The Logarithmic Potential and an Exponential Mass Function for Elementary Particles

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The assumption that elementary particles with nonzero rest mass consist of relativistic constituents moving with constant energy pc results in a logarithmic potential and exponential expression for particle masses. This approach is put to a test by assigning each elementary particles mass a position on a logarithmic spiral. Particles then accumulate on straight lines. It is discussed if this might be an indication for exponential mass quantization.

1 Introduction

The approach of fitting parts of elementary particle mass spectra involving logarithmic potentials has been subject to research in the past decades. In this paper the simple assumption of relativistic constituents moving with constant energy pc in a logarithmic potential is discussed. A similar approach has already been presented in one of the early papers by Y. Muraki et al. [1], where the additional assumption of circular quantized orbits results in an empiric logarithmic mass function with accurate fits for several meson resonance states.

Besides the basic assumption of constant energy pc of the constituents and a resulting logarithmic potential, however, the physical approach in this paper differs and results in an exponential mass function with elementary particle masses proportional to ϕ^n, where n are integers. ϕ is a constant factor derived and thus not empirical chosen to fit particle masses.

The mass function results in points on a logarithmic spiral lining up under a polar angle φ and being separated by the factor ϕ. Elementary particle masses following this exponential quantization thus would, when placed on the spiral, be found on straight lines. Even slight changes of the value ϕ would change the particle distribution on the spiral significantly. Linear distributions for particle masses on the spiral thus would give hints if the logarithmic potential is an approach worth being further investigated to explain the wide range of elementary particle masses.

2 Physical approach

Elementary particles with mass m consist of confined constituent particles, which are moving with constant energy pc within a sphere of radius R. For this derivation it is not essential to define further properties of the constituents, e.g. if they are rotating strings or particles in circular orbits.

The only assumption made is that the force F needed to counteract a supposed centrifugal force $F_Z \propto c^2/R$ acting on each constituent is equal or proportional to pc/R, thus $F = F_Z = a_1/R$, regardless of the origin of the interaction. The potential energy needed to confine a constituent therefore is

$$E = \int \frac{a_1}{R}\, dR = a_1 \int \frac{1}{R}\, dR = a_1 \ln \frac{R}{R_a}\,, \tag{2a}$$

where R_a is the integration constant and a_1 a parameter to be referred to later. The center of mass of the elementary particle as seen from the outside and thus the mass that is assigned to the system is

$$m = \frac{\hbar}{cR}\,. \tag{2b}$$

The logarithmic potential energy in Eq. (2a) is assumed to be proportional to m/R, yielding

$$E = \frac{a_2 m}{R}\,. \tag{2c}$$

Both parameters a_1 and a_2 are supposed not to be a function of R, but to depend on constituent particle properties and coupling constants, resp. For example, a_1/a_2 could be set equal c^2/γ (γ is the gravitational constant), but such a constraint is not required. Inserting m from Eq. (2b) into Eq. (2c) yields

$$E = a_2 \frac{\hbar}{cR^2}\,. \tag{2d}$$

The angular momentum of the system is assumed to be an integer multiple n of $\hbar$, with a ground state of radius R_0.

$$E_n = a_2 \frac{\hbar}{cR_n^2} = a_2 \frac{(n+1)\hbar}{cR_0^2}, \quad n = 0, 1, 2 \ldots \tag{2e}$$

From Eq. (2a) and Eq. (2e) it follows that

$$\ln \frac{R_a}{R_n} = -(n+1) \frac{R_a^2}{R_0^2} \quad \text{with} \quad R_a = \left(\frac{a_2 \hbar}{a_1 c}\right)^{\frac{1}{2}}, \tag{2f}$$

assigning the integration constant R_a a value. For $n = 0$ the value for R_n is set to R_0, allowing to calculate the ratio R_a/R_0 using Eq. (2f)

$$x = e^{-x^2} \quad \text{with} \quad x = \frac{R_a}{R_0}\,,$$

and with defining $\phi = 1/x$ resulting in

$$\phi = 1.53158. \tag{2g}$$

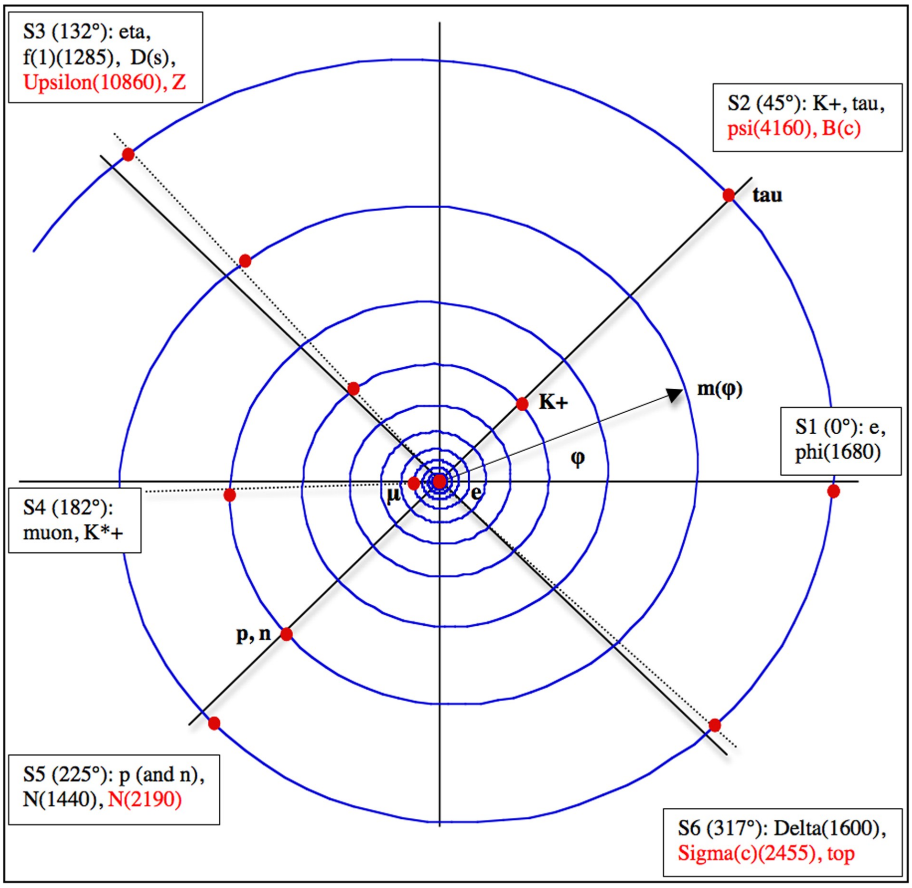

Fig. 1: The masses of elementary particles placed on the spiral and listed for each resulting sequence starting from the center. The solid lines are seperated by 45°. The red dot in the center is the electron at 0°. The outer limit of the spiral at 135° is about 2 GeV. Particles allocated on a sequence, but with masses too large for this scale are marked red in the attached listings of sequence particles. The top for example is far outside on S6 at 317°.

Since $\ln\phi = 1/\phi^2$ it follows that

$$R_n = R_a e^{(n+1)\ln\phi}\,. \tag{2h}$$

With Eq. (2b) and Eq. (2f) R_a can be written as

$$R_a = R_0\alpha\,, \tag{2i}$$

where

$$R_0 = \frac{\hbar}{m_0 c} \quad \text{with} \quad \alpha = m_0\left(\frac{a_2 c}{a_1\hbar}\right)^{\frac{1}{2}}$$

and inserting R_a into Eq. (2h) yields

$$R_n = R_0 e^{k\varphi_n} \quad \text{where} \quad k = \frac{1}{2\pi}\ln\phi\,, \tag{2j}$$

and

$$\varphi_n = 2\pi(n+1) + \varphi_s \quad \text{and} \quad \varphi_s = 2\pi\,\frac{\ln\alpha}{\ln\phi}\,.$$

Eq. (2j) applies to particle masses by inserting R_n into Eq. (2b). Thus with

$$m_n = \frac{\hbar}{R_n c} \quad \text{and} \quad m_0 = \frac{\hbar}{R_0 c}$$

it follows that

$$m_n = m_0 e^{k\varphi_n}\,. \tag{2k}$$

In Eq. (2k) $-k$ is substituted by k, which just determines to start with m_0 as the smallest instead of the biggest mass and thus turning the spiral from the inside to the outside instead vice versa. This has no influence on the results. m_n are elementary particle masses and points on a logarithmic spiral lining up at an angle φ_s as defined in Eq. (2j). These points are referred to as a particle sequence $S(\varphi_s)$. The angle φ_s should not be the same for all elementary particles since it is a function of the parameters a_1 and a_2.

To determine whether elementary particle masses tend to line up in sequences first of all a logarithmic spiral

$$m(\varphi) = m_0 e^{k\varphi}$$

with continues values for φ is calculated. m_0 is the initial mass and thus starting point of the spiral at $\varphi = 0$. The starting point $m_0 = m(\varphi = 0)$ is set so that as a result the electron is placed at the angle $\varphi = 0$.

One turn of the spiral $m(\varphi) \to m(\varphi + 2\pi)$ corresponds to multiplying $m(\varphi)$ by ϕ, yielding $m(\varphi)\phi = m(\varphi + 2\pi)$. Spiral points lining up at the same polar angle φ differ by a factor ϕ.

In a second step for each elementary particle mass pro-

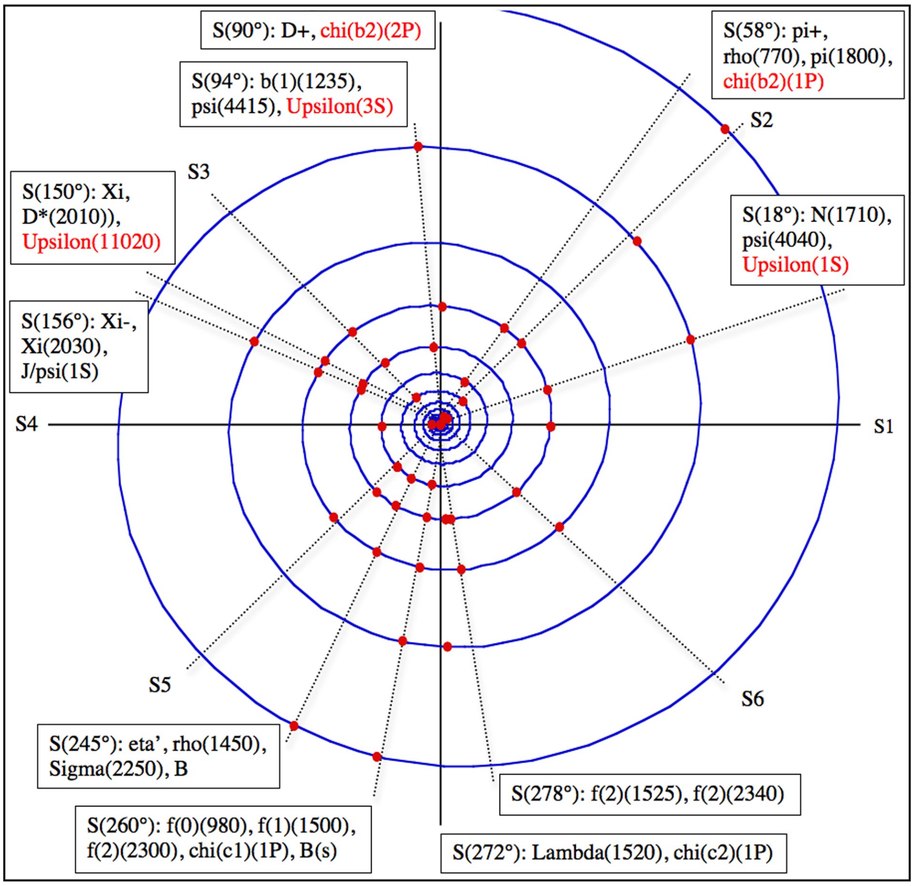

Fig. 2: Additional sequences shown within a mass range of 6.5 GeV. See Fig. 1 for listings of S1-S6.

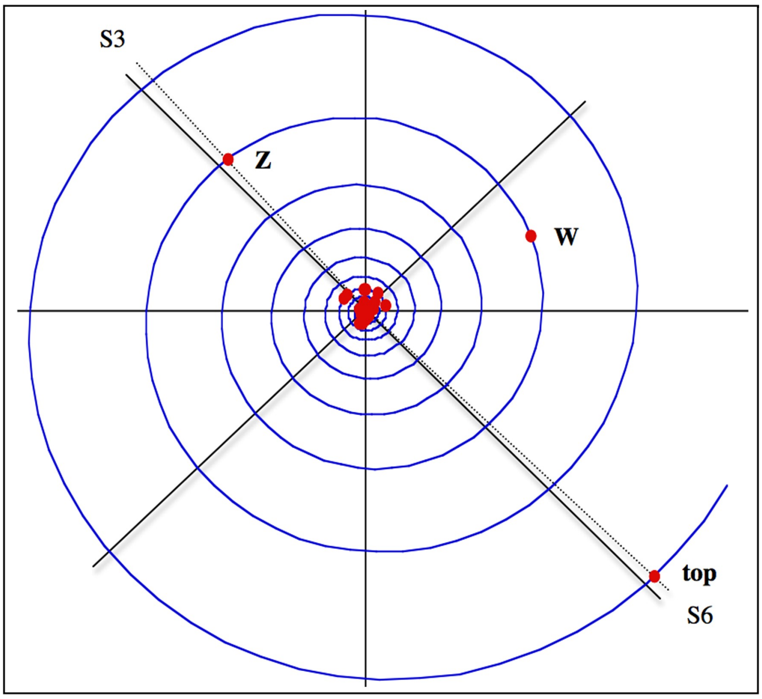

Fig. 3: At a mass range of 175 GeV the Z and top align with S3 and S6, resp., as listed in Fig. 1.

vided by the PDG table 2004 [2, 3] the resulting angle φ_s in the logarithmic spiral is calculated using Eq. (2k) with m_0 as the electron mass and φ_s as defined in Eq. (2j). This results in polar coordinates (m_n, φ_s) and thus a point on the spiral for each elementary particle.

After all elementary particles are entered as points into the spiral it is analyzed if sequences $S(\varphi_s)$, thus particle masses m_n lining up in the spiral in the same direction φ_s are found.

3 Results

The results for particle sequences are shown step by step for mass ranges from 2 GeV to 175 GeV to provide a clear overview. Elementary particles which are part of a sequence, but out of the shown mass range and thus not displayed as red dots in the spiral are marked red in the list of sequence particles, which is attached to each sequence.

All allocations of elementary particle masses to sequences are accurate within at least $\Delta m/m = 4{\times}10^{-3}$. All sequence positions are fitted and accurate within $\varphi_s \pm 0,5°$.

Fig. 1 shows the results within a mass range of 2 GeV from the center to the outer limit of the spiral. The position of the electron is set to 0° as the starting point of the spiral, the muon then is found to be at 182°. Also on these sequences are the phi (1680) and the K* (892), resp.

The K+, tau, psi (4160) and B (c) are at 45°. The proton, N (1440) and N (2190) opposite at 225°. The eta, f (1)(1285), D (s), Upsilon (10860), Z-boson are at 132° and the Delta (1600), Sigma (c)(2455) and the top opposite at 317°, resp. Calculating the Planck mass with $m_{pl} = (\hbar c/\gamma)^{1/2}$ results in a position on sequence S6.

In Fig. 2 additional sequences within a mass range of 6.5 GeV are shown, e.g. the pi+, rho (770), pi (1800) and chi (b2)(1P) are aligned at S (58°).

Also the f (0)(980), f (1)(1500), f (2)(2300), chi (b2)(1P) and B (s) are aligned precisely in a sequence at 260°. The f (2)(1525) and f (2)(2340) align at 278°.

Other sequences are as follows, at 150° (Xi, D* (2010), Upsilon (11020)), at 156° (Xi-, Xi (2030), J/psi (1S)) and at 245° (eta', rho (1450), Sigma (2250), B). Also the psi (4040), psi (4415), Upsilon (1S) and Upsilon (3S) are found in sequences.

A picture of the mass range of elementary particles at 175 GeV is shown in Fig. 3, with the Z and top aligning in the sequences S3 and S6, resp., as listed in Fig. 1.

4 Discussion and conclusion

In this simple model the mass distribution of elementary particles strongly depends on the derived quantization factor ϕ. Even slight changes $\Delta\phi/\phi \approx 5{\times}10^{-4}$ disrupt the particle sequences. Thus of interest are the symmetric sequences S1-S6 with precise positions for the electron, muon, kaon, proton and tau. Also the eta, K (892), D (s), B (c), Upsilon (10860), Z and top are placed on these sequences. Other sequences align particles like f's, pi's and Xi's.

The existence of more than one sequence implies that α in Eq. (2i), i.e. the ratio of parameters a_1 and a_2, has several values within the elementary particle mass spectrum.

Randomly chosen values for ϕ other than the derived one do not provide symmetric and precise results, but rather uniform distributions, as should be expected. The results of the precise and specific sequences in the derived logarithmic spiral still might be a pure coincidence. But they also could be an indication for constituent particles moving in a logarithmic potential, resulting in an exponential quantization for elementary particle masses. Then the results would suggest the logarithmic potential to be considered an approach worth being further investigated to explain the wide range of elementary particle masses.

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