In-Depth Development of Classical Electrodynamics

Yuri N. Keilman
646 3rd Street South East, Valley City, ND 58072, USA
E-mail: yurik6@peoplepc.com

There is hope that a properly developed Classical Electrodynamics (CED) will be able to play a rôle in a unified field theory explaining electromagnetism, quantum phenomena, and gravitation. There is much work that has to be done in this direction. In this article we propose a move towards this aim by refining the basic principles of an improved CED. Attention is focused on the reinterpretation of the E-M potential. We use these basic principles to obtain solutions that explain the interactions between a constant electromagnetic field and a thin layer of material continuum; between a constant electromagnetic field and a spherical configuration of material continuum (for a charged elementary particle); between a transverse electromagnetic wave and a material continuum; between a longitudinal aether wave (dummy wave) and a material continuum.

1 Introduction

The development of Classical Electrodynamics in the late 19th and early 20th century ran into serious trouble from which Classical Electrodynamics was not able to recover (see R. Feynman's *Lectures on Physics* [1]: Volume 2, Chapter 28). According to R. Feynman, this development "ultimately falls on its face" and "It is interesting, though, that the classical theory of electromagnetism is an unsatisfactory theory all by itself. There are difficulties associated with the *ideas* of Maxwell's theory which are not solved by and not directly associated with quantum mechanics". Further in the book he also writes: "To get a *consistent* picture, we must imagine that something holds the electron together", and "the extra non-electrical forces are also known by the more elegant name, the Poincare stresses". He then concludes: "— there have to be other forces in nature to make a consistent theory of this kind". CED was discredited not only by R. Feynman but also by many other famous physicists. As a result the whole of theoretical physics came to believe in the impossibility of explaining the stability of electron charge by classical means, claiming defect in the classical principles. But this is not true.

We showed earlier [2, 3, 4] and further elaborate here that there is nothing wrong with the basic classical *ideas* that Maxwell's theory is based upon. It simply needs further development. The work [2] opens the way to the natural (without singularities) development of CED. In this work it was shown that Poincare's claim in 1906 that the "material" part of the energy-momentum tensor, "Poincare stresses", has to be of a "nonelectromagnetic nature" (see Jackson, [5]) is incorrect. It was shown that the definite material part is expressed only through current desity (see formula (9) in [2]), and given a static solution: Ideal Particle, IP, see (19). The proper covariance of IP is manifest — the charges actually hold together and the energy inside an IP comes from the interior electric field (positive energy) and the interior charge density (negative energy, see formula (22) of [2]). The total energy inside an IP is zero, which means that the rest mass (total energy) corresponds to the vacuum energy only. The contributions to the "inertial mass" (linear momentum divided by velocity; R. Feynman called it "electromagnetic mass") can be calculated by making a Lorentz transformation and a subsequent integration. The total inertial mass is equal to the rest mass (which is in compliance with covariance) but the contributions are different: 4/3 comes from the vacuum electric field, 2/3 comes from the interior electric field, and -1 comes from the interior charge density. This is the explanation of the "anomalous factor of 4/3 in the inertia" (first found in 1881 by J. J. Thomson [5]).

Let us begin with Maxwell's equations:

$$j^i + \frac{c}{4\pi} F^{ik}_{|k} = 0\,, \qquad j^k_{|k} = 0\,; \tag{1}$$

$$\operatorname{div}\vec{E} = \frac{4\pi}{c} j^0\,, \qquad \oint_S \vec{E}\cdot d\vec{S} = \frac{4\pi}{c}\int_V j^0 dV\,; \tag{1a}$$

$$\operatorname{rot}\vec{H} = \frac{4\pi}{c}\vec{j} + \frac{1}{c}\frac{\partial\vec{E}}{\partial t}\,, \qquad \oint_\Gamma \vec{H}\cdot d\vec{\Gamma} = \frac{1}{c}\int_S \left(4\pi\vec{j} + \frac{\partial\vec{E}}{\partial t}\right)\cdot d\vec{S}\,; \tag{1b}$$

$$\frac{1}{c}\frac{\partial j^0}{\partial t} + \operatorname{div}\vec{j} = 0\,, \quad \frac{\partial}{c\partial t}\int_V j^0 dV = \oint_S \vec{j}\cdot d\vec{S}\,. \tag{1c}$$

The other half of Maxwell's equations is

$$F^{*ik}_{|k} = 0\,, \qquad F^{*ik} \equiv \frac{1}{2} e^{iklm} F_{lm}\,; \tag{2}$$

$$\operatorname{div}\vec{H} = 0\,, \qquad \oint_S \vec{H}\cdot d\vec{S} = 0\,; \tag{2a}$$

$$\operatorname{rot}\vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t}; \quad \oint_\Gamma \vec{E}\cdot d\vec{\Gamma} = -\frac{1}{c}\int_S \frac{\partial\vec{H}}{\partial t}\cdot d\vec{S}\,. \tag{2b}$$

The equations are given in 4D form, 3D form, and in an integral form. Equation (1) represents the interaction law between the electromagnetic field and the current density. Equation (2) applies only to the electromagnetic field. This whole system, wherein equation (1c) is not included, is definite for the 6 unknown components of the electromagnetic field on the condition that the currents (all the components) are given. This is the first order PDE system, the characteristics of which are the wave fronts.

What kind of currents can be given for this system? Not only can continuous fields of currents be prescribed. A jump in a current density is a normal situation. We can even go further and prescribe infinite (but the space integral has to be finite) current density. But in this case we have to check the results. In other words, the system allows that the given current density can contain Dirac's delta-functions if none of the integrals in (1) and (2) goes infinite. But this is not the end. There exists an energy-momentum tensor that gives us the energy density in space. The space integral of that density also has to be finite. Here arises the problem. If we prescribe a point charge (3D delta-function) then the energy integral will be infinite. If we prescribe a charged infinitely thin string (2D delta-function) then the energy will also be infinite. But if we prescribe an infinitely thin surface with a finite surface charge density on it (1-d delta-function) then the energy integral will be finite. It appears that this is the only case that we can allow. But we have to remember that it is possible that a **disruption surface** (where the charge/current density can be infinite) can be present in our physical system. This kind of surface allows the electromagnetic field to have a jump across this surface (this very important fact was ignored in conventional CED — see below). It is also very important to understand that all these delta-functions for the charge distribution are at our discretion: we can prescribe them or we can "hold out". If we choose to prescribe then we are taking on an additional responsibility. The major attempt to discredit CED (to remove any "obstacles" in the way of quantum theory) was right here. The detractors of CED (including celebrated names like R. Feynman in the USA and L. D. Landau in Russia but, remarkably, not A. Einstein) tried to convince us that a point charge is inherent to CED. With it comes the divergence of energy and the radiation reaction problem. This problem is solvable for the extended particle (which has infinite degrees of freedom) but is not solvable for the point particle. This is not an indication that the "classical theory of electromagnetism is an unsatisfactory theory by itself". Rather this means that we should not use the point charge model (or charged string model). Only a charged closed surface model is suitable.

We have another serious problem in conventional electrodynamics. As we have shown below, the variation procedure of conventional CED results in the requirement that the electromagnetic field must be continuous across any disruption surface. That actually implies the impossibility of a surface charge/current on a disruption surface. I changed the variation procedure of CED and arrived at a theory where the electromagnetic interaction (ultimately represented by Maxwell's equation (1)) is the only interaction. The so-called interaction term in the Lagrangian ($A^k j_k$) is abandoned. Also abandoned is the possibility introducing any other interactions (like the "strong" or "weak"). I firmly believe that all the experimental data for elementary particles, quantum phenomena, and gravitation can be explained starting only with the electromagnetic interaction (1).

What is the right expression for the energy-momentum tensor that corresponds to the system described by (1) and (2)? The classical principles require that this expression must be unique. Conventional electrodynamics provides us with the expression: $T^{ik} = \mu c u^i u^k \frac{ds}{dt}$ (for a "material" part containing free particles only: see Landau [6], formula 33.5) that contains density of mass, μ, and velocity only. No charge/current density is included. It seems that the mere presence of charge/current density has to contribute to the energy of the system. To correct the situation we took the simplest possible Lagrangian with charge density:

$$\Lambda = -\frac{1}{16\pi}\, g^{ab} g^{cd} F_{ac} F_{bd} - \frac{2\pi}{k_0^2 c^2}\, g^{ab} j_a j_b\,, \tag{3}$$

where k_0 is a new constant. No interaction term (like $A_k j^k$) is included.

2 Variation of metrics

Let us find the energy-momentum tensor that corresponds to the Lagrangian (3). The metric tensor in classical 4-space is $g_{ik} = \mathrm{diag}\,[1, -1, -1, -1]$ (we assume $c = 1$). Let us consider an arbitrary variation of a metric tensor but on the condition that this variation does not introduce any curvature in space. This variation is:

$$\delta g_{ik} = \xi_{i|k} + \xi_{k|i}\,, \tag{4}$$

where ξ^k is an arbitrary but small vector. One has to use the mathematical apparatus of General Relativity to check that with the variation (3) the Riemann curvature tensor remains zero to first order. Assuming that the covariant components of the physical fields are kept constant (then the contravariant components will be varied as a result of the variation of the metric tensor, but we do not use them — see (3) for an explanation) we can calculate the variation of the action. The variation of the square root of the determinant of the metric tensor is: $\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}\, g_{ik}\,\delta g^{ik}$ (this result can be found in textbooks on field theory). The variation of action becomes:

$$\begin{aligned} \delta S &= -\int \left\{ 2\frac{\partial \Lambda}{\partial g^{ik}} - \Lambda g_{ik} \right\} \xi^{i|k} \sqrt{-g}\, d\Omega = \\ &= -\int T_{ik}\, \xi^{i|k} \sqrt{-g}\, d\Omega\,, \end{aligned} \tag{5}$$

where

$$T_{ik} = -\frac{1}{4\pi}\, g^{ab} F_{ia} F_{kb} + \frac{1}{16\pi}\, F_{ab} F^{ab} g_{ik} - \\ - \frac{4\pi}{k_0^2}\, j_i j_k + \frac{2\pi}{k_0^2}\, j_a j^a g_{ik}\,.$$

If our system consists of two regions that are separated by a closed disruption surface S then the above procedure has to be applied to each region separately. We can write: $T_{ik}\xi^{i|k} = (T_{ik}\xi^i)^{|k} - T_{ik}^{|k}\xi^i$. The 4D volume integrals over divergence (the first term) can be expressed through 3D hypersurface integrals according to the 4D theorem of Gauss. The integral over some remote closed surface becomes zero due to the smallness of T_{ik} on infinity (usually assumed). The integral over a 3D volume at t_1 and t_2 becomes zero due to the assumption: $\xi_i = 0$ at these times. What is left is:

$$\delta S = -\int T_i^k \xi^i_{|k} \sqrt{-g}\, d\Omega = \int_S \left(T^k_{i\,\mathrm{out}} - T^k_{i\,\mathrm{in}}\right) \xi^i dS_k + \\ + \int_{\mathrm{in}} T^k_{i\,|k}\, \xi^i \sqrt{-g}\, d\Omega + \int_{\mathrm{out}} T^k_{i\,|k}\, \xi^i \sqrt{-g}\, d\Omega\,.$$

Since ξ_i are arbitrary small functions (between t_1 and t_2), the requirement $\delta S = 0$ yields:

$$T^{ia}_{|a} = 0\,. \tag{6}$$

This condition has to be fulfilled for the inside and the outside regions separately. And the additional requirement on the disruption surface S,

$$T^{ia} N_a, \tag{6a}$$

is continuous, where N_k is a normal to the surface.

We have found the unique definition of the energy-momentum tensor (5). If we want the action to be minimum with respect to the arbitrary variation of the metric tensor in flat space then (6) and (6a) should be satisfied. Let us rewrite the energy-momentum tensor in 3D form:

$$\left.\begin{aligned} T^{00} &= \frac{1}{8\pi}\left(E^2 + H^2\right) - \frac{2\pi}{k_0^2 c^2}\left[(j^0)^2 + (\vec{j})^2\right] \\ T^{11} &= \frac{1}{8\pi}\left(E^2 + H^2 - 2E_1^2 - 2H_1^2\right) - \\ &\quad - \frac{2\pi}{k_0^2 c^2}\left[(j^0)^2 - (\vec{j})^2 + 2(j^1)^2\right] \\ T^{01} &= \frac{1}{4\pi}\left(E_2 H_3 - E_3 H_2\right) - \frac{4\pi}{k_0^2 c^2}\, j^0 j^1 \\ T^{12} &= -\frac{1}{4\pi}\left(E_1 E_2 + H_1 H_2\right) - \frac{4\pi}{k_0^2 c^2}\, j^1 j^2 \end{aligned}\right\}. \tag{5a}$$

Notice that we have not used Maxwell's or any other field equations so far. It should also be noted that for the energy-momentum tensor (5), (5a) is not defined on the disruption surface itself, despite the fact that there can be a surface charge/current on a surface (infinite volume density but finite surface density).

Going further, **we are definitely stating that Maxwell's equation (1) is a universal law that should be fulfilled in all space without exceptions. It defines the interaction between the electromagnetic field and the field of current density. This law cannot be subjected to any variation procedure**. Maxwell's equation (2) we will confirm later as a result of a variation; see formula (9). Substituting (5) in (6) and using Maxwell's equation (1) and the antisymmetry of F_{ik}, we obtain:

$$\left.\begin{aligned} & j^a\left(\frac{k_0^2 c}{4\pi}\, F_{ai} + j_{a|i} - j_{i|a}\right) = 0 \\ & j^0\left(\frac{k_0^2 c}{4\pi}\, \vec{E} + \nabla j^0 + \frac{1}{c}\frac{\partial \vec{j}}{\partial t}\right) + \\ & \qquad + \vec{j} \times \left(\frac{k_0^2 c}{4\pi}\, \vec{H} - \mathrm{rot}\, \vec{j}\right) = 0 \end{aligned}\right\}. \tag{7}$$

This equation has to be fulfilled for the inside and outside regions separately because (6) is fulfilled separately in these regions. This is important. It is also important to realize that while the conservation of charge is fulfilled everywhere, including a disruption surface, the disruption surface itself is exempt from energy-momentum conservation (no surface energy, no surface tension). This arrangement is in agreement with the fact that we can integrate a delta-function (charge) but we cannot integrate its square (would be energy).

3 A new dynamics

Equation (7) we call a **Dynamics Equation.** It is a nonlinear equation. But it has to be fulfilled inside and outside the particle separately. This will allow us to reduce it to a linear equation inside these regions.

Definition: vacuum is a region of space where all the components of current density are zero.

Equation (7) is automatically satisfied in vacuum ($J^k = 0$). The other possibility ($J^k \neq 0$) will be the interior region of an elementary particle. The boundary between these regions will be a disruption surface. Inside the particle instead of (7) we have:

$$\left.\begin{aligned} & \frac{k_0^2 c}{4\pi}\, F_{ai} + j_{a|i} - j_{i|a} = 0 \\ & \frac{k_0^2 c}{4\pi}\, \vec{E} + \nabla j^0 + \frac{1}{c}\frac{\partial \vec{j}}{\partial t} = 0\,, \quad \frac{k_0^2 c}{4\pi}\, \vec{H} - \mathrm{rot}\, \vec{j} = 0 \end{aligned}\right\}. \tag{7a}$$

All the solutions of equation (7a) are also solutions of the nonlinear equation (7). At present we know nothing about the solutions of (7) that do not satisfy (7a). Inside the elementary particle the dynamics equation (7) or (7a) describes, as we call it, a **Material Continuum**. A Material Continuum cannot be divided into a system of material points. The

Relativistic (or Newtonian) Dynamics Equation of CED, that describes the behavior of the particle as a whole, completely disappears inside the elementary particle. There is no mass, no force, no velocity or acceleration inside the particle. The field of current density j^k defines a kinematic state of the Material Continuum. A world line of current j^k is not a world line of a material point. That allows us to deny any causal connection between the points on this line. In consequence, j^k can be space-like as well as time-like. That is in no contradiction with the fact that the boundary of the particle cannot exceed the speed of light. Equation (7a) is linear and allows superposition of different solutions. Using (1) we can obtain:

$$\left.\begin{array}{l} j^{k|a}_{|a} - k_0^2\, j^k = 0; \qquad j^k + k_0^2\, j^k = 0 \\ \Delta j^k - \dfrac{1}{c^2}\dfrac{\partial^2 j^k}{\partial t^2} + k_0^2\, j^k = 0 \end{array}\right\}. \tag{7b}$$

By equation (7) we have obtained something very important, but we are just on the beginning of a difficult and uncertain journey. Now the current density cannot be prescribed arbitrarily. Inside the particle it has to satisfy equation (7b). However, there are no provisions on the surface current density (if a surface current is different from zero then its density is necessarily expressed by a delta-function across the disruption surface).

4 The electromagnetic potential

Now we are going to vary the electromagnetic field F_{ik} in all the space, including a disruption surface. As usual, the variation is kept zero at t_1 and t_2 and also on a remote closed surface, at infinity. In this case the results of variation will be in force on the disruption surface itself. Still, we have to write the variation formulae for each region separately. We claim that equation (1) cannot be subjected to variation. It is the preliminary condition before any variation. In our system we have 10 unknown independent functions (4 functions in J_k and 6 functions in F_{ik}). These functions already have to satisfy 8 equations: 4 equations in (1) and 4 equations in (7). We have only 2 degrees of freedom left. We cannot vary F_{ik} by a straightforward procedure. Let us employ here the Lagrange method of indefinite factors. Let us introduce a modified Lagrangian:

$$\Lambda' = \Lambda + A^a \left(j_a + \frac{1}{4\pi} F^{|b}_{ab} \right), \tag{8}$$

where A^k are 4 indefinite Lagrange factors. Now we have $2+4=6$ degrees of freedom and we use them to vary F_{ik}. We have:

$$\delta S = -\int \left\{ \frac{\partial \Lambda'}{\partial F_{ik}}\, \delta F_{ik} + \frac{\partial \Lambda'}{\partial F_{ik|l}}\, \delta F_{ik|l} \right\} dV_4 =$$

$$-\int \left\{ \left(\frac{\partial \Lambda'}{\partial F_{ik|l}} \delta F_{ik} \right)_{|l} + \left[\frac{\partial \Lambda'}{\partial F_{ik}} - \left(\frac{\partial \Lambda'}{\partial F_{ik|l}} \right)_{l} \right] \delta F_{ik} \right\} dV_4 = 0.$$

The first term under integration is divergence and can be transformed to the hypersurface integral according to Gauss theorem. Since the variation is arbitrary, the square brackets term has to be zero in either case. It gives:

$$F_{ik} = A_{k|i} - A_{i|k}\,. \tag{9}$$

If V_4 is the inside region of the particle from t_1 to t_2 then the hypersurface integrals at t_1 and t_2 will be zero, but the hypersurface integral over the closed disruption surface will be

$$\frac{1}{4\pi} \int dt \oint \left(A^i g^{kl} - A^k g^{il} \right)_{\text{in}} \delta F_{ik}\, dS_l\,.$$

If V_4 is the outside vacuum then the hypersurface integrals at t_1 and t_2 will be zero. The hypersurface integral over the remote closed surface will be zero, but the hypersurface integral over the disruption surface will be

$$-\frac{1}{4\pi} \int dt \oint \left(A^i g^{kl} - A^k g^{il} \right)_{\text{out}} \delta F_{ik}\, dS_l\,.$$

These integrals will annihilate if the **potential A^k is continuous across the disruption surface**. The continuity of potential does not preclude the possibility of a surface charge/current and a jump of electromagnetic field as a consequence.

Claim: The variation procedure of conventional CED results in the impossibility of a surface charge/current on a disruption surface. The variation procedure of conventional CED begins with equation (9) replacing the electromagnetic field with a potential. It introduces the interaction term $A^k j_k$ in the Lagrangian and varies the potential δA^k. As a result of the least action it obtains Maxwell's equation (1). But it can be shown that the consideration of a disruption surface will produce the requirement of electromagnetic field continuity. This actually denies the possibility of a single layer surface charge/current (the double layers are not interesting and they will require the jump of potential and infinite electromagnetic field). Therefore, the conventional variation procedure is incorrect.

5 The physical meaning of potential

Now we learned that the electromagnetic potential, which was devoid of a physical meaning, has to be continuous across all the boundaries of disruption. This is a very important result. It allows me to reinterpret the physical meaning of potential. It is true that according to (9) we can add to the potential a gradient of some arbitrary function and the electromagnetic field won't change (gauge invariance). Yes, but this fact can be given another interpretation: **the potential is unique and it actually contains more information about physical reality than the electromagnetic field does**. To make the potential mathematically unique, besides initial data and boundary conditions we need only to impose the conservation equation (formerly Lorenz gauge).

$$A^{k}_{|k} = 0\,, \qquad A^{k|a}_{|a} = \frac{4\pi}{c}\, j^{k}, \qquad \Box A^{k} = -\frac{4\pi}{c}\, j^{k}. \quad (10)$$

This is true everywhere. Using (1), (7a), and (9) we can conclude that inside a material continuum the potential has to satisfy:

$$\left(A^{k}_{|b}{}^{|b} - k_0^2 A^{k}\right)^{|i} - \left(A^{i}_{|b}{}^{|b} - k_0^2 A^{i}\right)^{|k} = 0\,. \quad (7c)$$

If the equation:

$$A^{k}_{|b}{}^{|b} - k_0^2 A^{k} = 0 \quad \text{or} \quad \Box A^{k} + k_0^2 A^{k} = 0\,, \quad (11)$$

$$\Box \equiv \Delta - \frac{\partial^2}{c^2\,\partial t^2}$$

is satisfied then (7c) also satisfied. This type of equation is satisfied by the current density, see (7b). This equation can be called the "Generalized Helmholtz Equation". In static conditions (11) coincides with the Helmholtz equation. Equation (11) differs from the Klein-Gordon equation by the sign before the square of a constant.

The new interpretation of potential: A^0 represents the aether quantity (positive or negative), the 3-vector $\vec{A}$ represents the aether current. All together: the potential uniquely describes the existing physical reality — the aether. In general, the interpretation of potential doubles the interpretation of current.

6 The implications of the re-interpretation of potential

Let us suppose that the potential is equal to a gradient of some function G, which we call a "dummy generator":

$$\left.\begin{array}{l} A^{k} = g^{ka} G_{|a}\,, \quad A_0 = \dfrac{1}{c}\dfrac{\partial G}{\partial t}\,, \quad \vec{A} = -\nabla G \\[2ex] G^{|a}_{|a} = 0\,, \qquad \Delta G - \dfrac{1}{c^2}\dfrac{\partial^2 G}{\partial t^2} = 0 \end{array}\right\}, \quad (12)$$

G has to be the solution of a homogeneous wave equation. However, there are no requirements for G on a disruption surface that we know of at present. But now we won't say that G is devoid of a physical meaning (remember the mistake we made with potential).

What kind of a physical process is described here by the corresponding potential? There is no electromagnetic field and the energy-momentum tensor is equal to zero. These are the "dummy waves" — the longitudinal aether waves. These waves are physically significant only due to the boundary conditions on the disruption surfaces, which they affect. If this is the case, then G can be significant in physical experiment. It can be even unique under the laws (these laws are not completely clear) of another physical realm (realm of electromagnetic potential).

It is difficult to imagine an elementary particle without some oscillating electromagnetic field inside it. If we assume that the oscillating field is present inside the particle then the boundary conditions may require the corresponding oscillating electromagnetic field in vacuum that surrounds the particle. It is easy to show that the energy of this vacuum electromagnetic field will be infinite. However, it is possible that in vacuum only waves of the scalar potential take care of the necessary boundary conditions. Since the potential is not present in the energy-momentum tensor (5), there won't be any energy connected with it. **We are free to suggest that the massive elementary particles are the sources of these waves**. These waves are emitted continuously with an amplitude (or its square) that is proportional to the mass of the particle (this proposition seems to be reasonable). These waves are only outgoing waves. The incoming waves can only be plane incoherent waves (the spherical incoming coherent waves are impossible). We are not considering any incoming waves at this point.

We now show, by some examples, that the concept of the material continuum really works.

7 Obtaining solutions

Fortunately, all the equations for finding the solutions are linear. That allows us to seek a total solution as a superposition of the **particular solutions** which satisfy the equations and the boundary conditions separately. The only unlinear condition is (6a), which has to be fulfilled only on the disruption surface. Only the total solution can be used in (6a).

IP2 (Ideal Particle Second): Let us obtain the simplest static spherically symmetric solution with electric charge and electric field only. We have:

$$\left.\begin{array}{l} A^0_{\rm in} = \alpha\big(R_0(z) - R_0(z_1) + b z_1\big) \\[2ex] 0 \leqslant z \leqslant z_1\,, \qquad j^0 = \dfrac{k_0^2\, c}{4\pi}\,\alpha R_0(z) \end{array}\right\}, \quad (13a)$$

$$\left.\begin{array}{l} A^0_{\rm out} = \alpha\, b\, \dfrac{z_1^2}{z}\,, \qquad z_1 \leqslant z < \infty \\[2ex] b \equiv \sqrt{R_0^2(z_1) + R_1^2(z_1)}\,, \quad z = k_0 r \end{array}\right\}, \quad (13b)$$

$$\left.\begin{array}{l} E^r_{\rm in} = \alpha\, k_0\, R_1(z)\,, \qquad 0 \leqslant z \leqslant z_1 \\[2ex] E^r_{\rm out} = \alpha\, k_0\, b\, \dfrac{z_1^2}{z^2}\,, \qquad z_1 \leqslant z < \infty \end{array}\right\}, \quad (13c)$$

$$Q_{\rm tot} = \frac{\alpha}{k_0}\, z_1^2\, b\,, \quad Q_{\rm surf} = \frac{\alpha}{k_0}\, z_1^2\big(b - R_1(z_1)\big), \quad (13d)$$

$$mc^2 = \frac{\alpha^2}{2k_0}\big(-z_1^2\, R_0(z_1)\, R_1(z_1) + z_1^3\, R_0^2(z_1) + z_1^3\, R_1^2(z_1)\big) = \frac{\alpha^2}{2k_0}\big(z_1 - \sin z_1 \cos z_1\big), \quad (13e)$$

where $R_0(z)$ and $R_1(z)$ are spherical Bessel functions. In general, the electric field has a jump at the boundary of IP2. The position of the boundary z_1 is arbitrary, but only at $z_1 = n\pi$ (correspond to IP1) the surface charge is zero and the electric field continuous. The first term in the mass expression (with the minus sign) corresponds to the energy of the interior region of the particle. It can be positive or negative, depending on z_1 (at $z_1 = n\pi$ it is zero). The second and third terms together represent the vacuum energy, which is positive. The total energy/mass remains positive at all z_1.

It was confirmed that IP is an unstable "equilibrium". Given a small perturbation it will grow in time. We hope to find a stable solution among the more complicated solutions than IP. The first idea was to introduce a spin in a static solutions. Then we tried to introduce the steady-state oscillating solutions. It was confirmed that there exist oscillating solutions with oscillating potential in vacuum that does not produce any vacuum E-M field. Then we tried to introduce a spin that originates from the oscillating solutions. Also we tried to consider the cylindrically shaped particles that are moving with the speed of light (close to a photon, see [3]). All these attempts indicate that the boundary of a particle that separates the material continuum from vacuum is a key player in any solution.

8 The mechanism of interaction between a constant electric field and a static charge (simplified thin layer model)

The simplest solutions can be obtained in plane symmetry where all the physical quantities depend only on the third coordinate — z. Let us consider symmetry of the type, vacuum — material continuum — vacuum. The thin layer of material continuum from $z = 0$ to $z = a$ (a is of the order of the size of elementary particle) will represent a simplified model of an elementary particle. The boundaries at $z = 0$ and $z = a$ are deemed to be enforced by the particle and the whole deficit of energy or momentum on these boundaries is deemed to go directly to the particle. Actually, if we have a deficit of energy or momentum it means that we are missing a particular solution that brings this deficit to zero, according to (6a).

For further discussion we need to write down the integral form of the energy-momentum conservation:

$$\frac{\partial}{c\partial t}\int\limits_V T^{m0}dV = -\oint\limits_\Sigma T^{mq}d\Sigma_q\,, \tag{6b}$$

where V is a 3D volume (which is not moving — it is our choice), and Σ is a 3D closed surface around this volume (obviously also not moving). The index m can correspond to any coordinate, while the index q corresponds only to the terrestrial coordinates (1, 2, 3). If $m = 0$ then the left part of (6b) is the time rate of increase of the energy inside V. T^{0q} is the 3-dimensional Pointing vector (or the flow of energy through a square unit per unit of time). If $m = 3$ (in the plane symmetry only one coordinate is of interest) then the left part of (6b) is the time rate of increase of the linear momentum of the volume V (actually it is a force applied to the volume V). T^{3q} is the 3-vector (in general q can be 1, 2, 3; in our case $q = 3$) of the flow of linear momentum through a square unit per unit of time. It is obvious that when static (or in a steady state) the left part of (6b) must be zero if there is no source/drain of energy/linear momentum inside the said volume.

Suppose the constant electric field in the first vacuum region is E. The scalar potential (aether quantity), the electric field, and the charge density are:

$$\left.\begin{aligned}
&\Phi_1 = -Ez + C_1\,, \qquad E_1 = E\\
&\Phi_2 = -\frac{E}{k_0}\sin k_0 z + C_1\cos k_0 z\\
&C_1 = \frac{4\pi Q + E(1-\cos k_0 a)}{k_0\sin k_0 a}\\
&E_2 = E\cos k_0 z + k_0 C_1 \sin k_0 z\\
&\rho = \frac{k_0^2}{4\pi}\,\Phi_2\,, \quad \Phi_3 = -(E+4\pi Q)(z-a) + C_2\\
&C_2 = \frac{4\pi Q\cos k_0 a - E(1-\cos k_0 a)}{k_0 \sin k_0 a}\\
&E_3 = E + 4\pi Q
\end{aligned}\right\}. \tag{14}$$

Here the charge density is the solution of (7b) inside the second region. The potentials are the solutions of (10). All the physical quantities except ρ are continuous on the boundaries. That means that the jumps of the components of the energy-momentum tensor will be due to the jumps of the charge density only. The energy momentum tensor in this symmetry (and this particular case) is:

$$\left.\begin{aligned}
&T^{00} = \frac{1}{8\pi}\,E^2 - \frac{2\pi}{k_0^2}\,\rho^2 = T^{11} = T^{22}\\
&T^{03} = 0\,, \quad T^{33} = -\frac{1}{8\pi}\,E^2 - \frac{2\pi}{k_0^2\,c}\,\rho^2
\end{aligned}\right\}. \tag{15}$$

There is no energy flow in this system, but there is a flow of linear momentum. In the first vacuum region it is: $T^{33} = -E^2/8\pi$. Then it jumps on the first and on the second boundaries:

$$\left.\begin{aligned}
&T^{33}(z=0+) - T^{33}(z=0-) = -\frac{k_0^2\,C_1^2}{8\pi}\\
&T^{33}(z=a+) - T^{33}(z=a-) = \frac{k_0^2\,C_2^2}{8\pi}\\
&\frac{k_0^2\,C_1^2}{8\pi} - \frac{k_0^2\,C_2^2}{8\pi} = Q\left(E + \frac{4\pi Q}{2}\right)
\end{aligned}\right\}. \tag{16}$$

After that it is: $T^{33} = -(E+4\pi Q)^2/8\pi$. As we go from left to right the jump on the first boundary is negative. That means that the small volume that includes the first boundary gets negative outside (we always consider the outside normal to the closed surface Σ) flow of linear momentum. That means that the volume itself, according to expression (6b), gets the positive rate of linear momentum, which is the force in the positive direction of the z-axis. The first boundary is pushed in the positive direction of the z-axis. The second boundary is also pushed, but in the negative direction of the z-axis. The difference is exactly equal to the force with which the field acts on a particle; see (16). We see that electric field does not act on a charge *per se* but only on a whole particle and only through its boundaries. This picture is true only at $t = 0$ because the missing particular solution that makes the appearance of "free" sources and drains most definitely will depend on time (the particle will begin to accelerate). This is the actual success of the proposed modification of CED.

9 The mechanism of interaction between a constant electric field and a static spherical charge

Here we will confirm that the thin layer treatment corresponds to the more accurate but more complicated spherical charge treatment. Suppose we have a constant electric field E directed along the z-axis in vacuum. Also we have a sphere of radius r_1 that separates the material continuum inside the sphere, from vacuum. The situation is static at $t = 0$. The potential in general has to satisfy the equation $A^k_{|k} = 0$ (10) everywhere, and equation (7c) inside the material continuum. This last equation, with 3rd derivatives, has to be satisfied strictly inside a material continuum and not on the disruption surface itself (where a single layer of charge/current density is possible and the charge/current density, $j^k = \frac{c}{4\pi} A^{k\,|a}_{|a}$, can be infinite). In vacuum we have

$$A^{k\,|a}_{|a} = 0\,. \tag{17}$$

Let us define a "dummy" potential by:

$$\left.\begin{aligned} &D^k_{|k} = 0\,, \quad D^{i|k} - D^{k|i} = 0\,, \\ &\text{consequently:} \quad D^{k\,|a}_{|a} = 0\,. \end{aligned}\right\} \tag{18}$$

If we have a solution A^k of (10)+(7c) or a solution of (10)+(17) then $A^k + D^k$ will also be the solution of the same equations (it does not matter whether inside the material continuum or in vacuum).

Now we return to our particular case. The solution of (18) that we are interested in would be: $D^0 = \text{const}$. If there is no time dependence then (10) is satisfied for any A^0 if a vector potential is zero. Equation (7c) is a Laplace operator taken from a Helmholtz equation. The solutions of the Helmholtz equation being considered would be: $R_0(k_0 r)$ and $R_1(k_0 r)\cos\theta$ where R_n are the spherical Bessel functions. In vacuum we consider the solutions e/r, (where e is the total charge), $r\cos\theta$, and $(1/r^2)\cos\theta$. So, let us consider the potential

$$\left.\begin{aligned} A^0_{\text{in}} &= \alpha R_0(k_0 r) + \frac{e}{r_1} - \alpha R_0(k_0 r_1) \\ A^0_{\text{out}} &= \frac{e}{r} + E\left(\frac{r_1^3}{r^2} - r\right)\cos\theta \end{aligned}\right\}. \tag{19}$$

It is continuous at $r = r_1$. The corresponding electric field and charge density will be,

$$\left.\begin{aligned} &E_{r\ \text{in}} = \alpha k_0 R_1(k_0 r) \\ &E_{r\ \text{out}} = \frac{e}{r^2} + E\left(1 + 2\frac{r_1^3}{r^3}\right)\cos\theta \\ &E_{\theta\ \text{in}} = 0\,, \quad E_{\theta\ \text{out}} = E\left(\frac{r_1^3}{r^3} - 1\right)\sin\theta \\ &\rho = \frac{\alpha k_0^2}{4\pi} R_0(k_0 r) \end{aligned}\right\}. \tag{20}$$

We see that the radial component of the electric field has a jump while the θ component is continuous. The surface charge density and the total surface charge are:

$$\left.\begin{aligned} 4\pi\rho_{\text{surf}} &= -E_{r\ \text{in}}(r_1) + E_{r\ \text{out}}(r_1) = \\ &= \frac{e}{r_1^2} - \alpha k_0 R_1(k_0 r_1) + 3E\cos\theta \\ Q_{\text{surf tot}} &= e - \alpha k_0 r_1^2 R_1(k_0 r_1) \end{aligned}\right\}. \tag{21}$$

We see that it does not matter what the relation is between the constants α and e, the surface of the particle has a "surface charge polarization" $3E\cos\theta$. Only this polarization will result in the net force on the charge. The polarization in the volume of the particle can be introduced using the solution $R_1(k_0 r)\cos\theta$. But this polarization won't change the net force (it can be introduced with any constant factor). We've made the corresponding calculations that support this statement. We do not present them here, for simplification.

The double radial component of the energy-momentum tensor will be:

$$\left.\begin{aligned} &8\pi T^{rr} = E_\theta^2 - E_r^2 - \frac{16\pi^2}{k_0^2}\rho^2 \\ &8\pi T^{rr}_{\text{surf in}} = -\alpha^2 k_0^2 \left(R_0^2(k_0 r_1) + R_1^2(k_0 r_1)\right) \\ &8\pi T^{rr}_{\text{surf out}} = -\left(\frac{e^2}{r_1^4} + \frac{6e}{r_1^2} E\cos\theta + 9E^2\cos^2\theta\right) \\ &T^{\theta r}_{\text{surf in}} = T^{\theta r}_{\text{surf out}} = 0 \end{aligned}\right\}. \tag{22}$$

The force applied to the surface will be normal to the surface and equal to $T_{\text{surf in}} - T^{rr}_{\text{surf out}}$. This force is zero if $E = 0$. This case corresponds to the true static solution of our equations with (6a) satisfied. This solution enforces the spherical boundary. If E is not zero, then we do not know the actual solution because (6a) is not satisfied. The actual solution will be not static. But we can calculate the force at the moment when E was "turned on". To get the z component of this force we have to multiply the expression on $\cos\theta$. If we integrate this over the spherical surface then all the terms except the one with $\cos\theta$ are zero. The result of integration will be eE. This is exactly the force with which the electric field E acts on a charge e.

10 The transverse electromagnetic wave

Let us consider that the transverse electromagnetic wave is coming from the left and encounters the layer of material continuum. We expect to find the transmitted and reflected waves as well as the radiation pressure. "Behind" the transverse E-M wave we find that the transverse aether wave with only an x component (for x-polarized E-M wave) of the vector potential (aether current) is different from zero:

$$\left.\begin{aligned}
&{}_1A^1 = \Phi_1^+ + \Phi_1^- \\
&\Phi_1^+ = F_1^+ e^{-ikz}, \quad \Phi_1^- = F_1^- e^{ikz} \\
&{}_1E_1 = -ik \cdot {}_1A^1, \quad {}_1H_2 = -ik \cdot \left(\Phi_1^+ - \Phi_1^-\right) \\
&{}_2A^1 = \Phi_2^+ + \Phi_2^- \\
&\Phi_2^+ = F_2^+ e^{-ik'z}, \quad \Phi_2^- = F_2^- e^{ik'z} \\
&k = \frac{\omega}{c}, \quad (k')^2 = k_0^2 + k^2 \\
&{}_2E_1 = -ik \cdot {}_2A^1, \quad {}_2H_2 = -ik' \cdot \left(\Phi_2^+ - \Phi_2^-\right) \\
&j\,(z,t) = \frac{c\,k_0^2}{4\pi} \cdot {}_2A^1 \\
&{}_3A^1 = F_3^+ e^{-ikz} \\
&{}_3E_1 = -ik \cdot {}_3A^1, \quad {}_3H_2 = -ik \cdot {}_3A^1
\end{aligned}\right\}, \tag{23}$$

where the prefixes to the fields always denote the number of the region (we did not attach indexes to the current density j because it is different from zero only in the second region). We assume that all the functions depend on t through the factor $\exp(i\omega t)$. In the first region the given incoming wave F_1^+ and some reflected wave F_1^- are present. In the second region two waves are present. They satisfy the equations:

$$ {}_2A^{1\prime\prime} + k^2 \cdot {}_2A^1 = -\frac{4\pi}{c}\,j\,, \qquad \frac{\partial}{\partial x} \cdot {}_2A^1 = 0\,. \tag{24}$$

On the boundaries the vector potential (aether current) and its first derivative have to be continuous. We found that

$$\left.\begin{aligned}
&F_1^- = -F_1^+ \frac{2i\,k_0^2 \sin(k'a)}{D} \\
&F_3^+ e^{-ika} = F_1^+ \frac{4\,k\,k'}{D} \\
&D \equiv (k+k')^2 e^{ik'a} - (k-k')^2 e^{-ik'a} \\
&F_2^+ = F_1^+ \frac{2k\,(k+k')}{D}\, e^{ik'a} \\
&F_2^- = F_1^+ \frac{2k\,(k'-k)}{D}\, e^{-ik'a}
\end{aligned}\right\}. \tag{25}$$

Here we found the amplitudes of reflected and transmitted waves and the amplitudes of both waves in the second region (only F_1^+ is considered to be real and given).

We found previously that the energy-momentum tensor in a material continuum has the form (one-dimensional symmetry assumed):

$$\left.\begin{aligned}
&T^{00} = \frac{1}{8\pi}\left(E^2 + H^2\right) - \frac{2\pi}{k_0^2\,c^2}\,j^2 \\
&T^{33} = \frac{1}{8\pi}\left(E^2 + H^2\right) + \frac{2\pi}{k_0^2\,c^2}\,j^2 \\
&T^{11} = \frac{1}{8\pi}\left(-E^2 + H^2\right) - \frac{2\pi}{k_0^2\,c^2}\,j^2 = -T^{22} \\
&T^{03} = \frac{1}{4\pi}\,EH
\end{aligned}\right\}. \tag{26}$$

Since we use complex numbers — we have to take the real parts of the physical values, multiply them and then take the time average. The result will be the real part of the product of the first complex amplitude on the conjugate of the second complex amplitude. The result in the second region is:

$$\left.\begin{aligned}
&\frac{2\pi}{k_0^2\,c^2}\,j^2 = F_1^{+2}\,\frac{k_0^2\,k^2}{\pi|D|^2}\,\times \\
&\qquad \times \left(k_0^2 + 2k^2 + k_0^2 \cos 2k'(a-z)\right) \\
&T^{00} = -F_1^{+2}\,\frac{2k^2}{\pi|D|^2}\,\times \\
&\qquad \times \left(k_0^4 \cos 2k'(a-z) - k^2\,(k_0^2 + 2k^2)\right) \\
&T^{03} = F_1^{+2}\,\frac{4k^4 k'^2}{\pi|D|^2} \\
&T^{33} = F_1^{+2}\,\frac{2k^2 k'^2}{\pi|D|^2}\left(k_0^2 + 2k^2\right)
\end{aligned}\right\}. \tag{27}$$

The electric and magnetic fields are continuous in this system. The flow of energy appear to be independent of z in the second region. It is continuous on the boundaries (see (26); the currents are not included in T^{03}). This means that it is constant through the whole system. The flow of linear momentum (T^{33}) is positive in the first region and then jumps up on the first boundary due to the jump of the current j. It means that the surface integral in (6b) is positive and the first boundary is losing linear momentum. The surface is pulled in the negative direction of the z-axis. But this pull is less than another pull due to the jump on the second boundary; this can be determined from (27). We consider $k'a \sim \frac{\pi}{4}$, but it will be true for any $k'a$ different from π. Notice also that at $k'a = \pi$ the reflected wave is zero as can be seen from (25). Thus, the material continuum will experience the force (through its boundaries) in the positive direction of the z-axis. The numerical value of this force can be calculated from the jumps and it is equal to the force that we usually calculate from the linear momentum of incident transmitted and reflected waves.

11 The longitudinal aether (dummy) wave

Let us consider a longitudinal aether wave travelling from the left, encountering the layer of material continuum. There are no electromagnetic fields that accompany this wave in vacuum. Not so inside the material continuum. We have:

$$\left.\begin{aligned}
&{}_1A^0 = \Phi_1^+ + \Phi_1^- \\
&\Phi_1^+ = F_1^+ e^{-ikz}, \qquad \Phi_1^- = F_1^- e^{ikz}, \quad k = \frac{\omega}{c} \\
&{}_1A^3 = \Phi_1^+ - \Phi_1^-, \quad {}_2A^0 = \Phi_2^+ + \Phi_2^- \\
&\Phi_2^+ = F_2^+ e^{-ik'z}, \qquad \Phi_2^- = F_2^- e^{ik'z} \\
&{}_2A^3 = \frac{k}{k'}\left(\Phi_2^+ - \Phi_2^-\right), \quad (k')^2 = k_0^2 + k^2 \\
&j^0(z,t) = \frac{ck_0^2}{4\pi}\left(\Phi_2^+ + \Phi_2^-\right) \\
&j^3(z,t) = \frac{ck_0^2 k}{4\pi k'}\left(\Phi_2^+ - \Phi_2^-\right) \\
&E_3 = \frac{i k_0^2}{k'}\left(\Phi_2^+ - \Phi_2^-\right), \quad {}_3A^0 = {}_3A^3 = F_3^+ e^{-ikz}
\end{aligned}\right\} \qquad (28)$$

where we assume that all the functions depend on t through the factor $\exp(i\omega t)$. In the first region the given incoming wave F_1^+ and some reflected wave F_1^- are present (both are dummy waves). In the second region two waves are present. They satisfy the equations:

$$\left.\begin{aligned}
&{}_2A^{0\prime\prime} + k^2 \cdot {}_2A^0 = -\frac{4\pi}{c}\, j^0 \\
&{}_2A^{3\prime\prime} + k^2 \cdot {}_2A^3 = -\frac{4\pi}{c}\, j^3 \\
&ik \cdot {}_2A^0 + {}_2A^{3\prime} = 0\,.
\end{aligned}\right\}. \qquad (29)$$

To define all the waves we have to satisfy the conditions on the boundaries. The scalar potential (aether quantity) and the vector potential (aether current) should be continuous across the boundaries. We found that

$$\left.\begin{aligned}
&\text{on } z = a: \quad F_2^+ e^{-ik'a} = \frac{k + k'}{2k}\, F_3^+ e^{-ika} \\
&\qquad\qquad F_2^- e^{ik'a} = -\frac{k' - k}{2k}\, F_3^+ e^{-ika} \\
&\text{on } z = 0: \quad F_1^- = F_1^+ \frac{2ik_0^2 \sin(k'a)}{D} \\
&\qquad\qquad F_3^+ e^{-ika} = F_1^+ \frac{4kk'}{D} \\
&D \equiv (k + k')^2 e^{ik'a} - (k' - k)^2 e^{-ik'a} \\
&F_2^+ = F_1^+ \frac{2k'(k + k')}{D}\, e^{ik'a} \\
&F_2^- = -F_1^+ \frac{2k'(k' - k)}{D}\, e^{-ik'a}
\end{aligned}\right\}. \qquad (30)$$

Here we found the amplitudes of reflected and transmitted waves and the amplitudes of both waves in the second region (only F_1^+ is considered to be real).

From (28) we can calculate the derivatives:

$$\left.\begin{aligned}
&{}_1A^{0\prime} = -ik \cdot {}_1A^3, \quad {}_2A^{0\prime} = -\frac{ik'^2}{k} \cdot {}_2A^3 \\
&{}_1A^{3\prime} = -ik \cdot {}_1A^0, \quad {}_2A^{3\prime} = -ik \cdot {}_2A^0
\end{aligned}\right\}. \qquad (28a)$$

We see that the aether current (A^3) has a continuous derivative while the derivative of aether quantity (A^0) has a jump at the boundaries. This means that there are surface charges associated with the boundaries.

We notice from (28) that the electric field, charge density, and current density are different from zero inside the second region. This means that the material continuum produces a kind of physical response to the energy-less dummy waves. We also found previously that the energy-momentum tensor in a material continuum has the form (one-dimensional symmetry assumed),

$$\left.\begin{aligned}
&T^{00} = \frac{1}{8\pi}E^2 - \frac{2\pi}{k_0^2 c^2}\left(c^2\rho^2 + j^2\right) \\
&T^{11} = -\frac{1}{8\pi}E^2 - \frac{2\pi}{k_0^2 c^2}\left(c^2\rho^2 + j^2\right) \\
&T^{22} = T^{33} = \frac{1}{8\pi}E^2 - \frac{2\pi}{k_0^2 c^2}\left(c^2\rho^2 - j^2\right) \\
&T^{01} = -\frac{4\pi}{k_0^2 c^2}\, c\rho j
\end{aligned}\right\}. \qquad (31)$$

To actually calculate a time average of the energy-momentum tensor we have to take the real parts of the physical values, multiply them, and then take the time average. The result will be the real part of the product of the first complex amplitude on the conjugate of the second complex amplitude. The result of calculation is,

$$\left.\begin{aligned} T^{33} &= -(F_1^+)^2\,\frac{2\,k_0^4}{\pi|D|^2}\,(k_0^2+2k^2) \\ T^{03} &= -(F_1^+)^2\,\frac{4\,k_0^2\,k^2\,k'^2}{\pi|D|^2} \\ T^{00} &= (F_1^+)^2\,\frac{2\,k_0^6}{\pi|D|^2}\,\cos\bigl(2\,k'\,(a-z)\bigr) \end{aligned}\right\} . \qquad (32)$$

The first two time averages of the tensor components appear to be independent of z. The energy density depends on z. All these tensor components are zero in both vacuum regions. This means that all of them jump at the boundaries.

On the first boundary the jump of T^{33} is negative. It means that the first boundary will be pushed to the right. On the second boundary the jump will be positive and the same by its absolute value (because T^{33} is constant inside the second region). The second boundary will be pushed in the negative direction of the z-axis with the same force — we have equilibrium — no "free" force.

On the first boundary the jump of T^{03} is negative. It means that the first boundary will be getting energy. On the second boundary the jump will be positive and the same by its absolute value (because T^{03} is constant inside the second region). The second boundary will be losing the same amount of energy — no "free" energy.

It appears that the particular solutions that we have carry energy and momentum from the second boundary to the first, while the missing particular solution carries them back. If we imagine that the energy and momentum can be lost on the way from the sourse to the drain then we get a free linear momentum directed to the sourse of dummy waves (gravitational force). Also we get a free energy for heating stars. This unconservation proposition can be quite real if we consider that we obtained the conservation of energy-momentum from the requirement of minimum action. In the real physical world the action may has a small jitter around the exact minimum. Obviously this jitter is very small so that it can revile itself only on a cosmic scale.

At the present time we hesitate in proceeding further from these results because the meaning of these results has still to be clarified.

12 De Broglie's waves

Let us suppose, in addition (see Section 6), that the frequency of dummy waves (as well as the intensity) also proportional to the mass of the particle: $\omega = mc^2/\hbar$. The resting particles are present in abundance in the experimental arrangement itself. These resting particles can be partially synchronized in some proximity (the extent of this proximity is not known yet) of any point inside the experimental device. We can expect some standing scalar waves of a dummy generator that can be experienced by the moving particle independently of the direction of motion. In this case we can explain De Broglie's waves as beat frequency waves between the frequency of a resting particle and the Doppler shifted frequency of a moving particle. The rôle of the nonlinear device that is necessary to obtain the beat frequency wave, can be very well played by the boundary of the particle itself. This will explain "the wave properties of particles" by purely classical means, as first proposed in 1993 by Milo Wolff [7].

In the foregoing reformulation of conventional classical electrodynamics, we omitted the interaction term in the Lagrangian/Hamiltonian. Quantum Theory was undermined by this action. One should note that, historically, after the creation of quantum theory, there were attempts to legitimize the electromagnetic potential as a physically measurable value (see R. Feynman, [1]). Still, it is too early to try to find a classical basis for quantum theory, but the direction to go is that of the physical realm of the electromagnetic potential.

13 Conclusion

Probably it is not right to keep the disruption surface devoid from surface energy and surface tension. To introduce that correctly we have to consider some surface Lagrange density and add a surface integral to the action volume integral. That I hope to see in a future development.

Submitted on October 09, 2007
Accepted on October 22, 2007

References

1. Feynman R. Lectures on physics. Addison-Wesley, Reading, Massachusetts, 1966, v. 2, p.28–1.
2. Keilman Y. Nonelementary particles. *Phys. Essays*, 1998, v. 11 (1), 34.
3. Keilman Y. Cylindrical particles: the halfparticle and the particle of light of Classical Electrodynamics (CED). *Phys. Essays*, 2002, v. 15 (3), 257.
4. Keilman Y. The basic principles of Classical Electrodynamics. *Phys. Essays*, v. 16 (3).
5. Jackson J. D. Classical electrodynamics. John Wiley and Son, New York, 1975.
6. Landau L. D. and Lifshitz E. M. The classical theory of fields. Butterworth-Heinemann, Oxford, New York, 1975.
7. Wolff M. *Phys. Essays*, 1993, v. 6 (2), 190.