A Note on Computer Solution of Wireless Energy Transmit via Magnetic Resonance

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In the present article we argue that it is possible to find numerical solution of coupled magnetic resonance equation for describing wireless energy transmit, as discussed recently by Karalis (2006) and Kurs *et al.* (2007). The proposed approach may be found useful in order to understand the phenomena of magnetic resonance. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

In recent years there were some new interests in methods to transmit energy without wire. While it has been known for quite a long-time that this method is possible theoretically (since Maxwell and Hertz), until recently only a few researchers consider this method seriously.

For instance, Karalis et al [1] and also Kurs et al. [2] have presented these experiments and reported that efficiency of this method remains low. A plausible way to solve this problem is by better understanding of the mechanism of magnetic resonance [3].

In the present article we argue that it is possible to find numerical solution of coupled magnetic resonance equation for describing wireless energy transmit, as discussed recently by Karalis (2006) and Kurs *et al.* (2007). The proposed approach may be found useful in order to understand the phenomena of magnetic resonance.

Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

2 Numerical solution of coupled-magnetic resonance equation

Recently, Kurs *et al.* [2] argue that it is possible to represent the physical system behind wireless energy transmit using coupled-mode theory, as follows:

$$a_m(t) = (i\omega_m - \Gamma_m)\, a_m(t) + \sum_{n\neq m} i\kappa_{nm} a_n(t) - F_m(t)\,. \tag{1}$$

The simplified version of equation (1) for the system of two resonant objects is given by Karalis *et al.* [1, p. 2]:

$$\frac{da_1}{dt} = -\,i\,(\omega_1 - i\Gamma_1)\,a_1 + i\kappa a_2\,, \tag{2}$$

and

$$\frac{da_2}{dt} = -\,i\,(\omega_2 - i\Gamma_2)\,a_2 + i\kappa a_1\,. \tag{3}$$

These equations can be expressed as linear 1st order ODE as follows:

$$f'(t) = -\,i\,\alpha\, f(t) + i\,\kappa\, g(t) \tag{4}$$

and

$$g'(t) = -\,i\beta\, g(t) + i\,\kappa\, f(t)\,, \tag{5}$$

where

$$\alpha = (\omega_1 - i\,\Gamma_1) \tag{6}$$

and

$$\beta = (\omega_2 - i\Gamma_2) \tag{7}$$

Numerical solution of these coupled-ODE equations can be found using Maxima [4] as follows. First we find test when parameters (6) and (7) are set up to be 1. The solution is:

```
(%i5) 'diff(f(x),x)+%i*f=%i*b*g(x);
(%o5) 'diff(f(x),x,1)+%i*f=%i*b*g(x)
(%i6) 'diff(g(x),x)+%i*g=%i*b*f(x);
(%o6) 'diff(g(x),x,1)+%i*g=%i*b*f(x)
(%i7) desolve([%o5,%o6],[f(x),g(x)]);
```

The solutions for $f(x)$ and $g(x)$ are:

$$f(x) = \frac{\left[i\,g(0)\,b - i\,f(x)\right]\sin(bx)}{b} - \frac{\left[g(x) - f(0)\,b\right]\cos(bx)}{b} + \frac{g(x)}{b}\,, \tag{8}$$

$$g(x) = \frac{\left[i\,f(0)\,b - i\,g(x)\right]\sin(bx)}{b} - \frac{\left[f(x) - g(0)\,b\right]\cos(bx)}{b} + \frac{f(x)}{b}\,. \tag{9}$$

Translated back to our equations (2) and (3), the solutions for $\alpha = \beta = 1$ are given by:

$$a_1(t) = \frac{\left[i\,a_2(0)\kappa - i\,a_1\right]\sin(\kappa t)}{\kappa} - \frac{\left[a_2 - a_1(0)\,\kappa\right]\cos(\kappa t)}{\kappa} + \frac{a_2}{\kappa} \tag{10}$$

$$f(x) = e^{-(ic-ia)t/2}\left[\frac{\left[2i\,f(0)\,c + 2i\,g(0)\,b - f(0)(ic-i\,a)\right]\sin\left(\frac{\sqrt{c^2-2ac+4b^2+a^2}}{2}\,t\right)}{\sqrt{c^2-2ac+4b^2+a^2}} + \right.$$
$$\left. + \frac{f(0)\cos\left(\frac{\sqrt{c^2-2ac+4b^2+a^2}}{2}\,t\right)}{\sqrt{c^2-2ac+4b^2+a^2}}\right] \qquad (13)$$

$$g(x) = e^{-(ic-ia)t/2}\left[\frac{\left[2i\,f(0)\,c + 2ig(0)\,a - g(0)(ic-i\,a)\right]\sin\left(\frac{\sqrt{c^2-2ac+4b^2+a^2}}{2}\,t\right)}{\sqrt{c^2-2ac+4b^2+a^2}} + \right.$$
$$\left. + \frac{g(0)\cos\left(\frac{\sqrt{c^2-2ac+4b^2+a^2}}{2}\,t\right)}{\sqrt{c^2-2ac+4b^2+a^2}}\right] \qquad (14)$$

$$a_1(t) = e^{-(i\beta-i\,\alpha)t/2}\left(\frac{\left[2i\,a_1(0)\beta + 2i\,a_2(0)\,\kappa - (i\beta - i\alpha)\,a_1\right]\sin\left(\frac{\xi}{2}\,t\right)}{\xi} - \frac{a_1(0)\cos\left(\frac{\xi}{2}\,t\right)}{\xi}\right) \qquad (15)$$

$$a_2(t) = e^{-(i\beta-i\alpha)t/2}\left(\frac{\left[2i\,a_2(0)\beta + 2i\,a_1(0)\,\kappa - (i\beta - i\,\alpha)\,a_2\right]\sin\left(\frac{\xi}{2}\,t\right)}{\xi} - \frac{a_2(0)\cos\left(\frac{\xi}{2}\,t\right)}{\xi}\right) \qquad (16)$$

and

$$a_2(t) = \frac{\left[i\,a_1(0)\kappa - i\,a_2\right]\sin(\kappa t)}{\kappa} - $$
$$- \frac{\left[a_1 - a_2(0)\,\kappa\right]\cos(\kappa t)}{\kappa} + \frac{a_1}{\kappa}\,. \qquad (11)$$

Now we will find numerical solution of equations (4) and (5) when $\alpha \neq \beta \neq 1$. Using Maxima [4], we find:

```
(%i12) 'diff(f(t),t)+%i*a*f(t)=%i*b*g(t);
(%o12) 'diff(f(t),t,1)+%i*a*f(t)=%i*b*g(t)
(%i13) 'diff(g(t),t)+%i*c*g(t)=%i*b*f(t);
(%o13) 'diff(g(t),t,1)+%i*c*g(t)=%i*b*f(t)
(%i14) desolve([%o12,%o13],[f(t),g(t)]);
```

and the solution is found to be quite complicated: these are formulae (13) and (14).

Translated back these results into our equations (2) and (3), the solutions are given by (15) and (16), where we can define a new "ratio":

$$\xi = \sqrt{\beta^2 - 2\alpha\beta + 4\kappa^2 + \alpha^2}\,. \qquad (12)$$

It is perhaps quite interesting to remark here that there is no "distance" effect in these equations.

Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

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