# Spin Transport in Mesoscopic Superconducting-Ferromagnetic Hybrid Conductor

Walid A. Zein, Adel H. Phillips, and Omar A.Omar

Faculty of Engineering, Ain Shams University, Cairo, Egypt E-mail: adel\_phillips@yahoo.com

The spin polarization and the corresponding tunneling magnetoresistance (TMR) for a hybrid ferromagnetic/superconductor junction are calculated. The results show that these parameters are strongly depends on the exchange field energy and the bias voltage. The dependence of the polarization on the angle of precession is due to the spin flip through tunneling process. Our results could be interpreted as due to spin imbalance of carriers resulting in suppression of gap energy of the superconductor. The present investigation is valuable for manufacturing magnetic recording devices and nonvolatile memories which imply a very high spin coherent transport for such junction.

## 1 Introduction

Spintronics and spin-based quantum information processing explore the possibility to add new functionality to today's electronic devices by exploiting the electron spin in addition to its charge [1]. Spin-polarized tunneling plays an important role in the spin dependent transport of magnetic nanostructures [2]. The spin-polarized electrons injected from ferromagnetic materials into nonmagnetic one such as superconductor, semiconductor create a non equilibrium spin polarization in such nonmagnetic materials [3, 4, 5].

Ferromagnetic-superconductor hybrid systems are an attractive subject research because of the competition between the spin asymmetry characteristic of a ferromagnet and the correlations induced by superconductivity [1, 2, 6]. At low energies electronic transport in mesoscopic ferromagnetsuperconductor hybrid systems is determined by Andreevreflection [7]. Superconducting materials are powerful probe for the spin polarization of the current injected from ferromagnetic material [8, 9, 10]. Superconductors are useful for exploring how the injected spin-polarized quasiparticles are transported. In this case the relaxation time can be measured precisely in the superconducting state where thermal noise effects are small.

The present paper, spin-polarized transport through ferromagnetic/superconductor/ferromagnetic double junction is investigated. This investigation will show how Andreevreflection processes are sensitive to the exchange field energy in the ferromagnetic leads.

## 2 The model

A mesoscopic device is modeled as superconductor sandwiched between two ferromagnetic leads via double tunnel barriers. The thickness of the superconductor is smaller than the spin diffusion length and the magnetization of the ferromagnetic leads are aligned either parallel or antiparallel. The spin polarization of the conduction electrons due to Andreev reflection at ferromagnetic/superconductor interface could be determined through the following equation as:

$$P = \frac{\Gamma_{\uparrow}(E) - \Gamma_{\downarrow}(E)}{\Gamma_{\uparrow}(E) + \Gamma_{\downarrow}(E)}, \qquad (1)$$

where  $\Gamma_{\uparrow}(E)$  and  $\Gamma_{\downarrow}(E)$  are the tunneling probabilities of conduction electrons with up-spin and down-spin respectively. Since the present device is described by the following Bogoliubov-deGennes (BdG) equation [11]:

$$\begin{pmatrix} H_0 - h_{ex}(z)\sigma & \Delta(z) \\ \Delta^*(z) & -H_0 - \sigma h_{ex}(z) \end{pmatrix} \psi = E \psi, \quad (2)$$

where  $H_0$  is the single particle Hamiltonian and it is expressed as:  $t^2$ 

$$H_0 = -\frac{\hbar^2}{2m} \nabla^2 - \varepsilon_{nl} \,, \tag{3}$$

in which the energy,  $\varepsilon_{nl}$ , is expressed of the Fermi velocity  $v_F$ , Fermi-momentum  $P_F$ , the magnetic field *B* as [12]:

$$\varepsilon_{nl} = -(\alpha g + k_F D \sin \theta) \mu_B B \pm \\ \pm \left[ v_F^2 P_F^2 (1 - \sin \theta)^2 + \Delta^2 \right]^{1/2}.$$
(4)

In Eq. (4),  $\alpha = \pm 1/2$  for spin-up and spin down respectively,  $\mu_B$  is the Bohr magneton, g is the g-factor for electrons and  $\theta$  is the precession angle.

The interface between left ferromagnetic/superconductor and superconductor/right ferromagnetic leads are located at z = -L/2 and z = L/2 respectively. The parameter  $h_{ex}(z)$ represents the exchange field and is given by [13]:

$$h_{ex} = \left\{ \begin{array}{cc} h_0 & z \prec -L/2 \\ 0 & -L/2 \prec z \prec L/2 \\ \pm h_0 & z \succ L/2 \end{array} \right\} , \qquad (5)$$

where  $+h_0$  and  $-h_0$  represents the exchange fields for parallel and anti-parallel alignments respectively, the parameter  $\Delta(z)$  is the superconducting gap:

$$\Delta(z) = \left\{ \begin{array}{cc} 0 & z \prec -L/2, L/2 \prec z \\ \Delta & -L/2 \prec z \prec L/2 \end{array} \right\} .$$
(6)

The temperature dependence of the superconducting gap is given by [14]:

$$\Delta = \Delta_0 \tanh\left(1.74 \sqrt{\frac{T_c}{T}} - 1\right), \qquad (7)$$

where  $\Delta_0$  is the superconducting gap at T = 0 and  $T_c$  is the superconducting critical temperature. Now, in order to get the tunneling probability  $\Gamma_{\uparrow\downarrow}(E)$  for both up-spin and down-spin electrons by solving the Bogoliubov-deGennes Eqn. (2) as: The eigenfunction in the left ferromagnetic lead (z < -L/2) is given by:

$$\psi_{\sigma,nl}^{FM1}(r) = \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{iP_{\sigma,nl}^+(z+\frac{L}{2})} + a_{\sigma,nl} \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{iP_{\sigma,nl}^-(z+\frac{L}{2})} + b_{\sigma,nl} \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-P_{\sigma,nl}^+(z+\frac{L}{2})} \right] S_{nl}(x,y) .$$
(8)

In the superconductor (-L/2 < z < L/2), the eigenfunction is given by:

$$\begin{split} \psi^{SC}_{\sigma,nl}(r) &= \left[ \alpha_{\sigma,nl} \begin{pmatrix} u_0 \\ \nu_0 \end{pmatrix} e^{ik_{nl}^+(z+\frac{L}{2})} + \right. \\ &+ \beta_{\sigma,nl} \begin{pmatrix} \nu_0 \\ u_0 \end{pmatrix} e^{-ik_{nl}^-(z+\frac{L}{2})} + \\ &+ \xi_{\sigma,nl} \begin{pmatrix} u_0 \\ \nu_0 \end{pmatrix} e^{-ik_{nl}^+(z-\frac{L}{2})} + \\ &+ \eta_{\sigma,nl} \begin{pmatrix} u_0 \\ \nu_0 \end{pmatrix} e^{ik_{nl}^-(z-\frac{L}{2})} \right] S_{nl}(x,y) \,. \end{split}$$
(9)

And the eigenfunction in the right ferromagnetic lead (L/2 < z) is given by:

$$\psi_{\sigma,nl}^{FM2}(r) = \begin{bmatrix} C_{\sigma,nl} \begin{pmatrix} 1\\0 \end{bmatrix} e^{iq_{\sigma,nl}^+(z-\frac{L}{2})} + \\ + d_{\sigma,nl} \begin{pmatrix} 0\\1 \end{bmatrix} e^{-iq_{\sigma,nl}^-(z-\frac{L}{2})} \end{bmatrix} S_{nl}(x,y) \,.$$
(10)

Since the device is rectangular, the eigenfunction in the transverse (x&y) directions with channels n, l is given by;

$$S_{nl}(x,y) = \sin\left(\frac{n\pi x}{W}\right) \, \sin\left(\frac{l\pi y}{W}\right),$$
 (11)

where W is the width of the junction.

The wave numbers in the Eqs. (8), (9), (10) are given by:

$$P_{\sigma,nl}^{\pm} = \sqrt{\frac{2m}{\hbar^2} \left(\mu_F \pm E \pm \sigma h_{ex}\right)}, \qquad (12)$$

$$k_{\sigma,nl}^{\pm} = \sqrt{\frac{2m}{\hbar^2} \left(\mu_F \pm \Im\right) - \varepsilon_{nl}}, \qquad (13)$$

$$q_{\sigma,nl}^{\pm} = \sqrt{\frac{2m}{\hbar^2} \left(\mu_F \pm E \pm \sigma h_{ex} \pm \varepsilon_{nl}\right)}, \qquad (14)$$

where  $\Im = \sqrt{E^2 - \Delta^2}$ , and the energy  $\varepsilon_{nl}$  is given by Eq. (4). For the coherence factors of electron and holes  $u_0$  and  $\nu_0$  are related as [11]:

$$u_0^2 = 1 - \nu_0^2 = \frac{1}{2} \left[ 1 + \frac{\sqrt{E^2 - \Delta^2}}{E} \right].$$
 (15)

The coefficients in Eqs. (8), (9), (10) are determined by applying the boundary conditions at the interfaces and the matching conditions are:

$$\psi_{\sigma,nl}^{FM1}\left(z=-\frac{L}{2}\right) = \psi_{\sigma,nl}^{SC}\left(z=-\frac{L}{2}\right) \\ \psi_{\sigma,nl}^{SC}\left(z=\frac{L}{2}\right) = \psi_{\sigma,nl}^{FM2}\left(z=\frac{L}{2}\right) \end{cases} ,$$
(16)

$$\frac{d\psi_{\sigma,nl}^{SC}}{dz}\bigg|_{z=-\frac{L}{2}} - \frac{d\psi_{\sigma,nl}^{FM!}}{dz}\bigg|_{z=-\frac{L}{2}} = \frac{2mV}{\hbar^2}\psi_{\sigma,nl}^{FM1}\left(z=-\frac{L}{2}\right), \quad (17)$$

$$\frac{d\psi_{\sigma,nl}^{FM2}}{dz}\bigg|_{z=\frac{L}{2}} - \frac{d\psi_{\sigma,nl}^{SC!}}{dz}\bigg|_{z=\frac{L}{2}} = \frac{2mV}{\hbar^2}\psi_{\sigma,nl}^{FM2}\left(z=\frac{L}{2}\right). \quad (18)$$

Eqs. (14), (15), (16) are solved numerically [15] for the tunneling probabilities corresponding to up-spin and down-spin for the tunneled electrons. The corresponding polarization, P, Eq. (1) is determined at different parameters V,  $\theta$ , which will be discussed in the next section.

### 3 Results and discussion

Numerical calculations are performed for the present device, in which the superconductor is Nb and the ferromagnetic leads are of any one of ferromagnetic materials. The features of the present results are:

Fig. 1 shows the dependence of the polarization, P, on the bias voltage, V, at different parameters B, E, h and T. From the figure, the polarization has a peak at the value of Vnear the value of the energy gap  $\Delta_0$  for the present superconductor (Nb) ( $\Delta_0 = 1.5 \text{ meV}$ ) [16]. But for higher values of V, the polarization, P, decreases. As shown from Fig. 1a, the polarization does not change with the magnetic field, B, due to the Zeeman-energy. Some authors [17] observed the effect of magnetic field of values greater than 1 T, in this case the superconductivity will be destroyed (for Nb,  $B_c = 0.19$  T).

Now in order to observe the effect of the spin precession on the value of the polarization, P, this can be shown from Fig. 2. The dependence of the polarization, P, on the angle of precession,  $\theta$ , is strongly varies with the variation of the magnetic field, temperature, exchange field and the energy of

W. A. Zein, A. H. Phillips, and O. A. Omar. Spin Transport in Mesoscopic Superconducting-Ferromagnetic Hybrid Conductor



Fig. 1: The dependence of the polarization, P, on the bias voltage, V, at different B, E, h and T.



Fig. 2: The dependence of the polarization, P, on the angle of precession at different B, E, h and T.

W. A. Zein, A. H. Phillips, and O. A. Omar. Spin Transport in Mesoscopic Superconducting-Ferromagnetic Hybrid Conductor

44



Fig. 3: The variation of the TMR with the energy of the tunneled electrons at different parameters B, T, h and  $\theta$ .

the tunneled electrons. As shown from Fig. 2, the value of P is minimum at certain values of  $\theta$  also P is maximum at another values of  $\theta$ . This trend of the polarization with the angle of the precession is due to the flip of the electron spin when tunneling through the junction.

In order to investigate the spin injection tunneling through such hybrid magnetic system, we calculated the tunnel magnetoresistance (TMR) which is related to the polarization as [18]:  $p^{2}$ 

$$TMR = \frac{P^2}{1 - P^2 + \Gamma_s},$$
 (19)

where  $\Gamma_s$  is the relaxation parameter and is given by [18]:

$$\Gamma_s = \frac{e^2 N(0) R_T A L}{\tau_s} , \qquad (20)$$

where N(0) is the normal-state density of electrons calculated for both up-spin and down-spin distribution function  $f_{\sigma}(E)$ , which is expressed as [18]:

$$f_{\sigma}(E) \cong f_0(E) - \left(\frac{\partial f_0}{\partial E}\right) \sigma \,\delta\mu\,,$$
 (21)

where  $\sigma = \pm 1$  for both up and down spin of the electrons,  $\delta \mu$  is the shift of the chemical potential,  $\tau_s$  is the spin relaxation time, A is the junction area and  $R_T$  is the resistance at the interface of the tunnel junction.

Fig. 3 shows the variation of the TMR with the energy of the tunneled electrons at different parameters B, T, h and  $\theta$ . A peak is observed for TMR at a certain value which is in the near value of the gap energy  $\Delta_0$  for the superconductor (Nb). These results (Fig. 3) show the interplay between the spin polarization of electrons and Andreev-reflection process at the ferromagnetic/superconductor interface [19]. From our results; we can conclude that the spin-polarized transport depends on the relative orientation of magnetization in the two ferromagnetic leads. The spin polarization of the tunneled electrons through the junction gives rise to a nonequilibrium spin density in the superconductor. This is due to the imbalance in the tunneling currents carried by the spin-up and spindown electrons. The trend of the tunneling magnetoresistance (TMR) is due to the spin-orbit scattering in the superconductor. Our results are found concordant with those in literatures [20, 21, 22].

> Submitted on November 25, 2007 Accepted on November 30, 2007

### References

- 1. Zutic I., Fabian J. and Das Sarma S. *Review of Modern Physics*, 2004, v. 76, 323.
- Maekawa S. and Shinijo T. Spin dependent transport in magnetic nanostructures. Gordan and Breach. Sci. Publ., London, 2002.
- 3. Varet T. and Fert A. Phys. Rev. B, 1993, v. 48, 7099.
- 4. Jonson M. Appl. Phys. Lett., 1994, v. 65, 1460.
- 5. Nakamura Y., Pashkin Y. and Tsai J. S. *Nature*, 1999, v. 398, 786.
- Aumentado J. and Chandrasekhar V. Phys. Rev. B, 2001, v. 64, 54505.
- Andreev A.F. Zhurnal Eksperimantalnoii i Teoreticheskoii Fiziki, 1964, v. 46, 1823 (English translation published in Soviet Physics JETP-URSS, 1964, v. 19, 1228).

W. A. Zein, A. H. Phillips, and O. A. Omar. Spin Transport in Mesoscopic Superconducting-Ferromagnetic Hybrid Conductor

46

- 8. Meservey R. and Tedrow P. M. Phys. Rep., 1994, v. 238, 173.
- 9. Zutic I. and Valls O. T. Phys. Rev. B, 2000, v. 61, 1555.
- Awadalla A. A., Aly A. H. and Phillips A. H. International Journal of Nanoscience, 2007, v. 6(1), 41.
- deJang M. J. M and Beenakker C. W. J. *Pyhs. Rev. Lett.*, 1995, v. 74, 1657.
- 12. Tkachvandk Rikhter G. Phys. Rev. B, 2005, v. 71, 094517.
- 13. Benjamin C. and Citro R. Phys. Rev. B, 2005, v. 72, 085340.
- 14. Belzig W., Brataas A., Nazarov Yu. V. and Bauer G. E. W. *Phys. Rev. B*, 2000, v. 62, 9726.
- Kikuchi K., Imamura H., Takahashi S. and Maekawa S., *Phys. Rev. B*, 2001, v. 65, 020508 (R).
- deGennes P.G. Superconductivity of metals and alloys. Benjamin, New York, 1966.
- 17. Bergmann G., Lu J. and Wang D. Phys. Rev. B, 2005, v.71, 134521.
- 18. Takahashi S, Yamashita T., Imamura H. and Maekawa S. *J. Mag. Mat.*, 2002, v. 240, nos. 1–3, 100.
- Imamura H., Kikuchi K., Takahashi S. and Maekawa S. J. Appl. Phys., 2002, v. 91, 172509.
- Johansson J., Urech M., Haviland D. and Korenivski V. J. Appl. Phys., 2003, v. 93, 8650.
- 21. Kowalewski L., Wojciechowski R. and Wojtus P. *Physica Status* Solidi B, 2005, v. 243, 112.
- 22. Gonzalen E. M., Folgueras A. D., Escudero R., Ferrer J., Guinea F. and Vicent J. L. *New J. Phys.*, 2007, v. 9, 34.