Numerical Solution of Radial Biquaternion Klein-Gordon Equation

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In the preceding article we argue that biquaternionic extension of Klein-Gordon equation has solution containing imaginary part, which differs appreciably from known solution of KGE. In the present article we present numerical /computer solution of radial biquaternionic KGE (radialBQKGE); which differs appreciably from conventional Yukawa potential. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

In the preceding article [1] we argue that biquaternionic extension of Klein-Gordon equation has solution containing imaginary part, which differs appreciably from known solution of KGE. In the present article we presented here for the first time a numerical/computer solution of radial biquaternionic KGE (radialBQKGE); which differs appreciably from conventional Yukawa potential.

This biquaternionic effect may be useful in particular to explore new effects in the context of low-energy reaction (LENR) [2]. Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

2 Radial biquaternionic KGE (radial BQKGE)

In our preceding paper [1], we argue that it is possible to write biquaternionic extension of Klein-Gordon equation as follows:

$$\left[\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right) + i\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\right]\varphi(x,t) = = -m^2\,\varphi(x,t)\,, \tag{1}$$

or this equation can be rewritten as:

$$\left(\diamondsuit\bar{\diamondsuit} + m^2\right)\varphi(x,t) = 0, \tag{2}$$

provided we use this definition:

$$\diamondsuit = \nabla^q + i\,\nabla^q = \left(-i\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z}\right) + + i\left(-i\frac{\partial}{\partial T} + e_1\frac{\partial}{\partial X} + e_2\frac{\partial}{\partial Y} + e_3\frac{\partial}{\partial Z}\right), \tag{3}$$

where e_1, e_2, e_3 are *quaternion imaginary units* obeying (with ordinary quaternion symbols: $e_1 = i$, $e_2 = j$, $e_3 = k$):

$$i^2 = j^2 = k^2 = -1\,, \quad ij = -ji = k\,, \quad jk = -kj = i\,, \quad ki = -ik = j\,. \tag{4}$$

and quaternion *Nabla operator* is defined as [1]:

$$\nabla^q = -i\,\frac{\partial}{\partial t} + e_1\frac{\partial}{\partial x} + e_2\frac{\partial}{\partial y} + e_3\frac{\partial}{\partial z}\,. \tag{5}$$

(Note that (3) and (5) included partial time-differentiation.)

In the meantime, the standard Klein-Gordon equation usually reads [3, 4]:

$$\left(\frac{\partial^2}{\partial t^2} - \nabla^2\right)\varphi(x,t) = -m^2\varphi(x,t)\,. \tag{6}$$

Now we can introduce polar coordinates by using the following transformation:

$$\nabla = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{\ell^2}{r^2}\,. \tag{7}$$

Therefore, by substituting (7) into (6), the radial Klein-Gordon equation reads — by neglecting partial-time differentiation — as follows [3, 5]:

$$\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) - \frac{\ell(\ell+1)}{r^2} + m^2\right)\varphi(x,t) = 0\,, \tag{8}$$

and for $\ell = 0$, then we get [5]:

$$\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + m^2\right)\varphi(x,t) = 0\,. \tag{9}$$

The same method can be applied to equation (2) for radial biquaternionic KGE (BQKGE), which for the 1-dimensional situation, one gets instead of (8):

$$\left(\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r}\right) - i\,\frac{\partial}{\partial r}\left(\frac{\partial}{\partial r}\right) + m^2\right)\varphi(x,t) = 0\,. \tag{10}$$

In the next Section we will discuss numerical/computer solution of equation (10) and compare it with standard solution of equation (9) using Maxima software package [6]. It can be shown that equation (10) yields potential which differs appreciably from standard Yukawa potential. For clarity, all solutions were computed in 1-D only.

3 Numerical solution of radial biquaternionic Klein-Gordon equation

Numerical solution of the standard radial Klein-Gordon equation (9) is given by:

(%i1) diff(y,t,2)-'diff(y,r,2)+m^2*y;

(%o1) $m^2 \cdot y - \frac{d^2}{d^2 x} y$

(%i2) ode2 (%o1, y , r);

(%o2) $y = \%k_1 \cdot \%\mathtt{exp}(mr) + \%k_2 \cdot \%\mathtt{exp}(-mr)$ (11)

In the meantime, numerical solution of equation (10) for radial biquaternionic KGE (BQKGE), is given by:

(%i3) diff(y,t,2)- (%i+1)*'diff(y,r,2)+m^2*y;

(%o3) $m^2 \cdot y - (i+1)\,\frac{d^2}{d^2 r} y$

(%i4) ode2 (%o3, y , r);

(%o4) $y = \%k_1 \cdot \sin\left(\frac{|m|\, r}{\sqrt{-\%i-1}}\right) + \%k_2 \cdot \cos\left(\frac{|m|\, r}{\sqrt{-\%i-1}}\right)$ (12)

Therefore, we conclude that numerical solution of radial biquaternionic extension of Klein-Gordon equation yields different result compared to the solution of standard Klein-Gordon equation; and it *differs appreciably* from the well-known Yukawa potential [3, 7]:

$$u(r) = -\frac{g^2}{r}\, e^{-mr}. \qquad (13)$$

Meanwhile, Comay puts forth argument that the Yukawa lagrangian density has theoretical inconsistency within itself [3].

Interestingly one can find argument that biquaternion Klein-Gordon equation is nothing more than quadratic form of (modified) Dirac equation [8], therefore BQKGE described herein, i.e. equation (12), can be considered as a plausible solution to the problem described in [3]. For other numerical solutions to KGE, see for instance [4].

Nonetheless, we recommend further observation [9] in order to refute or verify this proposition of new type of potential derived from biquaternion Klein-Gordon equation.

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