $s = \int \sqrt{h_{\mu\nu}\, dx^\mu dx^\nu}$, is connected to that of the space-time $\mathbb{S}_1$, $\sigma = \int \sqrt{g_{\mu\nu}\, dx^\mu dx^\nu}$, through

$$ds = e^{2\varphi} d\sigma\,.$$

Furthermore, from the relation

$$g_\mu = \left(\delta^\nu_\mu + l\, \nabla_\mu \xi^\nu\right) h_\nu = e^\varphi h_\mu$$

we obtain the important relation

$$l\, \nabla_\nu \xi_\mu = (e^\varphi - 1)\, h_{\mu\nu}\,,$$

which means that

$$\Phi_{\mu\nu} = l\, \nabla_\nu \xi_\mu = \Phi_{\nu\mu}\,,$$

i.e., the quantum displacement gradient tensor field Φ is symmetric. Hence we may simply call Φ the *quantum strain tensor field*. We also see that the components of the quantum displacement field, $\phi^\mu = l\,\xi^\mu$, can now be described by the wave function ψ as

$$\phi_\mu = l\, \partial_\mu \psi$$

i.e.,

$$\psi = \psi_0 + \frac{1}{l} \int \phi_\mu dx^\mu$$

for an arbitrary initial value ψ_0 (which, most conveniently, can be chosen to be 0).

Furthermore, we note that the integrability condition $\Phi_{\mu\nu} = \Phi_{\nu\mu}$ means that the space-time $\mathbb{S}_1$ must now possess a symmetric, linear connection, i.e.,

$$\omega^\lambda_{\mu\nu} = \omega^\lambda_{\nu\mu} = \frac{1}{2}\, h^{\sigma\lambda}\, (\partial_\nu h_{\sigma\mu} - \partial_\sigma h_{\mu\nu} + \partial_\mu h_{\nu\sigma})\,,$$

which are just the Christoffel symbols $\{{}^{\,\lambda}_{\mu\nu}\}$ in the space-time $\mathbb{S}_1$. Hence ω is now none other than the symmetric Levi-Civita (Riemannian) connection. Using the metricity condition $\partial_\lambda g_{\mu\nu} = \Gamma_{\mu\nu\lambda} \;+\; \Gamma_{\nu\mu\lambda}$, i.e.,

$$\partial_\lambda\, g_{\mu\nu} = M h_{\mu\nu}\, \partial_\lambda \psi + (1 + M\,\psi)\, (\omega_{\mu\nu\lambda} \;+\; \omega_{\nu\mu\lambda})\,,$$

we obtain the mixed form

$$\omega_{\lambda\mu\nu} = \frac{1}{2}\, (1 + M\psi)^{-1}\, (\partial_\lambda g_{\mu\nu} - \partial_\mu g_{\nu\lambda} + \partial_\nu g_{\lambda\mu}) - $$
$$ - \frac{1}{2}\, M\, (1 + M\psi)^{-1}\, (h_{\mu\nu}\partial_\lambda \psi - h_{\nu\lambda}\partial_\mu \psi + h_{\lambda\mu}\partial_\nu \psi)$$

i.e.,

$$\omega^{\lambda}_{\mu\nu} = \frac{1}{2}\left(1 + M\psi\right)^{-1} h^{\lambda\rho}\left(\partial_\rho g_{\mu\nu} - \partial_\mu\, g_{\nu\rho} + \partial_\nu\, g_{\rho\mu}\right) - \\ - \frac{1}{2}\, M\left(1 + M\psi\right)^{-1}\left(\delta^{\lambda}_{\mu}\partial_\nu\psi + \delta^{\lambda}_{\nu}\partial_\mu\psi - h_{\mu\nu}h^{\lambda\rho}\partial_\rho\psi\right).$$

It may be noted that we have used the customary convention in which $\Gamma_{\lambda\mu\nu} = g_{\lambda\rho}\,\Gamma^{\rho}_{\mu\nu}$ and $\omega_{\lambda\mu\nu} = h_{\lambda\rho}\omega^{\rho}_{\mu\nu}$.

Now we shall see why we have made the particular choice $\varphi = \ln\,(1 + M\,\psi)^{\frac{1}{2}}$. In order to explicitly show that it now possess a *stochastic* part, let us rewrite the components of the metric tensor of the space-time $\mathbb{S}_2$ as

$$g_{\mu\nu} = \left(1 + M\psi\right) h_{\mu\nu}\,.$$

Combining this relation with the linearized relation $g_{\mu\nu} = h_{\mu\nu} + 2\,l\,\nabla_{(\mu}\,\xi_{\nu)}$ and contracting the resulting relation, we obtain

$$l D^2\psi = 2\left(e^{2\,\varphi} - 1\right) = 2M\psi\,,$$

where we have defined the differential operator $D^2 = h^{\mu\nu}\nabla_\mu\nabla_\nu$ such that

$$D^2\psi = h^{\mu\nu}\left(\partial_\mu\partial_\nu\psi - \omega^{\rho}_{\mu\nu}\,\partial_\rho\psi\right).$$

Expressing M explicitly, we obtain $D^2\psi = \mp\left(\frac{m_0\,c}{\hbar}\right)^2\psi$, i.e.,

$$\left(D^2 \pm \left(\frac{m_0 c}{\hbar}\right)^2\right)\psi = 0$$

which is precisely the *Klein-Gordon equation in the presence of gravitation*.

We may note that, had we combined the relation $g_{\mu\nu} = (1 + M\,\psi)\,h_{\mu\nu}$ with the fully non-linear relation

$$g_{\mu\nu} = h_{\mu\nu} + 2l\,\nabla_{(\mu}\xi_{\nu)} + l^2\,\nabla_\mu\xi^\lambda\nabla_\nu\xi_\lambda\,,$$

we would have obtained the following *non-linear* Klein-Gordon equation:

$$\left(D^2 \pm \left(\frac{m_0\,c}{\hbar}\right)^2\right)\psi = l^2 h^{\rho\sigma}h^{\mu\nu}\left(\nabla_\rho\nabla_\mu\psi\right)\left(\nabla_\sigma\nabla_\nu\psi\right).$$

Now, from the general relation between the connections Γ and ω given in Section 2, we obtain the following important relation:

$$\Gamma^{\lambda}_{[\mu\nu]} = -\frac{1}{2}\,l\left(\delta^{\lambda}_{\sigma} - l\,\bar{\nabla}_\sigma\bar{\xi}^\lambda\right)K^{\sigma}{}_{\rho\mu\nu}\xi^\rho,$$

which not only connects the torsion of the space-time $\mathbb{S}_2$ with the curvature of the space-time $\mathbb{S}_1$, but also describes the torsion as an intrinsic (geometric) quantum phenomenon. Note that

$$K^{\sigma}{}_{\rho\mu\nu} = \partial_\mu\begin{Bmatrix}\sigma\\ \rho\nu\end{Bmatrix} - \partial_\nu\begin{Bmatrix}\sigma\\ \rho\mu\end{Bmatrix} + \\ + \begin{Bmatrix}\lambda\\ \rho\nu\end{Bmatrix}\begin{Bmatrix}\sigma\\ \lambda\mu\end{Bmatrix} - \begin{Bmatrix}\lambda\\ \rho\mu\end{Bmatrix}\begin{Bmatrix}\sigma\\ \lambda\nu\end{Bmatrix}$$

are now the components of the Riemann-Christoffel curvature tensor describing the curvature of space-time in the standard general relativity theory.

Furthermore, using the relation between the two sets of basis vectors g_μ and h_μ, it is easy to see that the connection Γ is *semi-symmetric* as

$$\Gamma^{\lambda}_{\mu\nu} = \omega^{\lambda}_{\mu\nu} + \delta^{\lambda}_{\mu}\,\partial_\nu\varphi$$

or, written somewhat more explicitly,

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}\,h^{\sigma\lambda}\left(\partial_\nu h_{\sigma\mu} - \partial_\sigma h_{\mu\nu} + \partial_\mu h_{\nu\sigma}\right) + \\ + \frac{1}{2}\,\delta^{\lambda}_{\mu}\,\partial_\nu\big(\ln\left(1 + M\psi\right)\big).$$

We immediately obtain

$$\Gamma^{\lambda}_{(\mu\nu)} = \omega^{\lambda}_{\mu\nu} + \frac{1}{2}\left(\delta^{\lambda}_{\mu}\partial_\nu\varphi + \delta^{\lambda}_{\nu}\,\partial_\mu\varphi\right)$$

and

$$\Gamma^{\lambda}_{[\mu\nu]} = \frac{1}{2}\left(\delta^{\lambda}_{\mu}\partial_\nu\varphi - \delta^{\lambda}_{\nu}\partial_\mu\varphi\right).$$

Additionally, using the relation

$$\omega^{\nu}_{\nu\mu} = \omega^{\nu}_{\mu\nu} = \partial_\mu\left(\ln\sqrt{\det\left(h\right)}\right) = \\ = \partial_\mu\left(\ln\left(e^{-\varphi}\sqrt{\det\left(g\right)}\right)\right) = \partial_\mu\left(\ln\sqrt{\det\left(g\right)}\right) - \partial_\mu\varphi$$

we may now define two *semi-vectors* by the following contractions:

$$\Gamma_\mu = \Gamma^{\nu}_{\nu\mu} = \partial_\mu\left(\ln\sqrt{\det\left(h\right)}\right) + 4\,\partial_\mu\varphi$$

$$\Delta_\mu = \Gamma^{\nu}_{\mu\nu} = \partial_\mu\left(\ln\sqrt{\det\left(h\right)}\right) + \partial_\mu\varphi$$

or, written somewhat more explicitly,

$$\Gamma_\mu = \partial_\mu\left(\ln\sqrt{\det\left(h\right)} + \ln\left(1 + M\psi\right)^2\right)$$

$$\Delta_\mu = \partial_\mu\left(\ln\sqrt{\det\left(h\right)} + \ln\sqrt{1 + M\psi}\right).$$

We now define the *torsion vector* by

$$\tau_\mu = \Gamma^{\nu}_{[\nu\mu]} = \frac{3}{2}\,\partial_\mu\varphi\,.$$

In other words,

$$\tau_\mu = \frac{3}{4}\,\frac{M}{\left(1 + M\psi\right)}\,\partial_\mu\psi\,.$$

Furthermore, it is easy to show that the curvature tensors of our two space-times $\mathbb{S}_1$ and $\mathbb{S}_2$ are now identical:

$$R^{\sigma}{}_{\rho\mu\nu} = K^{\sigma}{}_{\rho\mu\nu}$$

which is another way of saying that the conformal transformation $g_\mu = e^\varphi\,h_\mu$ leaves the curvature tensor of the space-time $\mathbb{S}_1$ invariant. As an immediate consequence, we obtain the ordinary expression

$$R_{\rho\sigma\mu\nu} = \frac{1}{2}\left(\partial_\mu\partial_\sigma h_{\rho\nu} + \partial_\nu\partial_\rho h_{\sigma\mu} - \partial_\nu\partial_\sigma h_{\rho\mu} - \partial_\mu\partial_\rho h_{\sigma\nu}\right) + \\ + h_{\alpha\beta}\left(\omega^{\alpha}_{\rho\nu}\omega^{\beta}_{\sigma\mu} - \omega^{\alpha}_{\rho\mu}\,\omega^{\beta}_{\sigma\nu}\right).$$

Hence the following cyclic symmetry in Riemannian geometry:

$$R_{\rho\sigma\mu\nu} + R_{\rho\mu\nu\sigma} + R_{\rho\nu\sigma\mu} = 0$$

is preserved in the presence of torsion. In addition, besides the obvious symmetry $R_{\rho\sigma\mu\nu} = -R_{\rho\sigma\nu\mu}$, we also have the symmetry

$$R_{\rho\sigma\mu\nu} = -R_{\sigma\rho\mu\nu}$$

which is due to the metricity condition of the space-times $\mathbb{S}_1$ and $\mathbb{S}_2$. This implies the vanishing of the so-called Homothetic curvature as

$$H_{\mu\nu} = R^{\sigma}{}_{\sigma\mu\nu} = 0\,.$$

The Weyl tensor is given in the usual manner by

$$C_{\rho\sigma\mu\nu} = R_{\rho\sigma\mu\nu} - \frac{1}{2}\left(h_{\rho\mu}R_{\sigma\nu} + h_{\sigma\nu}R_{\rho\mu} - \right.$$
$$\left. - h_{\rho\nu}R_{\sigma\mu} - h_{\sigma\mu}R_{\rho\nu}\right) - \frac{1}{6}\left(h_{\rho\nu}h_{\sigma\mu} - h_{\rho\mu}h_{\sigma\nu}\right) R\,,$$

where $R_{\mu\nu} = R^{\sigma}{}_{\mu\sigma\nu}$ are the components of the symmetric Ricci tensor and $R = R^{\mu}{}_{\mu}$ is the Ricci scalar.

Now, by means of the conformal relation $g_{\mu\nu} = e^{2\varphi}\, h_{\mu\nu}$ we obtain the expression

$$R_{\rho\sigma\mu\nu} = e^{-2\varphi}\Big(\partial_\mu\partial_\sigma g_{\rho\nu} + \partial_\nu\partial_\rho g_{\sigma\mu} - \partial_\nu\partial_\sigma g_{\rho\mu}\partial_\mu\partial_\rho g_{\sigma\nu} +$$
$$+ g_{\alpha\beta}\left(\Gamma^{\alpha}_{\rho\nu}\Gamma^{\beta}_{\sigma\mu} - \Gamma^{\alpha}_{\rho\mu}\Gamma^{\beta}_{\sigma\nu}\right) + \left(\partial_\mu g_{\sigma\nu} - \partial_\nu g_{\sigma\mu}\right)\partial_\rho\varphi +$$
$$+ \left(\partial_\nu g_{\rho\mu} - \partial_\mu g_{\rho\nu}\right)\partial_\sigma\varphi + \left(\partial_\rho\, g_{\sigma\nu} - \partial_\sigma g_{\rho\nu}\right)\partial_\mu\varphi +$$
$$+ \left(\partial_\sigma g_{\rho\mu} - \partial_\rho g_{\sigma\mu}\right)\partial_\nu\varphi + g_{\sigma\nu}\partial_\mu\partial_\rho\varphi + g_{\rho\mu}\partial_\nu\partial_\sigma\varphi +$$
$$- g_{\rho\nu}\partial_\mu\partial_\sigma\varphi - g_{\mu\sigma}\partial_\nu\partial_\rho\varphi + 2\left(g_{\sigma\mu}\partial_\rho\varphi\,\partial_\nu\varphi + \right.$$
$$\left. + g_{\rho\nu}\partial_\mu\varphi\,\partial_\sigma\varphi - g_{\sigma\nu}\partial_\rho\varphi\,\partial_\mu\varphi - g_{\rho\mu}\partial_\sigma\varphi\,\partial_\nu\varphi\right) +$$
$$+ g_{\alpha\beta}\left(\left(\Gamma^{\alpha}_{\rho\mu}\partial_\nu\varphi - \Gamma^{\alpha}_{\rho\nu}\partial_\mu\varphi\right)\delta^{\beta}_{\sigma} - \left(\Gamma^{\beta}_{\sigma\mu}\partial_\nu\varphi - \Gamma^{\beta}_{\sigma\nu}\partial_\mu\varphi\right)\delta^{\alpha}_{\rho}\right)\Big).$$

Note that despite the fact that the curvature tensor of the space-time $\mathbb{S}_2$ is identical to that of the space-time $\mathbb{S}_1$ and that both curvature tensors share common algebraic symmetries, the Bianchi identity in $\mathbb{S}_2$ is not the same as the ordinary Bianchi identity in the torsion-free space-time $\mathbb{S}_1$. Instead, we have the following *generalized* Bianchi identity:

$$\bar\nabla_\lambda R_{\rho\sigma\mu\nu} + \bar\nabla_\mu R_{\rho\sigma\nu\lambda} + \bar\nabla_\nu R_{\rho\sigma\lambda\mu} =$$
$$= 2\left(\Gamma^{\eta}_{[\mu\nu]}R_{\rho\sigma\eta\lambda} + \Gamma^{\eta}_{[\nu\lambda]}R_{\rho\sigma\eta\mu} + \Gamma^{\eta}_{[\lambda\mu]}\, R_{\rho\sigma\eta\nu}\right).$$

Contracting the above relation, we obtain

$$\bar\nabla_\mu\left(R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu}R\right) = 2\, g^{\rho\nu}\Gamma^{\sigma}_{[\lambda\rho]}\, R^{\lambda}{}_{\sigma} + \Gamma^{\lambda}_{[\rho\sigma]}R^{\rho\sigma\nu}{}_{\lambda}\,.$$

Combining the two generalized Bianchi identities above with the relation $\Gamma^{\lambda}_{[\mu\nu]} = \frac{1}{2}\left(\delta^{\lambda}_{\mu}\,\partial_\nu\varphi - \delta^{\lambda}_{\nu}\partial_\mu\varphi\right)$, as well as recalling the definition of the torsion vector, and taking into account the symmetry of the Ricci tensor, we obtain

$$\bar\nabla_\lambda R_{\rho\sigma\mu\nu} + \bar\nabla_\mu R_{\rho\sigma\nu\lambda} + \bar\nabla_\nu R_{\rho\sigma\lambda\mu} =$$
$$= 2\left(R_{\rho\sigma\mu\nu}\partial_\lambda\varphi + R_{\rho\sigma\nu\lambda}\partial_\mu\varphi + R_{\rho\sigma\lambda\mu}\partial_\nu\,\varphi\right)$$

and

$$\bar\nabla_\nu\left(R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu}R\right) = -\,2\left(R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu}R\right)\partial_\nu\varphi$$

which, upon recalling the definition of the torsion vector, may be expressed as

$$\bar\nabla_\nu\left(R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu}R\right) = -\,\frac{4}{3}\left(R^{\mu\nu} - \frac{1}{2}\, g^{\mu\nu}R\right)\tau_\nu\,.$$

Apart from the above generalized identities, we may also give the ordinary Bianchi identities as

$$\nabla_\lambda R_{\rho\sigma\mu\nu} + \nabla_\mu R_{\rho\sigma\nu\lambda} + \nabla_\nu R_{\rho\sigma\lambda\mu} = 0$$

and

$$\nabla_\nu\left(R^{\mu\nu} - \frac{1}{2}\, h^{\mu\nu}R\right) = 0\,.$$

4 The electromagnetic sector of the conformal theory. The fundamental equations of motion

Based on the results obtained in the preceding section, let us now take the generator φ as describing the (quantum) electromagnetic field. Then, consequently, the space-time $\mathbb{S}_1$ is understood as being *devoid* of electromagnetic interaction. As we will see, in our present theory, it is the quantum evolution of the gravitational field that gives rise to electromagnetism. In this sense, the electromagnetic field is but an *emergent* quantum phenomenon in the evolution space $\mathbb{M}_4$.

Whereas the space-time $\mathbb{S}_1$ is purely gravitational, the *evolved* space-time $\mathbb{S}_2$ does contain an electromagnetic field. In our present theory, for reasonsthat will be clear soon, we shall define the electromagnetic field $F \in \mathbb{S}_2 \in \mathbb{M}_4$ in terms of the *torsion* of the space-time $\mathbb{S}_2$ by

$$F_{\mu\nu} = 2\,\frac{m_0c^2}{\bar e}\,\Gamma^{\lambda}_{[\mu\nu]}u_\lambda\,,$$

where $\bar e$ is the (elementary) charge of the electron and

$$u_\mu = g_{\mu\nu}\,\frac{dx^\nu}{ds} = e^{2\varphi}h_{\mu\nu}\frac{dx^\nu}{ds}$$

are the covariant components of the tangent velocity vector field satisfying $u_{mu}\, u^\mu = 1$.

We have seen that the space-time $\mathbb{S}_2$ possesses a manifest quantum structure through its evolution from the purely gravitational space-time $\mathbb{S}_1$. This means that $\bar e$ may be defined in terms of the fundamental Planck charge $\hat e$ as follows:

$$\bar e = N\hat e = N\,\sqrt{4\pi\varepsilon_0\hbar c}\,,$$

where N is a positive constant and ε_0 is the permittivity of free space. Further investigation shows that $N = \sqrt{\alpha}$ where $\alpha^{-1} \approx 137$ is the conventional fine structure constant.

Let us now proceed to show that the geodesic equation of motion in the space-time $\mathbb{S}_2$ gives the (generalized) Lorentz equation of motion for the electron. The result of paralleltransferring the velocity vector field u along the world-line (in the direction of motion of the electron) yields

$$\frac{\bar{D}\, u^{\mu}}{ds} = \left(\bar{\nabla}_{\nu} u^{\mu}\right) u^{\nu} = 0\,,$$

i.e.,

$$\frac{du^{\mu}}{ds} + \Gamma^{\mu}_{\rho\sigma} u^{\rho} u^{\sigma} = 0\,,$$

where, in general,

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}\, g^{\sigma\lambda} \left(\partial_{\nu} g_{\sigma\mu} - \partial_{\sigma} g_{\mu\nu} + \partial_{\mu} g_{\nu\sigma}\right) + \Gamma^{\lambda}_{[\mu\nu]} - \\ - g^{\lambda\rho} \left(g_{\mu\sigma} \Gamma^{\sigma}_{[\rho\nu]} + g_{\nu\sigma}\, \Gamma^{\sigma}_{[\rho\mu]}\right).$$

Recalling our expression for the components of the torsion tensor in the preceding section, we obtain

$$\Gamma^{\lambda}_{\mu\nu} = \frac{1}{2}\, g^{\sigma\lambda}\, \left(\partial_{\nu} g_{\sigma\mu} - \partial_{\sigma} g_{\mu\nu} + \partial_{\mu} g_{\nu\sigma}\right) + \\ + g_{\mu\nu} g^{\lambda\sigma} \partial_{\sigma}\varphi - \delta^{\lambda}_{\nu}\, \partial_{\mu}\varphi$$

which is completely equivalent to the previously obtained relation

$$\Gamma^{\lambda}_{\mu\nu} = \omega^{\lambda}_{\mu\nu} + \delta^{\lambda}_{\mu}\, \partial_{\nu}\varphi\,.$$

Note that

$$\Delta^{\lambda}_{\mu\nu} = \frac{1}{2}\, g^{\sigma\lambda} \left(\partial_{\nu} g_{\sigma\mu} - \partial_{\sigma} g_{\mu\nu} + \partial_{\mu} g_{\nu\sigma}\right)$$

are the Christoffel symbols in the space-time $\mathbb{S}_2$. These are not to be confused with the Christoffel symbols in the space-time $\mathbb{S}_1$ given by $\omega^{\lambda}_{\mu\nu}$.

Furthermore, we have

$$\frac{du^{\mu}}{ds} + \Delta^{\mu}_{\rho\sigma}\, u^{\rho} u^{\sigma} = 2 g^{\mu\rho} \Gamma^{\lambda}_{[\rho\sigma]} u_{\lambda} u^{\sigma}.$$

Now, since we have set $F_{\mu\nu} = 2 \frac{m_0\, c^2}{\bar{e}} \Gamma^{\lambda}_{[\mu\nu]} u_{\lambda}$, we obtain the equation of motion

$$m_0 c^2 \left(\frac{du^{\mu}}{ds} + \Delta^{\mu}_{\rho\sigma} u^{\rho} u^{\sigma}\right) = \bar{e}\, F^{\mu}{}_{\nu}\, u^{\nu},$$

which is none other than the Lorentz equation of motion for the electron in the presence of gravitation. Hence, it turns out that the electromagnetic field, which is non-existent in the space-time $\mathbb{S}_1$, is an *intrinsic geometric object* in the space-time $\mathbb{S}_2$. In other words, the space-time structure of $\mathbb{S}_2$ inherently contains both gravitation and electromagnetism.

Now, we see that

$$F_{\mu\nu} = \frac{m_0 c^2}{\bar{e}} \left(u_{\mu} \partial_{\nu}\varphi - u_{\nu} \partial_{\mu}\varphi\right).$$

In other words,

$$\bar{e}\, F^{\mu}{}_{\nu}\, u^{\nu} = m_0 c^2 \left(u^{\mu} \frac{d\varphi}{ds} - g^{\mu\nu} \partial_{\nu}\varphi\right).$$

Consequently, we can rewrite the electron's equation of motion as

$$\frac{du^{\mu}}{ds} + \Delta^{\mu}_{\rho\sigma} u^{\rho} u^{\sigma} = u^{\mu} \frac{d\varphi}{ds} - g^{\mu\nu}\, \partial_{\nu}\varphi\,.$$

We may therefore define an *asymmetric fundamental tensor of the gravoelectromagnetic manifold* $\mathbb{S}_2$ by

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} \frac{d\varphi}{ds} - \frac{\bar{e}}{m_0 c^2} F_{\mu\nu}$$

satisfying

$$\tilde{g}_{\mu\nu} u^{\nu} = \partial_{\mu}\varphi\,.$$

It follows immediately that

$$\left(\delta^{\mu}_{\nu} \frac{d\varphi}{ds} - \frac{\bar{e}}{m_0 c^2} F^{\mu}{}_{\nu}\right) u^{\nu} = g^{\mu\nu}\, \partial_{\nu}\varphi$$

which, when expressed in terms of the wave function ψ, gives the *Schrödinger-like* equation

$$u_{\mu} \frac{d\psi}{ds} = \frac{1}{M} \left(\partial_{\mu}\varphi + \frac{\bar{e}}{m_0 c^2} F_{\mu\nu} u^{\nu}\right) \psi\,.$$

We may now proceed to show that the electromagnetic current density given by the covariant expression

$$j^{\mu} = -\frac{c}{4\pi} \bar{\nabla}_{\nu} F^{\mu\nu}$$

is conserved in the present theory.

Let us first call the following expression for the covariant components of the electromagnetic field tensor in terms of the covariant components of the canonical electromagnetic four-potential A:

$$F_{\mu\nu} = \bar{\nabla}_{\nu} A_{\mu} - \bar{\nabla}_{\mu} A_{\nu}$$

such that $\bar{e}\, \bar{\nabla}_{\nu} A_{\mu} = m_0 c^2 u_{\mu} \partial_{\nu}\varphi$, i.e.,

$$m_0 c^2 \partial_{\mu}\varphi = \bar{e}\, u^{\nu} \bar{\nabla}_{\mu} A_{\nu}$$

which directly gives the equation of motion

$$m_0 c^2 \frac{d\varphi}{ds} = \bar{e}\, u^{\mu} u^{\nu} \bar{\nabla}_{\mu} A_{\nu}\,.$$

Hence, we obtain the following equation of state:

$$m_0 c^2\, \frac{d\psi}{ds} = 2\, \bar{e}\, \frac{(1 + M\psi)}{M}\, u^{\mu} u^{\nu} \bar{\nabla}_{\mu} A_{\nu}\,.$$

Another alternative expression for the electromagnetic field tensor is given by

$$F_{\mu\nu} = \partial_{\nu} A_{\mu} \; - \; \partial_{\mu} A_{\nu} 2\Gamma^{\lambda}_{[\mu\nu]} A_{\lambda} \; = \\ = \; \partial_{\nu} A_{\mu} - \partial_{\mu} A_{\nu} + A_{\nu}\, \partial_{\mu}\varphi - A_{\mu} \partial_{\nu}\varphi\,.$$

In the particular case in which the field-lines of the electromagnetic four-potential propagate in the direction of the electron's motion, we have

$$F_{\mu\nu} = \Lambda\, \frac{\bar{e}}{\left(1 - \frac{\beta^2}{c^2}\right)} \left(\partial_{\nu} u_{\mu} - \partial_{\mu} u_{\nu}\right)$$

where Λ is a proportionality constant and $\beta = \pm\, \bar{e} \sqrt{\frac{\Lambda}{m_0}}$. Then, we may define a vortical velocity field, i.e., a spin field, through the vorticity tensor which is given by

$$\omega_{\mu\nu} = \frac{1}{2} \left(\partial_{\nu} u_{\mu} - \partial_{\mu} u_{\nu}\right)$$

and hence

$$F_{\mu\nu} = 2\Lambda \, \frac{\bar{e}}{\left(1 - \frac{\beta^2}{c^2}\right)} \, \omega_{\mu\nu} \, ,$$

which describes an electrically charged spinning region in the *space-time continuum* $\mathbb{S}_2$.

Furthermore, we have the following generalized identity for the electromagnetic field tensor:

$$\bar{\nabla}_\lambda F_{\mu\nu} + \bar{\nabla}_\mu F_{\nu\lambda} + \bar{\nabla}_\nu F_{\lambda\mu} =$$
$$= 2\left(\Gamma^\sigma_{[\mu\nu]} F_{\sigma\lambda} + \Gamma^\sigma_{[\nu\lambda]} F_{\sigma\mu} + \Gamma^\sigma_{[\lambda\mu]} F_{\sigma\nu}\right)$$

which, in the present theory, takes the particular form

$$\bar{\nabla}_\lambda F_{\mu\nu} + \bar{\nabla}_\mu F_{\nu\lambda} + \bar{\nabla}_\nu F_{\lambda\mu} =$$
$$= 2\left(F_{\mu\nu}\partial_\lambda\varphi + F_{\nu\lambda}\partial_\mu\varphi + F_{\lambda\mu}\partial_\nu\varphi\right).$$

Contracting, we have

$$\bar{\nabla}_\mu \, j^\mu = -\frac{c}{4\pi} \, \bar{\nabla}_\mu \left(\Gamma^\mu_{[\rho\sigma]} F^{\rho\sigma}\right).$$

We therefore expect that the expression in the brackets indeed vanishes. For this purpose, we may set

$$j^\mu = -\frac{c}{4\pi} \, \Gamma^\mu_{[\rho\sigma]} \, F^{\rho\sigma}$$

and hence, again, using the relation

$$\Gamma^\lambda_{[\mu\nu]} = \frac{1}{2}\left(\delta^\lambda_\mu \partial_\nu\varphi - \delta^\lambda_\nu \partial_\mu\varphi\right),$$

we immediately see that

$$\bar{\nabla}_\mu j^\mu - \frac{c}{4\pi}\left(\partial_\nu\varphi \, \bar{\nabla}_\mu F^{\mu\nu} + F^{\mu\nu}\left(\partial_\nu\partial_\mu\varphi - \Gamma^\lambda_{[\mu\nu]}\partial_\lambda\varphi\right)\right) =$$
$$= -\,j^\mu \, \partial_\mu\varphi - \frac{c}{4\pi} \, \Gamma^\lambda_{[\mu\nu]} F^{\mu\nu} \, \partial_\lambda\varphi$$

i.e.,

$$\bar{\nabla}_\mu \, j^\mu = 0\,.$$

At this point, we may note the following: the fact that our theory employs torsion, from which the electromagnetic field is extracted, and at the same time guarantees electromagnetic charge conservation (in the form of the above continuity equation) in a natural manner is a remarkable property.

Now, let us call the relation

$$\Gamma^\lambda_{[\mu\nu]} = -\frac{1}{2} \, l \, \left(\delta^\lambda_\sigma - l \, \bar{\nabla}_\sigma \bar{\xi}^\lambda\right) R^\sigma{}_{\rho\mu\nu} \, \xi^\rho$$

obtained in Section 3 of this work (in which $R^\sigma{}_{\rho\mu\nu} = K^\sigma{}_{\rho\mu\nu}$). This can simply be written as

$$\Gamma^\lambda_{[\mu\nu]} = -\frac{1}{2} \, l \, e^{-\varphi} R^\lambda{}_{\rho\mu\nu} \, \xi^\rho$$

i.e.,

$$\Gamma^\lambda_{[\mu\nu]} = -\frac{1}{2} \, l \, e^{-\varphi} R^\lambda{}_{\rho\mu\nu} \, g^{\rho\sigma} \partial_\sigma\psi\,.$$

Hence, we obtain the elegant result

$$F_{\mu\nu} = -\,l \, \frac{m_0 c^2}{\bar{e}} \, e^{-\varphi} R^\lambda{}_{\rho\mu\nu} \, u_\lambda \, g^{\rho\sigma} \partial_\sigma\psi$$

i.e.,

$$F_{\mu\nu} = -\frac{l}{\bar{e}} \, \frac{m_0 c^2}{\sqrt{1 + M\psi}} \, R^\lambda{}_{\rho\mu\nu} \, u_\lambda \, g^{\rho\sigma} \partial_\sigma\psi$$

or, in terms of the components of the (dimensionless) microscopic displacement field ξ,

$$F_{\mu\nu} = -\,l \, \frac{m_0 c^2}{\bar{e}} \, e^{-\varphi} R^\lambda{}_{\rho\mu\nu} \, u_\lambda \, g^{\rho\sigma} \xi_\sigma$$

which further reveals *how the electromagnetic field originates in the gravitational field in the space-time* $\mathbb{S}_2$ *as a quantum field*. Hence, at last, we see a complete picture of the electromagnetic field as an emergent phenomenon. This completes the long-cherished hypothesis that the electromagnetic field itself is caused by a *massive* charged particle, i.e., when $m_0 = 0$ neither gravity nor electromagnetism can exist. Finally, with this result at hand, we obtain the following equation of motion for the electron in the gravitational field:

$$\frac{du^\mu}{ds} + \Delta^\mu_{\rho\sigma} u^\rho u^\sigma = -\,l \, e^{-\varphi} R^{\rho\sigma\mu}{}_\nu \, u_\rho \, \xi_\sigma \, u^\nu$$

i.e.,

$$\frac{du^\mu}{ds} + \Delta^\mu_{\rho\sigma} u^\rho u^\sigma = -\frac{l}{\sqrt{1 + M\psi}} \, R^{\rho\sigma\mu}{}_\nu \, u_\rho \, u^\nu \, \partial_\sigma\psi\,.$$

In addition, we note that the torsion tensor is now seen to be given by

$$\tau_\mu = -\frac{1}{2} \, l \, e^{-\varphi} R_{\mu\nu} \, \xi^\nu$$

or, alternatively,

$$\tau_\mu = -\frac{1}{2} \, l \, e^{-\varphi} R_{\mu\nu} \, g^{\nu\lambda} \, \partial_\lambda\psi\,.$$

In other words,

$$\tau_\mu = -\frac{1}{2} \, \frac{l}{\sqrt{1 + M\,\psi}} \, R_{\mu\nu} \, g^{\nu\lambda} \, \partial_\lambda\psi\,.$$

Hence, the second generalized Bianchi identity finally takes the somewhat more transparent form

$$\bar{\nabla}_\nu \left(R^{\mu\nu} - \frac{1}{2} \, g^{\mu\nu} R\right) =$$
$$= -\frac{2}{3} \, l \, e^{-\varphi} \left(R^{\mu\nu} R_{\nu\rho} - \frac{1}{2} \, R R^\mu{}_\rho\right) g^{\rho\sigma} \, \partial_\sigma\psi$$

i.e.,

$$\bar{\nabla}_\nu \left(R^{\mu\nu} - \frac{1}{2} \, g^{\mu\nu} R\right) =$$
$$= -\frac{2}{3} \, \frac{l}{\sqrt{1 + M\psi}} \left(R^{\mu\nu} R_{\nu\rho} - \frac{1}{2} \, R \, R^\mu{}_\rho\right) g^{\rho\sigma} \, \partial_\sigma\psi\,.$$

5 Final remarks

The present theory, in its current form, is still in an elementary state of development. However, as we have seen, the emergence of the electromagnetic field from the quantum evolution of the gravitational field is a remarkable achievement which deserves special attention. On another occasion, we shall expect to expound the structure of the generalized Einstein's equation in the present theory with a generally non-conservative energy-momentum tensor given by

$$T_{\mu\nu} = \pm \frac{c^4}{8\pi G} \left(R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R \right)$$

which, like in the case of self-creation cosmology, seems to allow us to attribute the creation and annihilation of matter directly to the scalar generator of the quantum evolution process, and hence the wave function alone, as

$$\bar{\nabla}_\nu T^{\mu\nu} = -\frac{2}{3} \frac{l}{\sqrt{1 + M\psi}}\, T^{\mu\nu} R_{\nu\rho}\, g^{\rho\sigma}\, \partial_\sigma \psi \neq 0\,.$$

6 Acknowledgements

The author is dearly indebted to Dmitri Rabounski and Stephen J. Crothers for their continuous support and sincere assistance.

Submitted on August 23, 2007
Accepted on September 24, 2007

References

1. Thiemann T. Introduction to Modern Canonical Quantum General Relativity. arXiv: gr-qc/0110034.
2. Barber G. A. The principles of self-creation cosmology and its comparison with General Relativity. arXiv: gr-qc/0212111.
3. Brans C. H. Consistency of field equations in “self-creation” cosmologies. *Gen. Rel. Grav.*, 1987, v. 19, 949–952.