On the Generalized Maxwell Equations and Their Prediction of Electroscalar Wave

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We have formulated the basic laws of electromagnetic theory in quaternion form. The formalism shows that Maxwell equations and Lorentz force are derivable from just one quaternion equation that only requires the Lorentz gauge. We proposed a quaternion form of the continuity equation from which we have derived the ordinary continuity equation. We introduce new transformations that produces a scalar wave and generalize the continuity equation to a set of three equations. These equations imply that both current and density are waves. Moreover, we have shown that the current can not circulate around a point emanating from it. Maxwell equations are invariant under these transformations. An electroscalar wave propagating with speed of light is derived upon requiring the invariance of the energy conservation equation under the new transformations. The electroscalar wave function is found to be proportional to the electric field component along the charged particle motion. This scalar wave exists with or without considering the Lorentz gauge. We have shown that the electromagnetic fields travel with speed of light in the presence or absence of free charges.

1 Introduction

Quaternions are mathematical construct that are generalization of complex numbers. They were introduced by Irish mathematician Sir William Rowan Hamilton in 1843 (Sweetser, 2005 [1]). They consist of four components that are represented by one real component (imaginary part) and three vector components (real part). Quaternions are closed under multiplication. Because of their interesting properties one can use them to write the physical laws in a compact way. A quaternion $\widetilde{A}$ can be written as $\widetilde{A} = A_0 + A_1 i + A_2 j + A_3 k$, where $i^2 = j^2 = k^2 = -1$ and $ij = k$, $ki = j, jk = i$, $ijk = -1$. A_0 is called the scalar component and A_1, A_2, A_3 are the vector components. Each component consists of real part and imaginary part. The real part of the scalar component vanishes. Similarly the imaginary part of the vector component vanishes too. This is the general prescription of quaternion representation.

In this paper we write the Maxwell equations in quaternion including the Lorentz force and the continuity equation. We have found that the Maxwell equations are derived from just one quaternion equation. The solution of these equations shows that the charge and current densities are waves traveling with speed of light. Generalizing the continuity equation resulted in obtaining three equations defining the charge and current densities. Besides, there exists a set of transformation that leave generalized continuity equation invariant. When these transformations are applied to the energy conservation law an electroscalar wave propagating with speed of light is obtained. Thus, the quaternionic Maxwell equation and continuity equation predict that there exist a scalar wave propagating with speed of light. This wave could possibly arise due to vacuum fluctuation. Such a wave is not included in the Maxwell equations. Therefore, the existence of the electroscalar is a very essential integral part of Maxwell theory. Expressions of Lorentz force and the power delivered to a charge particle are obtained from the quaternion Lorentz force.

Moreover, the current and charge density are solutions of a wave equation travelling with speed of light. Furthermore, we have shown that the electromagnetic field travels with speed of light in the presence and/or absence of charge. However, in Maxwell theory the electromagnetic field travels with speed of light only if there is no current (or free charge) in the medium. We have found here two more equations relating the charge and current that should supplement the familiar continuity equation. These two equations are found to be compatible with Maxwell equations. Hence, Maxwell equations are found to be invariant under these new transformations. This suggests that the extra two equations should be appended to Maxwell equations. Accordingly, we have found an electroscalar wave propagating at the speed of light. The time and space variation of this electroscalar wave induce a charge density and current density even in a source free. The electroscalar wave arises due to the invariance of the Maxwell equations under the new set of transformations. We have shown that such a scalar wave is purely electric and has no magnetic component. This is evident from the Poynting vector that has only two components, one along the particle motion and the other along the electric field direction. We remark that Maxwell equations are still exact and need no modifications. They steadily predict the existence of the a electroscalar wave if we impose the new transformation we obtained in this work.

2 Derivation of Maxwells' equations

The multiplication of two quaternions is given by

$$\tilde{A}\,\tilde{B} = (A_0, \vec{A})\,(B_0, \vec{B}) = \\ = (A_0 B_0 - \vec{A}\cdot\vec{B}, A_0\vec{B} + \vec{A}\,B_0 + \vec{A}\times\vec{B})\,. \qquad (1)$$

We define the quaternion D'Alembertian operator as

$$\tilde{\Box}^2 \equiv -|\nabla|^2 = -\tilde{\nabla}\tilde{\nabla}^* = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \vec{\nabla}\cdot\vec{\nabla}\,, \qquad (2)$$

where Nabla and its conjugate are defined by

$$\tilde{\nabla} = \left(\frac{i}{c}\frac{\partial}{\partial t}\,,\,\vec{\nabla}\right), \qquad \tilde{\nabla}^* = \left(\frac{i}{c}\frac{\partial}{\partial t}\,,\,-\vec{\nabla}\right). \qquad (3)$$

The wave equation of the quaternionic vector potential $\tilde{A} = (i\frac{\varphi}{c}\,, \vec{A})$ has the form

$$\tilde{\Box}^2\tilde{A} = \mu_0\tilde{J}\,, \qquad \tilde{J} = (ic\rho\,, \vec{J})\,. \qquad (4)$$

where ρ is the charge density.

The electric and magnetic fields are defined by (Jackson, 1967 [2])

$$\vec{E} = -\left(\vec{\nabla}\varphi + \frac{\partial\vec{A}}{\partial t}\right), \qquad \vec{B} = \vec{\nabla}\times\vec{A}\,. \qquad (5)$$

Using Eqs. (1)–(3), the scalar part of Eq. (4) now reads

$$-\frac{i}{c}\vec{\nabla}\cdot\left(\vec{\nabla}\varphi + \frac{\partial\vec{A}}{\partial t}\right) + \frac{i}{c}\frac{\partial}{\partial t}\left(\frac{1}{c^2}\frac{\partial\varphi}{\partial t} + \vec{\nabla}\cdot\vec{A}\right) - \\ - \vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{A}\right) = ic\mu_0\rho\,. \qquad (6)$$

Using Eq. (5) the above equation yields

$$\vec{\nabla}\cdot\vec{B} = 0\,, \qquad (7)$$

$$\frac{1}{c^2}\frac{\partial\varphi}{\partial t} + \vec{\nabla}\cdot\vec{A} = 0\,, \qquad (8)$$

and

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}\,, \qquad (9)$$

where $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$. This is the Gauss Law and is one of the Maxwell equations.

The vector part of the Eq. (4) can be written as

$$-\frac{i}{c}\left(\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t}\right) + \left(\vec{\nabla}\times\vec{B} - \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t}\right) - \\ - \vec{\nabla}\left(\frac{1}{c^2}\frac{\partial\varphi}{\partial t} + \vec{\nabla}\cdot\vec{A}\right) = \mu\vec{J}\,. \qquad (10)$$

This yields the two equations

$$\vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0 \qquad (11)$$

and

$$\vec{\nabla}\times\vec{B} - \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} = \mu_0\vec{J}\,. \qquad (12)$$

Eqs. (7), (9), (11) and (12) are the Maxwell equations.

By direct cancelation of terms, Eqs. (6) and (10), yield the wave equations of the scalar potential φ and the vector potential $\vec{A}$, viz., $\Box^2\varphi = \frac{\rho}{\varepsilon_0}$ and $\Box^2\vec{A} = \mu_0\vec{J}$.

We thus see that we are able to derive Maxwell equations from the wave equation of the quaternion vector potential. In this formalism only Lorentz gauge is required by the quaternion formulation to derive Maxwell equations. This would mean that Lorentz gauge is more fundamental. It is thus very remarkable that one are able to derive Maxwell equations from just one quaternion equation. Notice that with the 4-vector formulation Maxwell equation are written in terms of two sets of equations.

3 The quaternionic Lorentz force

The quaternionic Lorentz force can be written in the form

$$\left.\begin{array}{l} \tilde{F} = q\,\tilde{V}(\tilde{\nabla}\tilde{A})\,, \quad \tilde{V} = (ic\,, \vec{v})\,, \quad \tilde{F} = \left(i\,\frac{P}{c}\,,\,\vec{F}\right) \\ \tilde{A} = \left(\frac{i\varphi}{c}\,,\,\vec{A}\right), \ \tilde{\nabla} = \left(\frac{i}{c}\frac{\partial}{\partial t}\,,\,\vec{\nabla}\right) \end{array}\right\}, \qquad (13)$$

where P is the power. The scalar part of the above equation can be written in the form

$$-\,iq\,c\left[\left(\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial\varphi}{\partial t}\right) + \frac{\vec{v}}{c^2}\cdot\left(\vec{\nabla}\varphi + \frac{\partial\vec{A}}{\partial t}\right)\right] - \\ -\,q\,\vec{v}\cdot\vec{\nabla}\times\vec{A} = i\,\frac{P}{c}\,. \qquad (14)$$

Upon using Eqs. (5) and (8), one gets

$$q\,\vec{v}\cdot\vec{\nabla}\times\vec{A} = 0 \quad \Rightarrow \quad \vec{v}\cdot\vec{B} = 0\,, \qquad (15)$$

and

$$P = q\,\vec{v}\cdot\vec{E}\,. \qquad (16)$$

This is the usual power delivered to a charged particle in an electromagnetic field. Eq. (15) shows that the charged particle moves in a direction normal to the direction of the magnetic field.

Now the vector component of Eq. (13) is

$$q\left[-\frac{\partial\vec{A}}{\partial t} - \vec{\nabla}\varphi + ic\,\vec{\nabla}\times\vec{A} - \frac{\vec{v}}{c^2}\frac{\partial\varphi}{\partial t} - \right. \\ \left. -\,\vec{v}(\vec{\nabla}\cdot\vec{A}) + \vec{v}\times\left(\frac{i}{c}\frac{\partial\vec{A}}{\partial t} + \frac{i}{c}\,\vec{\nabla}\varphi + \vec{\nabla}\times\vec{A}\right)\right] = \vec{F}\,. \qquad (17)$$

This yields the two equations

$$\vec{F} = q\left(\vec{E} + \vec{v}\times\vec{B}\right), \tag{18}$$

and

$$\vec{B}_m \equiv \vec{B} = \frac{\vec{v}}{c^2}\times\vec{E}\,. \tag{19}$$

Eq. (18) is the familiar Lorentz force. Eq. (19) gives a new relation between the magnetic field of a moving charge due to an electric field. Thus, we are able to derive the power and the Lorentz force on a charged particle. This new magnetic field may be interpreted as the magnetic field seen in a frame moving with velocity $\vec{v}$ when $\vec{B}=0$ in the rest frame. It is thus an apparent field. This equation is compatible with Eq. (15), since $\vec{v}\cdot\vec{B}_m = \vec{v}\cdot(\frac{\vec{v}}{c^2}\times\vec{E}) = 0$, by vector property. Moreover, we notice that $\vec{E}\cdot\vec{B}_m = \vec{E}\cdot(\frac{\vec{v}}{c^2}\times\vec{E}) = 0$. This clearly shows that the magnetic field produced by the charged particle is perpendicular to the electric field applied on the particle. Thus, a charged particle when placed in an external electric field produces a magnetic field perpendicular to the direction of the particle motion and to the electric field producing it. As evident from Eq. (19), this magnetic field is generally very small due to the presence of the factor c^2 in the dominator. Hence, the reactive force arising from this magnetic field is

$$\vec{F}_m = q\,\vec{v}\times\vec{B}_m\,, \tag{20}$$

which upon using Eq. (19) yields

$$\vec{F}_m = q\,\vec{v}\times\left(\frac{\vec{v}}{c^2}\times\vec{E}\right). \tag{21}$$

Using Eq. (16) and the vector properties, this can be casted into

$$\vec{F}_m = \frac{P}{c^2}\,\vec{v} - \frac{v^2}{c^2}\,q\,\vec{E}\,. \tag{22}$$

This reactive force acts along the particle motion (longitudinal) and field direction. The negative sign of the second term is due to the back reaction of the charge when accelerates by the external electric field. The total force acting on the charge particle is $\vec{F}_{\text{total}} = q(\vec{E} + \vec{v}\times\vec{B}_{\text{total}})$, $\vec{B}_{\text{total}} = \vec{B} + \vec{B}_m$, $\vec{F}_{\text{total}} = q(1-\frac{v^2}{c^2})\vec{E} + q\vec{v}\times\vec{B} + \frac{P}{c^2}\vec{v}$. Notice that when $v \ll c$, this force reduces to the ordinary force and no noticeable difference will be observed. However, when $v\approx c$ measurable effects will be prominent.

4 Continuity equation

The quaternion continuity equation can be written in the form

$$\tilde{\nabla}\tilde{J} = 0\,, \qquad \tilde{J} = \left(i\rho c\,, \vec{J}\right), \tag{23}$$

so that the above equation becomes

$$\tilde{\nabla}\tilde{J} = \left[-\left(\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t}\right),\right.$$
$$\left.\frac{i}{c}\left(\frac{\partial\vec{J}}{\partial t} + \vec{\nabla}\rho\,c^2\right) + \vec{\nabla}\times\vec{J}\right] = 0\,. \tag{24}$$

which yields the following three equations

$$\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t} = 0\,, \tag{25}$$

and

$$\vec{\nabla}\rho + \frac{1}{c^2}\frac{\partial\vec{J}}{\partial t} = 0\,, \tag{26}$$

so that

$$\vec{\nabla}\times\vec{J} = 0\,. \tag{27}$$

Using the Stockes theorem one can write Eq. (27) to get, $\int\vec{J}\cdot d\vec{\ell} = 0$. Eqs. (26) and (27) are new equations for a flow. Eq. (27) states that a current emanating from a point in spacetime does not circulate to the same point. In comparison with a magnetic field, we know that the magnetic field lines have circulation.

Now take the dot product of both sides of Eq. (26) with $d\vec{S}$, where S is a surface, and integrate to get

$$\int\vec{\nabla}\rho\cdot d\vec{S} + \int\frac{1}{c^2}\frac{\partial\vec{J}\cdot d\vec{S}}{\partial t} = 0\,, \tag{28}$$

or

$$\int\vec{\nabla}\rho\cdot d\vec{S} + \frac{1}{c^2}\frac{\partial I}{\partial t} = 0\,, \qquad I = \int\vec{J}\cdot d\vec{S}\,. \tag{29}$$

But from Stokes' theorem $\int\vec{A}\cdot d\vec{S} = \int\vec{\nabla}\times\vec{A}\cdot d\vec{\ell}$. Therefore, one gets

$$\int\vec{\nabla}\rho\cdot d\vec{S} = \int\vec{\nabla}\times(\vec{\nabla}\rho)\cdot d\vec{\ell} = 0\,, \quad \vec{A} = \vec{\nabla}\rho\,. \tag{30}$$

This implies that $\frac{\partial I}{\partial t} = 0$ which shows that the current is conserved. This is a Kirchoff-type law of current loops. However, Eq. (25) represents a conservation of charge for electric current.

Eq. (27) suggests that one can write the current density as

$$\vec{J} = \vec{\nabla}\Lambda\,, \tag{31}$$

where Λ is some scalar field. It has a dimension of Henry (H). It thus represent a magnetic field intensity. We may therefore call it a magnetic scalar. Substituting this expression in Eq. (26) and using Eq. (42), one yields

$$\nabla^2\Lambda - \frac{1}{c^2}\frac{\partial^2\Lambda}{\partial t^2} = 0\,. \tag{32}$$

This means that the scalar function $\Lambda(r,t)$ is a wave traveling with speed of light.

Now taking the divergence of Eq. (26), one gets

$$\vec{\nabla}\cdot\vec{\nabla}(\rho\,c^2) + \frac{\partial\vec{\nabla}\cdot\vec{J}}{\partial t} = 0\,, \tag{33}$$

which upon using Eq. (25) becomes

$$\nabla^2(\rho\,c^2) + \frac{\partial\frac{-\partial\rho}{\partial t}}{\partial t} = 0\,, \tag{34}$$

or

$$\frac{1}{c^2}\frac{\partial^2\rho}{\partial t^2} - \nabla^2\rho = 0\,, \tag{35}$$

which states the the charge scalar (ρ) is a field propagating with speed of light.

Now take the curl of Eq. (27) to get

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{J}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{J}) - \nabla^2\vec{J} = 0\,, \tag{36}$$

and upon using Eq. (25) and (26) one gets

$$\vec{\nabla}\left(-\frac{\partial\rho}{\partial t}\right) - \nabla^2\vec{J} = \frac{\partial(-\vec{\nabla}\rho)}{\partial t} - \nabla^2\vec{J} = = \frac{\partial\frac{1}{c^2}\frac{\partial\vec{J}}{\partial t}}{\partial t} - \nabla^2\vec{J} = 0\,, \tag{37}$$

which states that the current density satisfies a wave that propagate with speed of light, i.e.,

$$\frac{1}{c^2}\frac{\partial^2\vec{J}}{\partial t^2} - \nabla^2\vec{J} = 0\,. \tag{38}$$

Therefore, both the current and charge densities are solutions of a wave equation traveling with a speed of light. This is a remarkable result that does not appear in Maxwell initial derivation. Notice however that if we take $\frac{\partial}{\partial t}$ of Eq. (12) and apply Eqs. (11) and (9), we get

$$\frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} - \nabla^2\vec{E} = -\frac{1}{\varepsilon_0}\left(\vec{\nabla}\rho + \frac{1}{c^2}\frac{\partial\vec{J}}{\partial t}\right). \tag{39}$$

Now take the curl of both sides of Eq. (12) and apply Eqs. (11) and (7), we get

$$\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} - \nabla^2\vec{B} = \mu_0\big(\vec{\nabla}\times\vec{J}\big)\,. \tag{40}$$

The left hand side of Eqs. (39) and (40) is zero according to Eqs. (26) and (27). Therefore, they yield electric and magnetic fields travelling with speed of light. However, Maxwell equations yield electric and magnetic fields propagating with speed of light only if $\vec{J}=0$ and $\rho=0$ (free space). Because of Eqs. (26) and (27) electromagnetic field travels with speed of light whether the space is empty or having free charges. It seems that Maxwell solution is a special case of the above two equations. Therefore, Eqs. (39) and (40) are remarkable.

Now we introduce the new gauge transformations of $\vec{J}$ and ρ as:

$$\rho' = \rho + \frac{1}{c^2}\frac{\partial\Lambda}{\partial t}\,, \qquad \vec{J}' = \vec{J} - \vec{\nabla}\Lambda\,, \tag{41}$$

leaving Eqs. (25) - (27) invariant, where Λ satisfies the wave equation

$$\frac{1}{c^2}\frac{\partial^2\Lambda}{\partial t^2} - \nabla^2\Lambda = -\left(\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho}{\partial t}\right). \tag{42}$$

These transformations are similar to gauge transformations endorse on the vector potential ($\vec{A}$) and the scalar potential (φ) leaving $\vec{E}$ and $\vec{B}$ invariant. It is interesting to see that the current $\vec{J}$ and density ρ are not unique, however. van Vlaenderen and Waser arrived at similar equations, but they attribute the Λ field to a longitudinal electroscalar wave in vacuum. Thus, even if there is no charge or current density present in a region, the scalar field Λ could act as a source for the electromagnetic field. Such a term could come from quantum fluctuations of the vacuum. This is a very intriguing result. Notice from Eq. (41) that the scalar wave (Λ) distribution induces a charge density, $\rho_{\text{vacuum}} = \frac{1}{c^2}\frac{\partial\Lambda}{\partial t}$, and a current $\vec{J}_{\text{vacuum}} = -\vec{\nabla}\Lambda$. It may help understand the Casimir force generated when two uncharged metallic plates in a vacuum, placed a few micrometers apart, without any external electromagnetic field attract each other(Bressi, *et al.*, 2002 [3]). Notice that this vacuum current and density satisfy the continuity equations, Eqs. (25)–(27). Note that these vacuum quantities could be treated as a correction of the current and charge, since in quantum electrodynamics all physical quantities have to be renormalized. It is interesting that the Maxwell equations expressed in Eqs. (39) and (40), are invariant under the transformation in Eq. (41) provided that $\vec{E}' = \vec{E}$, $\vec{B}' = \vec{B}$. It is thus remarkable to learn that Maxwell equations are invariant under the transformation,

$$\left.\begin{array}{ll} \rho' = \rho + \dfrac{1}{c^2}\dfrac{\partial\Lambda}{\partial t}\,, & \vec{J}' = \vec{J} - \vec{\nabla}\Lambda \\ \vec{E}' = \vec{E}, & \vec{B}' = \vec{B} \end{array}\right\}. \tag{43}$$

We notice from Eq. (42) that the electroscalar wave propagates with speed of light if the charge is conserved. However, if the charge is not conserve then Λ will have a source term equals to the charge violation term. In this case the electroscalar wave propagates with a speed less than the speed of light. Hence, charge conservation can be detected from the propagation speed of this electroscalar wave.

5 Poynting vector

The Poynting theorem, which represents the energy conservation law is given by (Griffiths, 1999 [4])

$$\frac{\partial u}{\partial t} + \vec{\nabla}\cdot\vec{S} = -\vec{J}\cdot\vec{E}\,, \tag{44}$$

where $\vec{S}$ is the Poynting vector, which gives the direction of energy flow and u is the energy density. However, in our present case we have

$$\frac{\partial u_{\text{total}}}{\partial t} + \vec{\nabla}\cdot\vec{S}_{\text{total}} = -\vec{J}\,'\cdot\vec{E}\,', \tag{45}$$

where $\vec{S}_{\text{total}} = \vec{S}_{\text{em}} + \vec{S}_m$ is the total Poynting vector, $\vec{S}_{\text{em}} = \frac{\vec{E}\times\vec{B}}{\mu_0}$, and $u_{\text{total}} = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}(\vec{B}+\vec{B}_m)^2$. Because of Eqs. (15) and (19), the cross term in the bracket vanishes. Hence,

$$u_{\text{total}} = \frac{1}{2}\varepsilon_0\left(1+\frac{v^2}{c^2}\right)E^2 + \frac{B^2}{2\mu_0} - \frac{1}{2}\varepsilon_0\left(\frac{\vec{v}}{c}\cdot\vec{E}\right)^2. \tag{46}$$

This implies that the excessive magnetic field of the charged particles contributes an energy, $u_m = \frac{1}{2}\,\varepsilon_0 \frac{v^2}{c^2}\,E^2 \times \times \left(1 - (\hat{n}\cdot\hat{e})^2\right)$, where $\hat{n}$ and $\hat{e}$ are two unit vectors along the motion of the particle and the electric field. This contribution is generally very small, viz., for $v \ll c$. When the charged particle moves parallel to the electric field, i.e., $\hat{n}\cdot\hat{e} = 1$, its energy density contribution vanishes.

Using Eq. (19), one finds

$$\vec{S}_m = \frac{\vec{E}\times\vec{B}_m}{\mu_0} = \frac{\vec{E}}{\mu_0}\times\left(\frac{\vec{v}\times\vec{E}}{c^2}\right) = = (\varepsilon_0 E^2)\,\vec{v} - (\vec{E}\cdot\vec{v})\,\varepsilon_0\,\vec{E}\,. \quad (47)$$

Using the vector identity, $\vec{\nabla}\cdot(f\vec{A}) = (\vec{\nabla}f)\cdot\vec{A} + (\vec{\nabla}\cdot\vec{A})f$ (Gradstein and Ryzik, 2002 [5]) and Eq. (19), the energy conservation law in Eq. (47) reads

$$\frac{\partial u_{\text{total}}}{\partial t} + \vec{\nabla}\cdot\left(\vec{S}_{\text{em}} + (\varepsilon_0 E^2)\,\vec{v}\right) = = -\vec{E}\cdot\vec{\nabla}\left(\Lambda - \varepsilon_0(\vec{E}\cdot\vec{v})\right). \quad (48)$$

The left hand side of the above equation vanishes when

$$\Lambda = \varepsilon_0\,(\vec{E}\cdot\vec{v})\,. \quad (49)$$

Thus, this scalar wave is not any arbitrary function. It is associated with the electric field of the electromagnetic wave. It is thus suitable to call this an electroscalar wave. Eq. (48) with the condition in Eq. (49) states that when Λ is defined as above, there is no work done to move the free charges, and that a new wave is generated with both energy density and having energy flow along the particle direction. Hence,

$$\frac{\partial u_{\text{total}}}{\partial t} + \vec{\nabla}\cdot\left(\vec{S}_{\text{em}} + (\varepsilon_0 E^2)\,\vec{v}\right) = 0\,. \quad (50)$$

In such a case, we see that no electromagnetic energy is converted (into neither mechanical energy nor heat). The medium acts as if it were empty of current. This shows that the scalar wave and the charged particle propagate concomitantly. However, in the de Broglie picture a wave is associated with the particle motion to interpret the wave particle duality present in quantum mechanics. Eq. (50) shows that there is no energy flow along along the magnetic field direction. Therefore, this electroscalar wave is a longitudinal wave. The transmission of such a wave does cost extra energy and it avails the electromagnetic energy accompany it. Notice that this scalar wave can be used to transmit and receive wireless signals (van Vlaenderen and Waser, 2001 [6]). It has an advantage over the electromagnetic wave, since it is a longitudinal wave and has no polarization properties. We will anticipate that this new scalar wave will bring about new technology of transmission that avails such properties.

We have seen that recently van Vlaenderen, 2003 [7], showed that there is a scalar wave associated with abandonment of Lorentz gauge. He called such a scalar field, S. We have shown that without such abandonment one can arrive at the same conclusion regarding the existence of such a scalar wave. We have seen that the scalar wave associated with the current $\vec{J}$ travels along the current direction. However, van Vlaenderen obtain such a scalar wave with the condition that $\vec{J} = \vec{B} = 0$. But our derivation here shows that this is not limited to such a case, and is valid for any value of $\vec{E}$, $\vec{B}$, and $\vec{J}$. We can obtain the scalar wave equation of van Vlaenderen if we apply our transformation in Eq. (41) to Maxwell equations.

Van Vlaenderen obtained a scalar field for $\vec{E} = 0$ and $\vec{B} = 0$. See, Eq. (25) and (26). These equations can be obtained from from Maxwell and continuity equation by requiring an invariance of Maxwell equations under our transformation in Eq. (41) without requiring $\vec{E} = \vec{B} = 0$. Therefore, our Eq. (42) is similar to van Vlaenderen equation, viz., Eq. (35).

Wesley and Monstein [9] claimed that the scalar wave (longitudinal electric wave) transmission has an energy density equals to $\frac{1}{2\mu_0}\,S^2$. However, if the violation of Lorentz condition is very minute then this energy density term will have a very small contribution and can be ignored in comparison with the linear term in the Poynting vector term. Notice, however, that in such a case the van Vlaenderen prediction will be indistinguishable from our theory with a valid Lorentz condition. Hence, the existence of the electroscalar wave is not very much associated with Lorentz condition invalidation. Ignatiev and Leus [10] have confirmed experimentally the existence of longitudinal vacuum wave without magnetic component. This is evident from Eq. (47) that the energy flows only along the particle motion and the electric field direction, without trace to any magnetic component. van Vlaenderen proposed source transformations to generalize electrodynamic force and power of a charge particle in terms of a scalar wave S. Therefrom, he obtain a Poynting vector due to this scalar to be $-\frac{S}{\mu_0}\vec{E}$. These transformations coincide with our new transformation that arising from the invariance of the continuity equations under these transformation. Hence, Eq. (35) of van Vlaenderen would become identical to our Eq. (43), by setting $\Lambda = \frac{S}{\mu_0}$, but not necessarily limited to $\vec{B} = 0$, as he assumed.

We summarize here the quaternion forms of the physical laws which we have studied so far we:

- Maxwell equation: $\tilde{\Box}^{\,2}\tilde{A} = \mu_0\tilde{J}$;
- Lorentz force: $\tilde{F} = q\tilde{V}(\tilde{\nabla}\tilde{A})$;
- continuity equation: $\tilde{\nabla}\tilde{J} = 0$.

6 Conclusion

I think that a new and very powerful idea drives this work, namely, that all events are nicely represented as a quaternion. This implies that any collection of event can be generated by an appropriate quaternion function. Scalar and vector mix under multiplication, so quaternions are mixed representation. Every event, function, operator can be written in terms of quaternions. We have shown in this paper that the four Maxwell equations emerge from just one quaternion equation. Moreover, Lorentz force and the power delivered by a charged particle stem from one quaternion equation. The quaternion form of the continuity equation gives rise to the ordinary continuity equation, in addition to two more equations. The invariance of Maxwell equations under our new transformation shown in Eq. (43) ushers in the existence of new wave. This wave is not like the ordinary electromagnetic wave we know. It is a longitudinal wave having their origin in the variation of the electric field. It is called an electroscalar wave, besides that fact that it has a dimension of magnetic field intensity. Thus, in this paper we have laid down the theoretical formulation of the electroscalar wave without spoiling the beauty of Maxwell equations (in addition to Lorentz force). This scalar wave is not like the scalar potential which is a wave with a source term represented by the density that travels at a speed less than that of light. If the electroscalar wave is found experimentally, it will open a new era of electroscalar communication, and a new technology is then required. We remark that one does not need to invalidate the Lorentz condition to obtain such wave as it is formulated by some authors. In this work, we have generalize the continuity equation to embody a set of three equations. These equations imply that both current and density are waves traveling at a speed of light. Urgent experimental work to disclose the validity of these predictions is highly needed.

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