Parameters for Viability Check on Gravitational Theories Regarding the Experimental Data

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Parameterized post-Newtonian formalism requires an existence of a symmetric metric in a gravitational theory in order to perform a viability check regarding the experimental data. The requirement of a symmetric metric is a strong constraint satisfied by very narrow class of theories. In this letter we propose a viability check of a theory using the corresponding theory equations of motion. It is sufficient that a connection exists, not necessarily a metrical one. The method is based on an analysis of the Lorentz invariant terms in the equations of motion. An example of the method is presented on the Einstein-Infeld-Hoffmann equations.

1 Introduction

The parameterized post-Newtonian (PPN) formalism is a tool used to compare classical theories of gravitation in the limit of weak field generated by objects moving slowly compared to c. It is applicable only for symmetric metric theories of gravitation that satisfy the Einstein equivalence principle.

Each parameter in PPN formalism is a measure of departure of a theory from Newtonian gravity represented by several parameters. Following the Will notation [1], there are ten parameters: $\gamma, \beta, \xi, \alpha_1, \alpha_2, \alpha_3, \zeta_1, \zeta_2, \zeta_3, \zeta_4$; γ is a measure of space curvature; β measures the nonlinearity in superposition of gravitational fields; ξ is a check for preferred location effects, i.e. a check for a violation of the strong equivalence principle (SEP) whether the outcomes of local gravitational experiments depend on the location of the laboratory relative to a nearby gravitating body; α_1, α_2, α_3 measure the extent and nature of preferred-frame effects, i.e. how much SEP is violated by predicting that the outcomes of local gravitational experiments may depend on the velocity of the laboratory relative to the mean rest frame of the universe; ζ_1, ζ_2, ζ_3, ζ_4 and α_3 measure the extent and nature of breakdowns in global conservation laws. The PPN metric components are

$$\begin{aligned} g_{00} = &-1+2U-2\beta U^2-2\xi\Phi_W+(2\gamma+2+\alpha_3+\zeta_1-2\xi)\,\Phi_1+ \\ &+2\,(3\gamma-2\beta+1+\zeta_2+\xi)\,\Phi_2+2\,(1+\zeta_3)\,\Phi_3+ \\ &+2\,(3\gamma+3\zeta_4-2\xi)\,\Phi_4-(\zeta_1-2\xi)\,A-(\alpha_1-\alpha_2-\alpha_3)w^2U- \\ &-\alpha_2 w^i w^j U_{ij}+(2\alpha_3-\alpha_1)\,w^i V_i+O(\epsilon^3)\,, \end{aligned} \tag{1.1}$$

$$\begin{aligned} g_{0i} = &-\tfrac{1}{2}\,(4\gamma+3+\alpha_1-\alpha_2+\zeta_1-2\eta)\,V_i- \\ &-\tfrac{1}{2}\,(1+\alpha_2-\zeta_1+2\xi)\,W_i- \\ &-\tfrac{1}{2}\,(\alpha_1-2\alpha_2)\,w^i U-\alpha_2 w^j U_{ij}+O(\epsilon^{5/2})\,, \end{aligned} \tag{1.2}$$

$$g_{ij} = (1+2\gamma U)\,\delta_{ij}+O(\epsilon^2)\,, \tag{1.3}$$

where w^i is the coordinate velocity of the PPN coordinate system relative to the mean rest-frame of the universe and U, U_{ij}, Φ_W, A, Φ_1, Φ_2, Φ_3, Φ_4, V_i and W_i are the metric potentials constructed from the matter variables and have similar form as the Newtonian gravitational potential [1, 2].

The theories that can be compared using PPN formalism are straightforward alternatives to GR. The bounds on the PPN parameters are not the ultimate criteria for viability of a gravitational theory, because many theories can not be compared using PPN formalism. For example, Misner et al. [3] claim that Cartan's theory is the only non-metric theory to survive all experimental tests up to that date and Turyshev [4] lists Cartan's theory among the few that have survived all experimental tests up to that date. There are general viability criteria [5] for a gravitational theory: (i) is it self-consistent? (ii) is it complete? (iii) does it agree, to within several standard deviations, with all experiments performed to date?

For a symmetric metric theory, the answer of (iii) is consisted in checking the PPN parameters. But, for a non-symmetric or a non-metric theory there is not a convenient method. So, we propose a method for checking (iii) even in the cases when the PPN formalism can not be applied such as non-symmetric metric and non-metric theories. It is based on a Lorentz invariance analysis of all terms in the equations of motion of the corresponding theory. Since there is no general equations of motion formula for all theories, we give an example of the method on the Einstein-Infeld-Hoffmann (EIH) equations. However, the general principle of the method can be applied to any other theory in which the equations of motion can be derived, no matter whether the theory includes a metric or not.

2 Lorentz invariant terms in the EIH equations

Given a system of n bodies, the equations of motion of the j-th body is

$$\frac{d^2\vec{r}_j}{dt^2} = \sum_{i\neq j} \frac{(\vec{r}_i - \vec{r}_j)Gm_i}{r_{ij}^3}\Bigg[1 - \frac{3}{2c^2}\frac{[\dot{\vec{r}}_i \cdot (\vec{r}_j - \vec{r}_i)]^2}{r_{ij}^2} - \frac{2(\beta+\gamma)}{c^2}\sum_{k\neq j}\frac{Gm_k}{r_{jk}} - \frac{2\beta-1}{c^2}\sum_{k\neq i}\frac{Gm_k}{r_{ik}} + \frac{1}{2c^2}(\vec{r}_i - \vec{r}_j)\dot{\vec{v}}_i - \frac{2(1+\gamma)}{c^2}\dot{\vec{r}}_i\dot{\vec{r}}_j + \gamma\left(\frac{v_j}{c}\right)^2 + (1+\gamma)\left(\frac{v_i}{c}\right)^2\Bigg] + \frac{1}{c^2}\sum_{i\neq j}\frac{Gm_i}{r_{ij}^3}\Big((\vec{r}_j - \vec{r}_i)\cdot\big((2+2\gamma)\dot{\vec{r}}_j - (1+2\gamma)\dot{\vec{r}}_i\big)\Big) \times (\dot{\vec{r}}_j - \dot{\vec{r}}_i) + \frac{3+4\gamma}{2c^2}\sum_{i\neq j}\frac{Gm_i}{r_{ij}}\dot{\vec{v}}_i\,, \tag{2.1}$$

where $\vec{r}_s$ is the radius-vector of the s-th body, $\vec{v}_s = \dot{\vec{r}}_s$ is the velocity of the s-th body and upper dot marks the differentiation with time. Formula (2.1) can be rearranged in the form

$$\frac{d^2\vec{r}_j}{dt^2} = \sum_{i\neq j} \frac{(\vec{r}_i - \vec{r}_j)Gm_i}{r_{ij}^3}\Bigg[1 - \frac{3}{2c^2}\frac{[\dot{\vec{r}}_i \cdot (\vec{r}_j - \vec{r}_i)]^2}{r_{ij}^2} + \frac{1}{c^2}\sum_{k\neq i}\frac{Gm_k}{r_{ik}} + \frac{1}{2c^2}(\vec{r}_i - \vec{r}_j)\dot{\vec{v}}_i - \frac{2}{c^2}\dot{\vec{r}}_i\dot{\vec{r}}_j + \left(\frac{v_i}{c}\right)^2\Bigg] + \frac{1}{c^2}\sum_{i\neq j}\frac{Gm_i}{r_{ij}^3}\Big((\vec{r}_j - \vec{r}_i)\cdot(2\dot{\vec{r}}_j - \dot{\vec{r}}_i)\Big)(\dot{\vec{r}}_j - \dot{\vec{r}}_i) + \frac{3}{2c^2}\sum_{i\neq j}\frac{Gm_i}{r_{ij}}\dot{\vec{v}}_i + \gamma\Bigg[\sum_{i\neq j}\frac{(\vec{r}_i - \vec{r}_j)Gm_i}{r_{ij}^3} \times \left(-\frac{2}{c^2}\sum_{k\neq j}\frac{Gm_k}{r_{jk}} + \frac{1}{2c^2}(\vec{r}_i - \vec{r}_j)\dot{\vec{v}}_i + \frac{1}{c^2}(\dot{\vec{r}}_i - \dot{\vec{r}}_j)^2\right) + \frac{2}{c^2}\sum_{i\neq j}\frac{Gm_i}{r_{ij}^3}\Big((\vec{r}_j - \vec{r}_i)\cdot(\dot{\vec{r}}_j - \dot{\vec{r}}_i)\Big)(\dot{\vec{r}}_j - \dot{\vec{r}}_i) + \frac{2}{c^2}\sum_{i\neq j}\frac{Gm_i}{r_{ij}}\dot{\vec{v}}_i\Bigg] - \beta\,\frac{2}{c^2}\Bigg[\sum_{i\neq j}\frac{(\vec{r}_i - \vec{r}_j)Gm_i}{r_{ij}^3} \times \left(\sum_{k\neq j}\frac{Gm_k}{r_{jk}} + \sum_{k\neq i}\frac{Gm_k}{r_{ik}}\right)\Bigg]. \tag{2.2}$$

The second and the third term are of order c^{-2} and each of them is Lorentz invariant, neglecting the terms of order c^{-4} and smaller, i.e. they take same values in all inertial systems. So, (2.2) means

$$\frac{d^2\vec{r}_j}{dt^2} = \sum_{i\neq j}\Bigg\{-\frac{(\vec{r}_j - \vec{r}_i)Gm_i}{r_{ij}^3}\Bigg[1 - \frac{3}{2}\frac{[\vec{v}_i\cdot(\vec{r}_j - \vec{r}_i)]^2}{r_{ij}^2c^2} + \frac{v_i^2}{c^2} - 2\frac{\vec{v}_i\cdot\vec{v}_j}{c^2}\Bigg] + \frac{Gm_i}{r_{ij}^3c^2}(\vec{v}_j - \vec{v}_i)\big[(\vec{r}_j - \vec{r}_i)\cdot\vec{v}_j\big]\Bigg\} + \text{Lorentz invariant terms.} \tag{2.3}$$

Every single Lorentz invariant term in (2.2), i.e. in (2.3), can be replaced by a term proportional to the corresponding Lorentz invariant term, so

$$\frac{d^2\vec{r}_j}{dt^2} = \sum_{i\neq j}\Bigg\{-\frac{(\vec{r}_j - \vec{r}_i)Gm_i}{r_{ij}^3}\Bigg[1 - \frac{3}{2}\frac{[\vec{v}_i\cdot(\vec{r}_j - \vec{r}_i)]^2}{r_{ij}^2c^2} + \frac{v_i^2}{c^2} - 2\frac{\vec{v}_i\cdot\vec{v}_j}{c^2}\Bigg] + \frac{Gm_i}{r_{ij}^3c^2}(\vec{v}_j - \vec{v}_i)[(\vec{r}_j - \vec{r}_i)\cdot\vec{v}_j] + A\,\frac{(\vec{r}_i - \vec{r}_j)Gm_i}{r_{ij}^3}\frac{(\vec{v}_i - \vec{v}_j)^2}{c^2} + B\,\frac{(\vec{r}_i - \vec{r}_j)Gm_i}{r_{ij}^3c^2} \times [(\vec{r}_i - \vec{r}_j)\cdot\dot{\vec{v}}_i] + C\,\frac{Gm_i}{r_{ij}c^2}\dot{\vec{v}}_i + D\,\frac{Gm_i}{r_{ij}^3c^2}(\vec{v}_j - \vec{v}_i) \times [(\vec{r}_j - \vec{r}_i)\cdot(\vec{v}_j - \vec{v}_i)] + E\sum_{k\neq i,j}\frac{(\vec{r}_i - \vec{r}_j)Gm_i}{r_{ij}^3}\frac{Gm_k}{r_{ki}c^2} + F\sum_{k\neq i,j}\frac{(\vec{r}_i - \vec{r}_j)Gm_i}{r_{ij}^3}\frac{Gm_k}{r_{kj}c^2}\Bigg\}. \tag{2.4}$$

The bounds on the parameters A, B, C, D, E and F can be determined directly from the experimental data. Now, the viability check of any gravitational theory regarding the agreement on the experimental data would be consisted in checking how the theory fits in the bounds of the new parameters.

3 Conclusion

In this letter we introduced a new approach of viability check of gravitational theories regarding the experimental data, based on the analysis of the Lorentz invariance of the equations of motion. An example is given for the EIH equations. This method can be applied on any theory that has a connection regardless it is metrical or not. The bounds of the new parameters can be determined directly from the experimental data.

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