On the New Gauge Transformations of Maxwell's Equations

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We have found new gauge transformations that are compatible with Maxwell's equations and Lorentz gauge. With these transformations, we have formulated the electrodynamic equations that are shown to be invariant. New generalized continuity equations are derived that are also compatible with Maxwell's equations. Moreover, we have shown that the electromagnetic wave travels with speed of light in vacuum or a medium with free charge or current if the generalized continuity equations are satisfied. Magnetic monopoles don't show up in ordinary experiments because the Lorentz force acting on the magnetic charge is zero.

1 Introduction

Maxwell's equations describing the electric ($\vec{E}$) and magnetic ($\vec{B}$) fields reveal that when these fields are written in terms of a vector and scalar potentials, the equations of motion of these potential are generally solutions of wave equation with a source term. However, there is no unique way to define these potentials. A set of new potentials satisfying the Lorentz gauge can be solutions as well. Thus, Maxwell's equations are also invariant under these gauge transformations. Maxwell's equations are invariant under Lorentz transformation. Since the motion of charged particles is governed by the continuity equation, Maxwell's equations determine the motion of the charged particles in conformity with this equation.

Using quaternions, we have recently shown that Maxwell's equations can be written as a single quaternionic equation (Arbab and Satti, 2009 [1]). It is a wave equation. This immediately shows that the electromagnetic fields are waves. Similarly, by writing the continuity equation in a quaternionic form, we have shown that this equation yields three set of equations. We call these equations the generalized continuity equations (GCEs). Besides, we have found that the magnetic field arised from the charge motion (with speed $\vec{v}$) acted by an electric field is given by $\vec{B}=\frac{\vec{v}}{c^2}\times\vec{E}$. Because of this feature, the magnetic monopoles postulated by Dirac (Dirac, 1931 [2]) couldn't show up, because the Lorentz force component acting on this magnetic charge vanishes (Moulin, 2001 [3], Wolfgang, 1989 [4]). Hence, magnetic monopole can only be detected indirectly.

In the present paper, we have introduced new gauge transformations that leave Maxwell's equations, Lorentz gauge and the continuity equations invariant. Moreover, we know that according to Maxwell's theory the electromagnetic fields travel with speed of light in vacuum, i.e., when no free charge or current exists. However, in our present formulation, we have shown that the electromagnetic fields travel with speed of light in vacuum or free charged medium if the GCEs are satisfied.

2 Continuity equation

The flow of any continuous medium is governed by the continuity equation. The quaternionic continuity equation reads, (Arbab and Satti, 2009 [1]),

$$\tilde{\nabla}\tilde{J}=\left[-\left(\vec{\nabla}\cdot\vec{J}+\frac{\partial\rho}{\partial t}\right)\frac{i}{c}\left(\frac{\partial\vec{J}}{\partial t}+\vec{\nabla}\rho\, c^2\right)+\vec{\nabla}\times\vec{J}\right]=0\,, \quad (1)$$

where

$$\tilde{\nabla}=\left(\frac{i}{c}\frac{\partial}{\partial t}\,,\vec{\nabla}\right), \qquad \tilde{J}=\left(i\rho c\,,\vec{J}\right). \quad (2)$$

This implies that

$$\vec{\nabla}\cdot\vec{J}+\frac{\partial\rho}{\partial t}=0\,, \quad (3)$$

$$\vec{\nabla}\rho+\frac{1}{c^2}\frac{\partial\vec{J}}{\partial t}=0\,, \quad (4)$$

and

$$\vec{\nabla}\times\vec{J}=0\,. \quad (5)$$

We call Eqs. (3)–(5) the *generalized continuity equations* (GCEs). Equation (5) states the current density $\vec{J}$ is irrotational.

In a covariant form, Eqs. (3)–(5) read

$$\partial_\mu J^\mu=0\,, \qquad N_{\mu\nu}\equiv\partial_\mu J_\nu-\partial_\nu J_\mu=0\,. \quad (6)$$

Notice that the tensor $N_{\mu\nu}$ is an antisymmetric tensor. It is evident from Eq. (6) that Eqs. (3)–(6) are Lorentz invariant. Now differentiate Eq. (3) partially with respect to time and use Eq. (4), we obtain

$$\frac{1}{c^2}\frac{\partial^2\rho}{\partial t^2}-\nabla^2\rho=0\,. \quad (7)$$

Similarly, take the divergence of Eq. (4) and use Eq. (3), we obtain

$$\frac{1}{c^2}\frac{\partial^2 \vec{J}}{\partial t^2} - \nabla^2 \vec{J} = 0\,, \tag{8}$$

where $\rho = \rho(\vec{r}, t)$ and $\vec{J} = \vec{J}(\vec{r}, t)$. Therefore, both the current density and charge density satisfy a wave equation propagating with speed of light. In covariant form, Eqs. (7) and (8) now read

$$\Box^2 J^\nu \equiv \partial_\mu \partial^\mu J^\nu = 0\,. \tag{9}$$

We remark that the GCEs are applicable to any flow whether created by charged particles or neutral ones.

3 Maxwell's equations

We have recently shown that quaternion equation (Arbab and Satti, 2009 [1])

$$\tilde{\Box}^2 \tilde{A} = \mu_0 \tilde{J}\,, \qquad \tilde{A} = \left(i\,\frac{\varphi}{c}\,,\ \vec{A}\right) \tag{10}$$

yields the Maxwell's equations (Arbab and Satti, 2009 [1])

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}\,, \tag{11}$$

$$\vec{\nabla}\times\vec{E} + \frac{\partial \vec{B}}{\partial t} = 0\,, \tag{12}$$

$$\vec{\nabla}\times\vec{B} - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J}\,, \tag{13}$$

and

$$\vec{\nabla}\cdot\vec{B} = 0\,. \tag{14}$$

The electric and magnetic fields are defined by the vector potential (A) and the scalar potential (φ) as follows

$$\vec{E} = -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t}\,, \qquad \vec{B} = \vec{\nabla}\times\vec{A}\,, \tag{15}$$

such that the Lorentz gauge

$$\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial \varphi}{\partial t} = 0\,, \tag{16}$$

is satisfied. We know that the electric and magnetic fields are invariant under the following gauge transformations

$$\vec{A}\,' = \vec{A} - \vec{\nabla}\chi\,, \qquad \varphi\,' = \varphi + \frac{\partial \chi}{\partial t}\,. \tag{17}$$

The invariance of the Lorentz gauge implies that

$$\frac{1}{c^2}\frac{\partial^2 \chi}{\partial t^2} - \nabla^2 \chi = 0\,. \tag{18}$$

The 4-vector potential, A_μ, can be written as

$$A_\mu = \left(\frac{\varphi}{c}, -\vec{A}\right)\,. \tag{19}$$

In a covariant form, Eq. (17) becomes

$$A'_\mu = A_\mu + \partial_\mu \chi\,. \tag{20}$$

Eq. (15) can be written in a covariant form as

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,. \tag{21}$$

In a covariant form, Maxwell's equations, Eqs. (11)–(14), read

$$\partial_\mu F^{\mu\nu} = \mu_0 J^\nu\,, \qquad \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} + \partial_\lambda F_{\mu\nu} = 0\,. \tag{22}$$

Notice however that if we take $\frac{\partial}{\partial t}$ of Eq. (12) and apply Eqs. (13) and (14), we get

$$\frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} - \nabla^2 \vec{B} = \mu_0\left(\vec{\nabla}\times\vec{J}\right). \tag{23}$$

Now take the curl of both sides of Eq. (12) and apply Eqs. (11) and (13), we get

$$\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \nabla^2 \vec{E} = -\frac{1}{\varepsilon_0}\left(\vec{\nabla}\rho + \frac{1}{c^2}\frac{\partial \vec{J}}{\partial t}\right). \tag{24}$$

We remark that, according to our GCEs, the electric and magnetic waves propagate with speed of light whether $\vec{J} = \rho = 0$ or not, as long as Eqs. (4) and (5) are satisfied.

In a covariant form, Eqs. (23) and (24) read

$$\Box^2 F_{\mu\nu} = \mu_0\left(\partial_\mu J_\nu - \partial_\nu J_\mu\right)\,. \tag{25}$$

This can be casted in the form

$$\partial^\alpha\left(\mu_0^{-1}\partial_\alpha F_{\mu\nu} + g_{\nu\alpha}J_\mu - g_{\mu\alpha}J_\nu\right) \equiv \partial^\alpha C_{\alpha\mu\nu} = 0\,, \tag{26}$$

where

$$C_{\alpha\mu\nu} = \mu_0^{-1}\partial_\alpha F_{\mu\nu} + g_{\nu\alpha}J_\mu - g_{\mu\alpha}J_\nu\,, \tag{27}$$

where $g_{\mu\nu}$ is the metric tensor. Notice that the current tensor $C_{\alpha\mu\nu}$ is antisymmetric in the indices μ, ν and is a conserved quantity. Likewise the total momentum and energy of the electrodynamics system (fields + particles) is conserved, we found here that the total current of the system, one arising from the electromagnetic fields and the other from the particles motion, is conserved. The first term in Eq. (27) represents the electromagnetic current, the second term represents the electronic current and the last term represents the vacuum current (with negative sign) as suggested by Eq. (28).

4 New gauge transformations

Now we introduce the current density transformations (CDTs) for $\vec{J}$ and ρ, viz.,

$$\rho' = \rho + \frac{1}{c^2}\frac{\partial \Lambda}{\partial t}\,, \qquad \vec{J}\,' = \vec{J} - \vec{\nabla}\Lambda\,, \tag{28}$$

leaving the generalized continuity equations (GCEs) invariant. In a covariant form, Eq. (28) reads

$$J_\mu{}' = J_\mu + \partial_\mu \Lambda\,. \tag{29}$$

Applying this transformation in Eq. (6), one finds that

$$\partial_\mu (J^\mu + \partial^\mu \Lambda) = \partial_\mu J^{\mu\,\prime} = 0\,, \qquad N'_{\mu\nu} = N_{\mu\nu}\,. \tag{30}$$

It is thus evident that the GCEs are invariant under the CDTs. Moreover, the application of the current transformation in the continuity equation, Eq. (3), yields

$$\frac{1}{c^2}\frac{\partial^2 \Lambda}{\partial t^2} - \nabla^2 \Lambda = -\left(\vec{\nabla}\cdot\vec{J} + \frac{\partial \rho}{\partial t}\right). \tag{31}$$

We thus that $\vec{J}$ and ρ in the GCEs are not unique and any new set of $\vec{J}\,'$ and ρ' will lead to the same GCEs provided that Λ is gauged by Eq. (31). Since the right hand side of Eq. (31) vanishes, Λ is a solution of a wave equation traveling with speed of light in vacuum. This equation is similar to Eq. (18). Notice also that Eqs. (23) and (24) are invariant under the following CDTs

$$\rho' = \rho + \frac{1}{c^2}\frac{\partial \Lambda}{\partial t}\,, \quad \vec{J}\,' = \vec{J} - \vec{\nabla}\Lambda\,, \quad \vec{E}\,' = \vec{E}, \quad \vec{B}\,' = \vec{B}\,. \tag{32}$$

In a covariant form, these read

$$J'_\mu = J_\mu + \partial_\mu \Lambda\,, \qquad F'_{\mu\nu} = F_{\mu\nu}\,. \tag{33}$$

Now let us introduce new gauge transformations (NGTs) as follows

$$\vec{A}\,' = \vec{A} + \alpha\vec{J}\,, \;\; \varphi\,' = \varphi + \alpha\rho\, c^2, \;\; \alpha = \mu_0\lambda^2, \;\; \lambda = \text{const.} \tag{34}$$

In a covariant form, Eq. (34) reads

$$A'_\mu = A_\mu + \alpha\, J_\mu\,, \tag{35}$$

so that the electromagnetic tensor

$$F'_{\mu\nu} = F_{\mu\nu} + \alpha\,(\partial_\mu J_\nu - \partial_\nu J_\mu)\,, \tag{36}$$

using Eq. (6), is invariant under the NGTs and hence, Maxwell's equations are invariant too. Moreover, notice that the Lorentz gauge

$$\vec{\nabla}\cdot\vec{A} + \frac{1}{c^2}\frac{\partial \varphi}{\partial t} = 0\,, \qquad \text{or} \qquad \partial_\mu A^\mu = 0\,, \tag{37}$$

is also invariant under the NGTs provided that the continuity equation, Eq. (3), is satisfied. The covariant derivative is defined by

$$D_\mu = \partial_\mu - \frac{ie}{\hbar}\, A_\mu\,. \tag{38}$$

The quantum electrodynamics Lagrangian of a particle of spinor ψ is given by

$$\mathcal{L} = \overline{\psi}\,(i\hbar\, c\,\gamma^\mu\, D_\mu - mc^2)\psi - \frac{1}{4\mu_0} F_{\mu\nu}F^{\mu\nu}\,. \tag{39}$$

so that the Eq. (39) is invariant under the local gauge transformation of the spinor ψ (Bjorken, 1964 [5]). In terms of this derivative, one has

$$F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu\,, \tag{40}$$

and Maxwell's equations become

$$D_\mu F^{\mu\nu} = \mu_0 J^\nu, \quad D_\mu F_{\nu\lambda} + D_\nu F_{\lambda\mu} + D_\lambda F_{\mu\nu} = 0\,. \tag{41}$$

Upon using Eq. (6), Eq. (41) is invariant under NGTs.

Applying the NGTs into the above Lagrangian yields

$$\mathcal{L}' = \mathcal{L} + \alpha J_\mu J^\mu\,. \tag{42}$$

The current density is defined by $J^\mu = ec\overline{\psi}\gamma^\mu\psi$. This extra interaction term has already appeared in the Fermi theory of beta decay. It is written in the form $\frac{G_F}{\sqrt{2}} J_\mu J^\mu$, i.e., $\alpha = \frac{G_F}{\sqrt{2}}$, where G_F is the Fermi constant. We anticipate that this term is related to the mass of the photon (propagator). This term term couldn't be added to the initial Lagrangian because, it breaks the ordinary gauge invariance. However, the NGTs could rise to the mass of the photon. It is something like Higg's mechanism that gives the elementary particles their masses. Such a term may be related to an interaction of two electrons closed to each other like in Cooper pairs in superconductivity. The behavior of superconductors suggests that electron pairs are coupling over a range of hundreds of nanometers, three orders of magnitude larger than the lattice spacing. These coupled electrons can take the character of a boson and condense into the ground state.

5 Symmetrized Maxwell's equation

Dirac was the first to suggest the possibility of a particle that carries magnetic charge. At the present time there is no experimental evidence for the existence of magnetic charges or monopoles. This can be formulated in the context of Maxwell's equations. Maxwell's equations can be written in a symmetric form by invoking the idea of monopole. Let us denote the magnetic charge by q_m and its density and current by ρ_m and J_m, so that symmetrized Maxwell's equations are written as follows

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_e}{\varepsilon_0}\,, \tag{43}$$

$$\vec{\nabla}\times\vec{E} = -\mu_0\vec{J}_m - \frac{\partial \vec{B}}{\partial t}\,, \tag{44}$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}_e + \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t}\,, \tag{45}$$

and

$$\vec{\nabla}\cdot\vec{B} = \mu_0\rho_m\,. \tag{46}$$

Lorentz force will have the form (Moulin, 2001 [3])

$$F = q_e(\vec{E} + \vec{v}\times\vec{B}) + q_m(\vec{B} - \frac{\vec{v}}{c^2}\times\vec{E})\,. \tag{47}$$

But since (Arbab and Satti, 2009 [1])

$$\vec{B} = \frac{\vec{v}}{c^2} \times \vec{E} \tag{48}$$

the Lorentz force does not affect the magnetic charge whether it exists or not. Hence, the magnetic monopole does not manifest its self via Lorentz force. The magnetic field generated by the charged particle is in such a way that it does not influence the magnetic charge. Note also that the magnetic field created by the charged particle does not do work because $\vec{v} \cdot \vec{B}$. The above symmetrized Maxwell's equations have the duality transformations, i.e., $\vec{E} \to \vec{B}$, $\vec{B} \to -\vec{E}$.

Using the vector identity $\vec{\nabla} \cdot (\vec{A} \times \vec{C}) = \vec{C} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{C})$, it is interesting to notice that the divergence of Eq. (48) vanishes, viz.,

$$\vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot \left(\frac{\vec{v}}{c^2} \times \vec{E} \right) = = \frac{1}{c^2} \left[\vec{E} \cdot (\vec{\nabla} \times \vec{v}) - \vec{v} \cdot (\vec{\nabla} \times \vec{E}) \right] = 0, \tag{49}$$

for a motion with constant velocity, where $\vec{\nabla} \times \vec{v} = 0$ and $\vec{v}$ is perpendicular to $\vec{\nabla} \times \vec{E}$.

6 The Biot-Savart law

We can now apply Eq. (48) to calculate the magnetic field acted on the electron in Hydrogen-like atoms. This magnetic field is produced by the moving electron due to the presence of an electric field created by the nucleus at a distance r, as seen by the electron. Therefore,

$$\vec{B} = \frac{\vec{v}}{c^2} \times \vec{E}\,, \tag{50}$$

where $\vec{E}$ is the electric produced by the nucleus at the electron site. The magnetic field due to a single moving charged particle (q) is given by the Biot-Savart law as

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\,\vec{v} \times \vec{r}}{r^3}\,. \tag{51}$$

Comparing Eq. (50) with Eq. (51) and using the fact that $\mu_0 \varepsilon_0 c^2 = 1$, one gets

$$\vec{E} = \frac{q}{4\pi\varepsilon_0} \frac{\vec{r}}{r^3} \tag{52}$$

which is the familiar definition of the electric field of a single charged particle. Hence, Eq. (50) is one variant of Biot-Savart law. This law was not included in the original formulation of Maxwell's theory. Hence, Maxwell's equations were missing this law and thus were incomplete.

Since the electric field produced by the nucleus is perpendicular to the electron velocity, Eq. (50) yields

$$B = \frac{v}{c^2}\,E\,. \tag{53}$$

But for Hydrogen-like atoms

$$E = \frac{1}{4\pi\varepsilon_0} \frac{Ze}{r^2}\,, \tag{54}$$

so that one has

$$B = \frac{Zev}{4\pi\varepsilon_0 r^2}\,. \tag{55}$$

In terms of the orbital angular momentum (L) where $L = mvr$, one has

$$\vec{B} = \frac{Ze}{4\pi\varepsilon_0 m r^3} \vec{L}\,. \tag{56}$$

However, this is the same equation that is obtained using the Biot-Savart law. This is a remarkable result, and suggests that the relation $\vec{B} = \frac{\vec{v}}{c^2} \times \vec{E}$ is truly fundamental in electrodynamics. This term gives rise to the spin-orbit interaction described by

$$E_{\text{int}} = \frac{1}{4\pi\varepsilon_0} \frac{Ze^2}{m_e^2 c^2 r^3}\, \vec{S} \cdot \vec{L}\,. \tag{57}$$

A factor of $1/2$ correcting the above expression is introduced by Thomas leading to

$$E_{\text{int}} = \frac{1}{8\pi\varepsilon_0} \frac{Ze^2}{m_e^2 c^2 r^3}\, \vec{S} \cdot \vec{L}\,. \tag{58}$$

We now use the Biot-Savart law to demonstrate that $\vec{\nabla} \cdot \vec{B} = 0$. This law is written in the form

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3}\, d^3 r'\,. \tag{59}$$

Using the vector identity $\vec{\nabla} \cdot (\vec{A} \times \vec{C}) = \vec{C} \cdot (\vec{\nabla} \times \vec{A}) - \vec{A} \cdot (\vec{\nabla} \times \vec{C})$, one has

$$\vec{\nabla} \cdot \int \left(\frac{\vec{J}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \right) d^3 r' = = \int \frac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \cdot (\vec{\nabla} \times \vec{J})\, d^3 r' - \int \vec{J} \cdot \left(\vec{\nabla} \times \frac{(\vec{r} - \vec{r}\,')}{|\vec{r} - \vec{r}\,'|^3} \right) d^3 r'. \tag{60}$$

Because of Eq. (5) and the fact that the curl of any pure radial function is zero, i.e. $\vec{\nabla} \times (f(r)\,\hat{r}) = 0$, the first and the second term vanish, so that above equation yields

$$\vec{\nabla} \cdot \vec{B} = 0\,. \tag{61}$$

Now let us calculate the magnetic field at a distance r from the wire produced by an infinitely long wire carrying a current I. Using Ampere's law, this is given by

$$B = \frac{\mu_0 I}{2\pi r}\,. \tag{62}$$

However, using Eqs. (50) and (51) and the fact that $It = q$ and $\vec{v}$ is perpendicular to $\vec{r}$, one finds that the magnetic field sets up at a point P at a distance r is *not* instantaneous, but reaches after a passage of time

$$\Delta t = \frac{2r}{v}\,. \qquad (63)$$

Placing a detector at a distance r from the wire, one can measured this time experimentally. $2\,r$ is the round trip distance covered by the mediator (photon) traveling with speed v to send the magnetic induction at a point P. This exhibits the causal behavior associated with the wave disturbance. This shows that an effect observed at the point r at time t is caused by the action of the source a distant r away at an earlier or retarded time $t' = t - r/c$.The time r/c is the time of propagation of the disturbance from the source to the point r. Because of this Maxwell's equations satisfy the causality principle. Notice that this magnetic field is not changing with time. This may help understand that photons are emitted and absorbed by electron continuously, asserting that the electromagnetic interaction is exchanged by a mediator, as advocated by the quantum field theory.

7 Concluding remarks

We have shown in this paper the importance of the new gauge transformations, and how they leave Maxwell's equations invariant. These are the continuity equations, the current-density transformations and the current-gauge field transformations. According to Noether's theorem, invariance of a Lagrangian under any transformation will give rise to a conserved quantity. Hence, we trust that there must be some deep connections of these transformations with other electrodynamics phenomena. We emphasize here how the relation $\vec{B} = \frac{\vec{v}}{c^2} \times \vec{E}$ is important in calculating magnetic fields produced by moving charged particle. This equation was missing in the derivation of Maxwell's equations. Note that this field is always perpendicular to the velocity of the particle, i.e., $\vec{v} \cdot \vec{B} = 0$. We have also found that $\vec{B} = \frac{\vec{v}}{c^2} \times \vec{E}$ is equivalent to Biot-Savart law. Thus, the quaternionic form of Maxwell's equations generalizes the ordinary Maxwell's equations and unified the Biot-Savart law with other electromagnetic laws. The magnetic charge (monopole) proposed by Dirac could exist in principle, but it doesn't feel the electromagnetic force. The generalized continuity equations are in agreement with Newton's second law of motion. Moreover, we have obtained the Euler and energy conservation equations from the quaternionic Newton's law. Application of these new gauge transformations in quantum field theory will be one of our future endeavor.

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