A Model of Electron-Positron Pair Formation

Bo Lehnert

Alfvén Laboratory, Royal Institute of Technology, S-10044 Stockholm, Sweden
E-mail: Bo.Lehnert@ee.kth.se

The elementary electron-positron pair formation process is considered in terms of a revised quantum electrodynamic theory, with special attention to the conservation of energy, spin, and electric charge. The theory leads to a wave-packet photon model of narrow line width and needle-radiation properties, not being available from conventional quantum electrodynamics which is based on Maxwell's equations. The model appears to be consistent with the observed pair production process, in which the created electron and positron form two rays that start within a very small region and have original directions along the path of the incoming photon. Conservation of angular momentum requires the photon to possess a spin, as given by the present theory but not by the conventional one. The nonzero electric field divergence further gives rise to a local intrinsic electric charge density within the photon body, whereas there is a vanishing total charge of the latter. This may explain the observed fact that the photon decays on account of the impact from an external electric field. Such a behaviour should not become possible for a photon having zero local electric charge density.

1 Introduction

During the earliest phase of the expanding universe, the latter is imagined to be radiation-dominated, somewhat later also including particles such as neutrinos and electron-positron pairs. In the course of the expansion the "free" states of highly energetic electromagnetic radiation thus become partly "condensed" into "bound" states of matter as determined by Einstein's energy relation.

The pair formation has for a long time both been studied experimentally [1] and been subject to theoretical analysis [2]. When a high-energy photon passes the field of an atomic nucleus or that of an electron, it becomes converted into an electron and a positron. The orbits of these created particles form two rays which start within a very small volume and have original directions along the path of the incoming photon.

In this paper an attempt is made to understand the elementary electron-positron pair formation process in terms of a revised quantum electrodynamic theory and its application to a wave-packet model of the individual photon [3, 4, 5, 6]. The basic properties of the latter will be described in Section 2, the intrinsic electric charge distribution of the model in Section 3, the conservation laws of pair formation in Section 4, some questions on the vacuum state in Section 5, and the conclusions are finally presented in Section 6.

2 A photon model of revised quantum electrodynamics

The detailed deductions of the photon model have been reported elsewhere [3, 4, 5, 6] and will only be summarized here. The corresponding revised Lorentz and gauge invariant theory represents an extended version which aims beyond Maxwell's equations. Here the electric charge density and the related electric field divergence are nonzero in the vacuum state, as supported by the quantum mechanical vacuum fluctuations and the related zero-point energy. The resulting wave equation of the electric field $\mathbf{E}$ then has the form

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)\mathbf{E} + \left(c^2\nabla + \mathbf{C}\frac{\partial}{\partial t}\right)(\operatorname{div}\mathbf{E}) = 0\,, \quad (1)$$

which includes the effect of a space-charge current density $\mathbf{j} = \varepsilon_0(\operatorname{div}\mathbf{E})\mathbf{C}$ that arises in addition to the displacement current $\varepsilon_0\,\partial\mathbf{E}/\partial t$. The velocity $\mathbf{C}$ has a modulus equal to the velocity c of light, as expressed by $\mathbf{C}^2 = c^2$. The induction law still has the form

$$\operatorname{curl}\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \quad (2)$$

with $\mathbf{B}$ standing for the magnetic field strength.

The photon model to be discussed here is limited to axisymmetric normal modes in a cylindrical frame (r, φ, z) where $\partial/\partial\varphi = 0$. A form of the velocity vector

$$\mathbf{C} = c\,(0, \cos\alpha, \sin\alpha) \quad (3)$$

is chosen under the condition $0 < |\cos\alpha| \ll 1$, such as not to get into conflict with the Michelson-Morley experiments, i.e. by having phase and group velocities which only differ by a very small amount from c. The field components can be expressed in terms of a generating function

$$G_0 \cdot G = E_z + (\cot\alpha)E_\varphi\,, \qquad G = R(\rho)\,e^{i(-\omega t + kz)}, \quad (4)$$

where G_0 is an amplitude factor, $\rho = r/r_0$ with r_0 as a characteristic radial distance of the spatial profile, and ω and k standing for the frequency and wave number of a normal mode. Such modes are superimposed to form a wave-packet having the spectral amplitude

$$A_k = \left(\frac{k}{k_0^2}\right) \exp\left[-z_0^2 (k - k_0)^2\right], \tag{5}$$

where k_0 and $\lambda_0 = \frac{2\pi}{k_0} = \frac{c}{\nu_0}$ are the main wave number and wave length, and $2z_0$ represents the effective axial length of the packet. According to experimental observations, the packet must have a narrow line width, as expressed by $k_0 z_0 \gg 1$. The spectral averages of the field components in the case $|\cos\alpha| \ll 1$ are then

$$\bar{E}_r = -iE_0 R_5 , \tag{6}$$

$$\bar{E}_\varphi = E_0 (k_0 r_0)(\sin\alpha)(\cos\alpha) R_3 , \tag{7}$$

$$\bar{E}_z = E_0 (k_0 r_0)(\cos\alpha)^2 R_4 \tag{8}$$

and

$$\bar{B}_r = -\frac{1}{c}\frac{1}{\sin\alpha}\bar{E}_\varphi , \tag{9}$$

$$\bar{B}_\varphi = \frac{1}{c}(\sin\alpha)\bar{E}_r , \tag{10}$$

$$\bar{B}_z = \frac{1}{c}(\cos\alpha)\frac{R_8}{R_5}\bar{E}_r . \tag{11}$$

Here

$$R_3 = \rho^2 D_\rho R , \quad R_4 = R - R_3 , \quad R_5 = \frac{d}{d\rho}(R - R_3) , \tag{12}$$

$$R_8 = \left(\frac{d}{d\rho} + \frac{1}{\rho}\right) R_3 , \qquad D_\rho = \frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{d\rho} \tag{13}$$

and

$$E_0 = e_0 \bar{f} , \quad e_0 = \frac{g_0\sqrt{\pi}}{k_0^2 r_0 z_0} , \quad G_0 = g_0(\cos\alpha)^2, \tag{14}$$

$$\bar{f} = \left[\cos(k_0\bar{z}) + i\sin(k_0\bar{z})\right] \exp\left[-\left(\frac{\bar{z}}{2z_0}\right)^2\right], \tag{15}$$

where $\bar{z} = z - c(\sin\alpha)t$.

Choosing the part of the normalized generating function G which is symmetric with respect to the axial centre $\bar{z} = 0$ of the moving wave packet, the components $(\bar{E}_\varphi, \bar{E}_z, \bar{B}_r)$ become symmetric and the components $(\bar{E}_r, \bar{B}_\varphi, \bar{B}_z)$ antisymmetric with respect to the same centre. Then the integrated electric charge and magnetic moment vanish.

The equivalent total mass defined by the electromagnetic field energy and the energy relation by Einstein becomes on the other hand

$$m \cong 2\pi\frac{\varepsilon_0}{c^2} r_0^2 W_m e_0^2 \int_{-\infty}^{+\infty} f^2 \, d\bar{z} , \tag{16}$$

$$W_m = \int \rho R_5^2 \, d\rho ,$$

where expression (15) has to be replaced by the reduced function

$$f = \left[\sin(k_0\bar{z})\right] \exp\left[-\left(\frac{\bar{z}}{2z_0}\right)^2\right] \tag{17}$$

due to the symmetry condition on G with respect to $\bar{z} = 0$. Finally the integrated angular momentum is obtained from the Poynting vector, as given by

$$s \cong -2\pi\varepsilon_0 \int_{-\infty}^{+\infty}\int r^2 \bar{E}_r \bar{B}_z \, dr d\bar{z} =$$

$$= 2\pi\frac{\varepsilon_0}{c}(\cos\alpha) r_0^3 W_s e_0^2 \int_{-\infty}^{+\infty} f^2 \, d\bar{z} , \tag{18}$$

$$W_s = -\int \rho^2 R_5 R_8 \, d\rho .$$

Even if the integrated (total) electric charge of the photon body as a whole vanishes, there is on account of the nonzero electric field divergence a local nonzero electric charge density

$$\bar{\rho} = e_0 f \frac{\varepsilon_0}{r_0}\frac{1}{\rho}\frac{d}{d\rho}(\rho R_5) . \tag{19}$$

Due to the factor $\sin(k_0\bar{z})$ this density oscillates rapidly in space as one proceeds along the axial direction. Thus the electric charge distribution consists of two equally large positive and negative oscillating contributions of total electric charge, being mixed up within the volume of the wave packet.

To proceed further the form of the radial function $R(\rho)$ has now to be specified. Since the experiments clearly reveal the pair formation to take place within a small region of space, the incoming photon should have a strongly limited extension in its radial (transverse) direction, thus having the character of "needle radiation". Therefore the analysis is concentrated on the earlier treated case of a function R which is divergent at $\rho = 0$, having the form

$$R(\rho) = \rho^{-\gamma} e^{-\rho} , \qquad \gamma > 0 . \tag{20}$$

In the radial integrals of equations (16) and (18) the dominant terms then result in $R_8 \cong -R_5$ and

$$W_m = \int_{\rho_m}^{\infty} \rho R_5^2 \, d\rho = \frac{1}{2}\gamma^5 \rho_m^{-2\gamma} , \tag{21}$$

$$W_s = \int_{\rho_s}^{\infty} \rho^2 R_5^2 \, d\rho = \frac{1}{2}\gamma^5 \rho_s^{-2\gamma+1} , \tag{22}$$

where $\rho_m \ll 1$ and $\rho_s \ll 1$ are small nonzero radii at the origin $\rho = 0$. To compensate for the divergence of W_m and W_s when ρ_m and ρ_s approach zero, we now introduce the shrinking parameters

$$r_0 = c_r \cdot \varepsilon , \qquad g_0 = c_g \cdot \varepsilon^\beta , \tag{23}$$

where c_r and c_g are positive constants and the dimensionless smallness parameter ε is defined by $0 < \varepsilon \ll 1$. From relations

(14)–(18), (21)–(23), the energy relation $mc^2 = h\nu_0$, and the quantum condition of the angular momentum, the result becomes

$$m = \pi^2 \frac{\varepsilon_0}{c^2} \gamma^5 \left(\frac{1}{k_0^2 z_0}\right)^2 c_g^2 \frac{\varepsilon^{2\beta}}{\rho_m^{2\gamma}} J_m = \frac{h}{\lambda_0 c}, \tag{24}$$

$$s = \pi^2 \frac{\varepsilon_0}{c} \gamma^5 \left(\frac{1}{k_0^2 z_0}\right)^2 c_g^2 c_r (\cos\alpha) \frac{\varepsilon^{2\beta+1}}{\rho_s^{2\gamma-1}} J_m = \frac{h}{2\pi} \tag{25}$$

with

$$J_m = \int_{-\infty}^{+\infty} f^2\, d\bar{z} \cong z_0 \sqrt{2\pi}\,. \tag{26}$$

Here we are free to choose $\beta = \gamma \gg 1$ which leads to

$$\rho_s \cong \rho_m = \varepsilon\,. \tag{27}$$

The lower limits ρ_m, and ρ_s of the integrals (21) and (22) then decrease linearly with ε and with the radius r_0. This forms a "similar" set of geometrical configurations, having a common shape which is independent of ρ_m, ρ_s, and ε in the range of small ε.

Taking $\hat{r} = r_0$ as an effective radius of the configuration (20), combination of relations (23)–(25) finally yields a photon diameter

$$2\hat{r} = \frac{\varepsilon\lambda_0}{\pi|\cos\alpha|} \tag{28}$$

being independent of γ. Thus the individual photon model becomes strongly needle-shaped when $\varepsilon \leqslant |\cos\alpha|$.

It should be observed that the photon spin of expression (25) disappears when $\operatorname{div}\mathbf{E}$ vanishes and the basic relations reduce to Maxwell's equations. This is also the case under more general conditions, due to the behaviour of the Poynting vector and to the requirement of a finite integrated field energy [3, 4, 5, 6].

3 The intrinsic electric charge distribution

We now turn to the intrinsic electric charge distribution within the photon wave-packet volume, representing an important but somewhat speculative part of the present analysis. It concerns the detailed process by which the photon configuration and its charge distribution are broken up to form a pair of particles of opposite electric polarity. Even if electric charges can arise and disappear in the vacuum state due to the quantum mechanical fluctuations, it may be justified as a first step to investigate whether the total intrinsic photon charge of one polarity can become sufficient as compared to the electric charges of the electron and positron.

With the present strongly oscillating charge density in space, the total intrinsic charge of either polarity can be estimated with good approximation from equations (17) and (19). This charge appears only within half of the axial extension of the packet, and its average value differs by the factor $\frac{2}{\pi}$ from the local peak value of its sinusoidal variation. From equation (19) this intrinsic charge is thus given by

$$q = \frac{z_0}{\pi} \int_{\rho_q}^{\infty} 2\pi r \frac{\bar{\rho}}{f}\, dr = 2\sqrt{\pi} z_0 \varepsilon_0 \gamma^3 \frac{1}{k_0^2 z_0} c_g \frac{\varepsilon^\beta}{\rho_q^\gamma}, \tag{29}$$

where the last factor becomes equal to unity when $\beta = \gamma$ and the limit $\rho_q = \varepsilon$ for a similar set of geometrical configurations. Relations (29) and (24) then yield

$$q^2 = \frac{8}{\pi^3} \varepsilon_0 c^2 \gamma z_0 m = \frac{8}{\pi^3} \varepsilon_0\, c\, h\, \gamma \frac{z_0}{\lambda_0} \cong \cong 45{\times}10^{-38}\, \gamma \frac{z_0}{\lambda_0} \tag{30}$$

and

$$\frac{q}{e} \cong 4.2 \left(\frac{\gamma z_0}{\lambda_0}\right)^{1/2}. \tag{31}$$

With a large γ and a small line width leading to $\lambda_0 \ll z_0$, the total intrinsic charge thus substantially exceeds the charge of the created particle pair. However, the question remains how much of the intrinsic charge becomes available during the disintegration process of the photon.

A much smaller charge would become available in a somewhat artificial situation where the density distribution of charge is perturbed by a 90 degrees phaseshift of the sinusoidal factor in expression (17). This would add a factor $2\exp\left[-4\pi^2(z_0/\lambda_0)^2\right]$ to the middle and right-hand members of equation (30), and makes $q \gtrsim e$ only for extremely large values of γ and for moderately narrow line widths.

4 Conservation laws of pair formation

There are three conservation laws to be taken into account in the pair formation process. The first concerns the total energy. Here we limit ourselves to the marginal case where the kinetic energy of the created particles can be neglected as compared to the equivalent energy of their rest masses. Conservation of the total energy is then expressed by

$$mc^2 = \frac{hc}{\lambda_0} = 2m_e c^2. \tag{32}$$

Combination with equation (28) yields an effective photon diameter

$$2\hat{r} = \frac{\varepsilon h}{2\pi m_e c|\cos\alpha|}. \tag{33}$$

With $\varepsilon \leqslant |\cos\alpha|$ we have $2\hat{r} \leqslant 3.9{\times}10^{-13}$ m being equal to the Compton wavelength and representing a clearly developed form of needle radiation.

The second conservation law concerns the preservation of angular momentum. It is satisfied by the spin $\frac{h}{2\pi}$ of the photon in the capacity of a boson particle, as given by expression (25). This angular momentum becomes equal to the sum of the spin $\frac{h}{4\pi}$ of the created electron and positron being fermions. In principle, the angular momenta of the two created particles could also become antiparallel and the spin of the photon zero, but such a situation would contradict all other experience about the photon spin.

The third conservation law deals with the preservation of the electric charge. This condition is clearly satisfied by the vanishing integrated photon charge, and by the opposite polarities of the created particles. In a more detailed picture where the photon disintegrates into the charged particles, it could also be conceived as a splitting process of the positive and negative parts of the intrinsic electric charge distributions of the photon.

Magnetic moment conservation is satisfied by having parallel angular momenta and opposite charges of the electron and positron, and by a vanishing magnetic moment of the photon [5, 6].

5 Associated questions of the vacuum state concept

The main new feature of the revised quantum electrodynamical theory of Section 2 is the introduction of a nonzero electric field divergence in the vacuum, as supported by the existence of quantum mechanical fluctuations. In this theory the values of the dielectric constant and the magnetic permeability of the conventional empty-space vacuum have been adopted. This is because no electrically polarized and magnetized atoms or molecules are assumed to be present, and that the vacuum fluctuations as well as superimposed regular phenomena such as waves take place in a background of empty space.

As in a review by Gross [7], the point could further be made that a "vacuum polarization" screens the point-charge-like electron in such a way that its effective electrostatic force vanishes at large distances. There is, however, experimental evidence for such a screening not to become important at the scale of the electron and photon models treated here. In the vacuum the electron is thus seen to be subject to scattering processes due to its full electrostatic field, and an electrically charged macroscopic object is also associated with such a measurable field. This would be consistent with a situation where the vacuum fluctuations either are small or essentially independent as compared to an external disturbance, and where their positive contributions to the local electric charge largely cancel their negative ones.

To these arguments in favour of the empty-space values of the dielectric constant and the magnetic permeability two additional points can also be added. The first is due to the Heisenberg uncertainty relation which implies that the vacuum fluctuations appear spontaneously during short time intervals and independently of each other. They can therefore hardly have a screening effect such as that due to Debye in a quasi-neutral plasma. The second point is based on the fact that static measurements of the dielectric constant and the magnetic permeability result in values the product of which becomes equal to the inverted square of the measured velocity of light.

6 Conclusions

The basis of the conservation laws in Section 4 is rather obvious, but it nevertheless becomes nontrivial when a comparison is made between conventional quantum electrodynamics based on Maxwell's equations on one hand and the present revised theory on the other. Thereby the following points should be observed:

- The needle-like radiation of the present photon model is necessary for understanding the observed creation of an electron-positron pair which forms two rays that start within a small region, and which have original directions along the path of the incoming photon. Such needle radiation does not come out of conventional theory [3, 4, 5, 6];
- The present revised theory leads to a nonzero spin of the photon, not being available from conventional quantum electrodynamics based on Maxwell's equations; [3, 4, 5, 6]. The present model is thus consistent with a photon as a boson which decays into two fermions;
- The nonzero divergence of the electric field in the present theory allows for a local nonzero electric charge density, even if the photon has a vanishing net charge. This may indicate how the intrinsic electric photon charges can form two charged particles of opposite polarity when the photon structure becomes disintegrated. Such a process is supported by the experimental fact that the photon decays into two charged particles through the impact of the electric field from an atomic nucleus or from an electron. This could hardly occur if the photon body would become electrically neutral at any point within its volume. Apart from such a scenario, the electromagnetic field configuration of the photon may also be broken up by nonlinear interaction with a strong external electric field;
- The present approach to the pair formation process has some similarity with the breaking of the stability of vacuum by a strong external electric field, as being investigated by Fradkin et al. [8].

Submitted on November 01, 2007
Accepted on November 19, 2007

References

1. Richtmyer F. K. and Kennard E. H. Introduction to modern physics. McGraw-Hill Book Comp. Inc., New York and London, 1947, pages 600 and 699.
2. Heitler W. The quantum theory of radiation. Oxford, Clarendon Press, 1954, Ch. 5, paragraph 26.
3. Lehnert B. Photon physics of revised electromagnetics. *Progress in Physics*, 2006, v. 2, 78–85.

4. Lehnert B. Momentum of the pure radiation field. *Progress in Physics*, 2007, v. 1, 27–30.
5. Lehnert B. Revised quantum electrodynamics with fundamental applications. In: *Proceedings of 2007 ICTP Summer College on Plasma Physics* (Edited by P. K. Shukla, L. Stenflo, and B. Eliasson), World Scientific Publishers, Singapore, 2008.
6. Lehnert B. A revised electromagnetic theory with fundamental applications. Swedish Physics Archive (Edited by D. Rabounski), The National Library of Sweden, Stockholm, 2007; and Bogoljubov Institute for Theoretical Physics (Edited by A. Zagorodny), Kiev, 2007.
7. Gross D. J. The discovery of asymptotic freedom and the emergence of QCD. Les Prix Nobel, 2004, Edited by T. Frängsmyr, Almquist och Wiksell International, Stockholm 2004, 59–82.
8. Fradkin E. S., Gavrilov S. P., Gitman D. M., and Shvartsman Sh. M. Quantum electrodynamics with unstable vacuum. (Edited by V. L. Ginzburg), Proceedings of the Lebedev Physics Institute, Academy of Sciences of Russia, v. 220, 1995, Nova Science Publishers, Inc., Commack, New York.