The Solar System According to General Relativity: The Sun's Space Breaking Meets the Asteroid Strip

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This study deals with the exact solution of Einstein's field equations for a sphere of incompressible liquid without the additional limitation initially introduced in 1916 by Schwarzschild, by which the space-time metric must have no singularities. The obtained exact solution is then applied to the Universe, the Sun, and the planets, by the assumption that these objects can be approximated as spheres of incompressible liquid. It is shown that gravitational collapse of such a sphere is permitted for an object whose characteristics (mass, density, and size) are close to the Universe. Meanwhile, there is a spatial break associated with any of the mentioned stellar objects: the break is determined as the approaching to infinity of one of the spatial components of the metric tensor. In particular, the break of the Sun's space meets the Asteroid strip, while Jupiter's space break meets the Asteroid strip from the outer side. Also, the space breaks of Mercury, Venus, Earth, and Mars are located inside the Asteroid strip (inside the Sun's space break).

The main task of this paper is to study the possibilities of applying condensed matter models in astrophysics and cosmology. A cosmic object consisting of condensed matter has a constant volume and a constant density. A sphere of incompressible liquid, being in the weightless state (as any cosmic object), is a kind of condensed matter. Thus, assuming that a star is a sphere of incompressible liquid, we can study the gravitational field of the star inside and outside it.

The Sun orbiting the center of the Galaxy meets the weightless condition (see [1] for detail)

$$\frac{GM}{r} = v^2,$$

where $G = 6.67 \times 10^{-8} \text{ cm}^3/\text{g} \times \text{sec}^2$ is the Newtonian gravitational constant, *M* is the mass of the Galaxy, *r* is the distance of the Sun from the center of the Galaxy, and v is the Sun's velocity in its orbit. The planets of the Solar System also satisfy the weightless condition. Assuming that the planets have a similar internal constitution as the Sun, we can consider these objects as spheres of incompressible liquid being in a weightless state.

I will consider the problems by means of the General Theory of Relativity. First, it is necessary to obtain the exact solution of the Einstein field equations for the space-time metric induced by the gravitational field of a sphere of incompressible liquid.

The regular field equations of Einstein, with the λ -field neglected, have the form

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = -\varkappa T_{\alpha\beta} , \qquad (1)$$

where $R_{\alpha\beta}$ is the Ricci tensor, *R* is the Riemann curvature scalar, $\varkappa = \frac{8\pi G}{c^2} = 18.6 \times 10^{-28}$ cm/g is the Einstein gravitational constant, $T_{\alpha\beta}$ is the energy-momentum tensor, and $\alpha, \beta =$

0, 1, 2, 3 are the space-time indices. The gravitational field of spherical island of substance should possess spherical symmetry. Thus, it is described by the metric of spherical kind

$$ds^{2} = e^{\nu}c^{2}dt^{2} - e^{\lambda}dr^{2} - r^{2}(d\theta^{2} + \sin^{2}\theta \, d\varphi^{2}), \qquad (2)$$

where e^{v} and e^{λ} are functions of *r* and *t*.

In the case under consideration the energy-momentum tensor is that of an ideal liquid (incompressible, with zero viscosity), by the condition that its density is constant, i.e. $\rho = \rho_0 = const$. As known, the energy-momentum tensor in this case is

$$T^{\alpha\beta} = \left(\rho_0 + \frac{p}{c^2}\right) b^{\alpha} b^{\beta} - \frac{p}{c^2} g^{\alpha\beta},\tag{3}$$

where p is the pressure of the liquid, while

$$b^{\alpha} = \frac{dx^{\alpha}}{ds}, \qquad b_{\alpha}b^{\alpha} = 1$$
 (4)

is the four-dimensional velocity vector, which determines the reference frame of the given observer. Also, the energymomentum tensor should satisfy the conservation law

$$\nabla_{\!\sigma} T^{\alpha\sigma} = 0, \qquad (5)$$

where ∇_{σ} is the four-dimensional symbol of covariant differentiation.

Formally, the problem we are considering is a generalization of the Schwarzschild solution produced for an analogous case (a sphere of incompressible liquid). Karl Schwarzschild [2] solved the Einstein field equations for this case, by the condition that the solution must be regular. He assumed that the components of the fundamental metric tensor $g_{\alpha\beta}$ must satisfy the signature conditions (the space-time metric must have no singularities). Thus, the Schwarzschild solution, according to his initial assumption, does not include space-time singularities.

Larissa Borissova. The Solar System According to General Relativity: The Sun's Space Breaking Meets the Asteroid Strip

This limitation of the space-time geometry, initially introduced in 1916 by Schwarzschild, will not be used by me in this study. Therefore, we will be able to study the singular properties of the space-time metric associated with a sphere of incompressible liquid. Then I will apply the obtained results to the cosmic objects such as the Sun and the planets.

The exact solution of the field equations (1) is obtained for the spherically symmetric metric (2) inside a sphere of incompressible liquid, which is described by the energy-momentum tensor (3). I consider here the reference frame which accompanies to the observer, consequently the components of his four-velocity vector are [3]

$$b^0 = \frac{1}{\sqrt{g_{00}}}, \quad b^i = 0, \quad i = 1, 2, 3,$$
 (6)

while the physically observed components of the energymomentum tensor $T_{\alpha\beta}$ has the form

$$\rho = \frac{T_{00}}{g_{00}} = \rho_0 \,, \ J^i = \frac{c \, T_0^i}{\sqrt{g_{00}}} = 0 \,, \ U^{ik} = c^2 T^{ik} = p h^{ik}, \quad (7)$$

where ρ is the density of the medium, J^i is the density of the momentum in the medium, U^{ik} is the stress-tensor, h^{ik} is the observable three-dimensional fundamental metric tensor [3].

Because we do not limit the solution by that the metric must be regular, the obtained metric has two singularities: 1) collapse by $g_{00} = 0$, and 2) break of the space by $g_{11} \rightarrow \infty$. It will be shown then that these singularities are irremovable, because the strong signature condition is also violated in both cases.

In order to obtain the exact internal solution of the Einstein field equations with respect to a given distribution of matter, it is necessary to solve two systems of equations: the Einstein field equations (1), and the equations of the conservation law (5).

After algebra we obtain the Einstein field equations in the spherically symmetric space (2) inside a sphere of incompressible liquid. The obtained equations, in component notation, are

$$e^{-\nu} \left(\ddot{\lambda} - \frac{\dot{\lambda}\dot{\nu}}{2} + \frac{\dot{\lambda}^2}{2} \right) - c^2 e^{-\lambda} \left[\nu'' - \frac{\lambda'\nu'}{2} + \frac{2\nu'}{r} + \frac{(\nu')^2}{2} \right] = = -\varkappa \left(\rho_0 c^2 + 3p \right), \quad (8)$$
$$\frac{\dot{\lambda}}{r} e^{-\lambda - \frac{\nu}{2}} = \varkappa J^1 = 0, \quad (9)$$

$$e^{\lambda-\nu}\left(\ddot{\lambda}-\frac{\dot{\lambda}\dot{\nu}}{2}+\frac{\dot{\lambda}^2}{2}\right)-c^2\left[\nu''-\frac{\lambda'\nu'}{2}+\frac{(\nu')^2}{2}\right]+\frac{2c^2\lambda'}{r}=$$
$$=\varkappa\left(\rho_0c^2-p\right)e^{\lambda},\quad(10)$$

$$\frac{c^2 \left(\lambda' - \nu'\right)}{r} e^{-\lambda} + \frac{2c^2}{r^2} \left(1 - e^{-\lambda}\right) = \varkappa \left(\rho_0 c^2 - p\right). \tag{11}$$

The second equation manifests that $\lambda = 0$ in this case. Hence, the space inside the sphere of incompressible liquid does not deform. Taking this circumstance into account, and also that the stationarity of λ , we reduce the field equations (8–11) to the final form

$$c^{2}e^{-\lambda}\left[\nu^{\prime\prime} - \frac{\lambda^{\prime}\nu^{\prime}}{2} + \frac{2\nu^{\prime}}{r} + \frac{(\nu^{\prime})^{2}}{2}\right] = \varkappa\left(\rho_{0}c^{2} + 3p\right)e^{\lambda}, \quad (12)$$

$$-c^{2}\left[\nu'' - \frac{\lambda'\nu'}{2} + \frac{(\nu')^{2}}{2}\right] + \frac{2c^{2}\lambda'}{r} = \varkappa \left(\rho_{0}c^{2} - p\right)e^{\lambda}, \quad (13)$$

$$\frac{c^2 \left(\lambda' - \nu'\right)}{r} e^{-\lambda} + \frac{2c^2}{r^2} \left(1 - e^{-\lambda}\right) = \varkappa \left(\rho_0 c^2 - p\right) e^{\lambda}.$$
 (14)

To solve the equations (12–14), a formula for the pressure p is necessary. To find the formula, we now deal with the conservation equations (5). Because, as was found, $J^i = 0$ we obtain, this formula reduces to only a single nontrivial equation

$$p'e^{-\lambda} + \left(\rho_0 c^2 + p\right) \frac{\nu'}{2} e^{-\lambda} = 0, \qquad (15)$$

where $p' = \frac{dp}{dr}$, $v' = \frac{dv}{dr}$, $e^{\lambda} \neq 0$. Dividing both parts of (15) by $e^{-\lambda}$, we arrive at

$$\frac{dp}{\rho_0 c^2 + p} = -\frac{d\nu}{2},\tag{16}$$

which is a plain differential equation with separable variables. It can be easily integrated as

$$p_0 c^2 + p = B e^{-\frac{\nu}{2}}, \qquad B = const.$$
 (17)

Thus we have to express the pressure p as the function of the variable v,

$$p = Be^{-\frac{\nu}{2}} - \rho_0 c^2. \tag{18}$$

In look for an *r*-dependent function p(r), we integrate the field equations (12–14), taking into account (18). We find finally expressions for e^{λ} and e^{ν}

$$g_{00} = e^{\nu} = \frac{1}{4} \left(3e^{\frac{\nu_a}{2}} - \sqrt{1 - \frac{\kappa\rho_0 r^2}{3}} \right)^2, \tag{19}$$

$$e^{\lambda} = -g_{11} = \frac{1}{1 - \frac{\kappa \rho_0 r^2}{3}},$$
(20)

where $e^{\frac{v_a}{2}} = \sqrt{1 - \frac{2GM}{c^2 a}} = \sqrt{1 - \frac{r_g}{r}}$ is obtained from the boundary conditions, while r_g is the Hilbert radius.

Thus the space-time metric of the gravitational field inside a sphere of incompressible liquid is, since the formulae of ν and λ have already been obtained, as follows

$$ds^{2} = \frac{1}{4} \left(3e^{\frac{\nu_{a}}{2}} - \sqrt{1 - \frac{\varkappa \rho_{0}r^{2}}{3}} \right)^{2} c^{2} dt^{2} - \frac{dr^{2}}{1 - \frac{\varkappa \rho_{0}r^{2}}{3}} - r^{2} \left(d\theta^{2} + \sin^{2}\theta \, d\varphi^{2} \right).$$
(21)

Taking into account that $M = \frac{4\pi a^3 \rho_0}{3}$ and $r_g = \frac{2GM}{c^2}$, we rewrite (21) in the form

$$ds^{2} = \frac{1}{4} \left(3 \sqrt{1 - \frac{r_{g}}{a}} - \sqrt{1 - \frac{r^{2}r_{g}}{a^{3}}} \right)^{2} c^{2} dt^{2} - \frac{dr^{2}}{1 - \frac{r^{2}r_{g}}{a^{3}}} - r^{2} \left(d\theta^{2} + \sin^{2}\theta \, d\varphi^{2} \right).$$
(22)

It is therefore obvious that this "internal" metric completely coincides with the Schwarzschild metric in emptiness on the surface of the sphere of incompressible liquid (r = a). This study is a generalization of the originally Schwarzschild solution for such a sphere [2], and means that Schwarzschild's requirement to the metric to be free of singularities will not be used here. Naturally, the metric (22) allows singularities. This problem will be solved by analogy with the singular properties of the Schwarzschild solution in emptiness [4] (a mass-point's field), which already gave black holes.

Consider the collapse condition for the space-time metric of the gravitational field inside a sphere of incompressible liquid (21). The collapse condition $g_{00} = 0$ in this case is

$$3e^{\frac{\nu_a}{2}} = \sqrt{1 - \frac{\varkappa \rho_0 r^2}{3}},$$
 (23)

or, in terms of the Hilbert radius, when the metric takes the form (22), the collapse condition is

$$3 \sqrt{1 - \frac{r_g}{a}} = \sqrt{1 - \frac{r_g r^2}{a^3}} .$$
 (24)

We obtain that the numerical value of the radial coordinate r_c , by which the sphere's surface meets the surface of collapse, is

$$r_c = a \sqrt{9 - \frac{8a}{r_g}} \,. \tag{25}$$

Because we keep in mind really cosmic objects, the numerical value of r_c should be real. This requirement is obviously satisfied by

$$a < 1.125 r_g$$
. (26)

If this condition holds not $(a \ge r_g)$, the sphere, which is a spherical liquid body, has not the state of collapse. It is obvious that the condition $a = r_g$ satisfies to (26). It is obvious that r_c is imaginary for $r_g \ll a$, so collapse of such a sphere of incompressible liquid is impossible.

For example, consider the Universe as a sphere of incompressible liquid (the liquid model of the Universe). Assuming, according to the numerical value of the Hubble constant (17), that the Universe's radius is $a = 1.3 \times 10^{28}$ cm, we obtain the collapse condition, from (26),

$$r_q > 1.2 \times 10^{28} \,\mathrm{cm},$$
 (27)

and immediately arrive at the following conclusion:

The observable Universe as a whole, being represented in the framework of the liquid model, is completely located inside its gravitational radius. In other words, the observable Universe is a collapsar — a huge black hole.

In another representation, this result means that a sphere of incompressible liquid can be in the state of collapse only if its radius approaches the radius of the observable Universe.

Let's obtain the condition of spatial singularity — space breaking. As is seen, the metric (21) or its equivalent form (22) has space breaking if its radial coordinate r equals to

$$r_{br} = \sqrt{\frac{3}{\kappa\rho_0}} = a\sqrt{\frac{a}{r_g}}.$$
 (28)

For example, considering the Sun as a sphere of incompressible liquid, whose density is $\rho_0 = 1.4 \text{ g/cm}^3$, we obtain

$$r_{br} = 3.4 \times 10^{13} \text{ cm},$$
 (29)

while the radius of the Sun is $a = 7 \times 10^{10}$ cm and its Hilert radius $r_g = 3 \times 10^5$ cm. Therefore, the surface of the Sun's space of breaking is located outside the surface of the Sun, far distant from it in the near cosmos.

Another example. Assume our Universe to be a sphere of incompressible liquid, whose density is $\rho_0 = 10^{-31}$ g/cm³. The radius of its space breaking, according to (28), is

$$r_{br} = 1.3 \times 10^{29} \text{ cm.}$$
 (30)

Observational astronomy provides the following numerical value of the Hubble constant

$$H = \frac{c}{a} = (2.3 \pm 0.3) \times 10^{18} \text{ sec}^{-1}, \qquad (31)$$

where a is the observed radius of the Universe. It is easily obtain from here that

$$a = 1.3 \times 10^{28} \text{ cm.}$$
 (32)

This value is comparable with (30), so the Universe's radius may meet the surface of its space breaking by some conditions. We calculate the mass of the Universe by $M = \frac{4\pi a^3 \rho_0}{3}$, where *a* is (32). We have $M = 5 \times 10^{54}$ g. Thus, for the liquid model of the Universe, we obtain $r_g = 7.4 \times 10^{26}$ cm: the Hilbert radius (the radius of the surface of gravitational collapse) is located inside the liquid spherical body of the Universe.

A few words more on the singularities of the liquid sphere's internal metric (21). In this case, the determinant of the fundamental metric tensor equals

$$g = -\frac{1}{4} \left(3e^{\frac{\nu_a}{2}} - \sqrt{1 - \frac{\varkappa \rho_0 r^2}{3}} \right)^2 \frac{r^4 \sin^2 \theta}{\sqrt{1 - \frac{\varkappa \rho_0 r^2}{3}}},$$
 (33)

so the strong signature condition g < 0 is always true for a sphere of incompressible liquid, except in two following

Larissa Borissova. The Solar System According to General Relativity: The Sun's Space Breaking Meets the Asteroid Strip

cases: 1) in the state of collapse $(g_{00} = 0)$, 2) by the breaking of space $(g_{11} \rightarrow \infty)$. These particular cases violate the weak signature conditions $g_{00} > 0$ and $g_{11} < 0$ correspondingly. If both weak signature conditions are violated, g has a singularity of the kind $\frac{0}{0}$. If collapse occurs in the absence of the space breaking, we have g = 0. If no collapse, while the space breaking is present, we have $g \rightarrow \infty$. In all the cases, the singularity is non-removable, because the strong singular condition q < 0 is violated.

So, as was shown above, a spherical object consisting of incompressible liquid can be in the state of gravitational collapse only if it is as large and massive as the Universe. Meanwhile, the space breaking realizes itself in the fields of all cosmic objects, which can be approximated by spheres of incompressible liquid. Besides, since $r_{br} \sim \frac{1}{\sqrt{\rho_0}}$, the r_{br} is then greater while smaller is the ρ_0 . Assuming all these, we arrive at the following conclusion:

A regular sphere of incompressible liquid, which can be observed in the cosmos or an Earth-bound laboratory, cannot collapse but has the space breaking a singular surface, distantly located around the liquid sphere.

First, we are going to consider the Sun as a sphere of incompressible liquid. Schwarzschild [2] was the first person who considered the gravitational field of a sphere of incompressible liquid. He however limited this consideration by an additional condition that the space-time metric should not have singularities. In this study the metric (21) will be used. It allows singularities, in contrast to the limited case of Schwarzschild: 1) collapse of the space, and 2) the space breaking.

Calculating the radius of the space breaking by formula (28), where we substitute the Sun's density $\rho_0 = 1.41$ g/cm³, we obtain

$$r_{br} = 3.4 \times 10^{13} \text{ cm} = 2.3 \text{ AU},$$
 (34)

where $1 \text{ AU} = 1.49 \times 10^{13}$ cm (Astronomical Unit) is the average distance between the Sun and the Earth. So, we have obtained that the spherical surface of the Sun's space breaking is located inside the Asteroid strip, very close to the orbit of the maximal concentration of substance in it (as is known, the Asteroid strip is hold from 2.1 to 4.3 AU from the Sun). Thus we conclude that:

The space of the Sun (its gravitational field), as that of a sphere of incompressible liquid, has a breaking. The space breaking is distantly located from the Sun's body, in the space of the Solar System, and meets the Asteroid strip near the maximal concentration of the asteroids.

In addition to it, we conclude:

The Sun, approximated by a mass-point according to the Schwarzschild solution for a mass-point's field in emptiness, has a space breaking located inside the Sun's body. This space breaking coincides with the Schwarzschild sphere — the sphere of collapse.

What is the Schwarzschild sphere? It is an imaginary spherical surface of the Hilbert radius $r_g = \frac{2GM}{c^2}$, which is not a radius of a physical body in a general case (despite it can be such one in the case of a black hole — a physical body whose radius meets the Hilbert radius calculated for its mass). The numerical value of r_g is determined only by the mass of the body, and does not depend on its other properties. The physical meaning of the Hilbert radius in a general case is as follows: this is the boundary of the region in the gravitational field of a mass-point M, where real particles exist; particles in the boundary (the Hilbert radius) bear the singular properties. In the region wherein $r \leq r_g$, real particles cannot exist.

Let us turn back to the Sun approximated by a sphere of incompressible liquid. The space-time metric is (21) in this case. Substituting into (25) the Sun's mass $M = 2 \times 10^{33}$ g, radius $a = 7 \times 10^7$ cm, and the Hilbert radius $r_g = 3 \times 10^5$ cm calculated for its mass, we obtain that the numerical value of the radial coordinate r_c by which the Sun's surface meets the surface of collapse of its mass is imaginary. Thus, we arrive at the conclusion that a sphere of incompressible liquid, whose parameters are the same as those of the Sun, cannot collapse.

Thus, we conclude:

A Schwarzschild sphere (collapsing space breaking) exists inside any physical body. The numerical value of its radius r_g is determined only by the body's mass M. We refer to the space-time inside the Schwarzschild sphere ($r < r_g$) as a "black hole". This space-time does not satisfy the singular conditions of the space-time where real observers exist. Schwarzschild sphere (internal black hole) is an internal characteristic of any gravitating body, independent on its internal constitution.

One can ask: then what does the Hilbert radius r_g mean for the Sun, in this context? Here is the answer: r_g is the photometric distance in the radial direction, separating the "external" region inhabited with real particles and the "internal" region under the radius wherein all particles bear imaginary masses. Particles which inhabit the boundary surface (its radius is r_g) bear singular physical properties. Note that no one real (external) observer can register events inside the singularity.

What is a sphere of incompressible liquid of the radius $r = r_c$? This is a "collapsar" — the object in the state of gravitational collapse. As it was shown above, not any sphere of incompressible liquid can be collapsar: the possibility of its collapse is determined by the relation between its radius *a* and its Hilbert radius r_g , according to formula (25). It was shown above that the Universe considered as a sphere of incompressible liquid is a collapsar.

Now we apply this research method to the planets of the Solar System. Thus, we approximate the planets by spheres of incompressible liquid. The numerical values of r_c , calculated for the planets according to the same formula (25) as that for the liquid model of the Sun, are imaginary. Therefore, the planets being approximated by spheres of incompressible liquid cannot collapse as well as the Sun.

The Hilbert radius r_g calculated for the planets is much smaller than the sizes of their physical bodies, and is in the order of 1 cm. This means that, given any of the planets of the Solar System, the singulary surface separating our world and the imaginary mass particles world in its gravitational field draws the sphere of the radius about one centimetre around its centre of gravity.

The numerical values of the radius of the space breaking are calculated for each of the planets through the average density of substance inside the planet according to the formula (28).

The results of the summarizing and substraction associated with the planets lead to the next conclusions:

- The spheres of the singularity breaking of the spaces of Mercury, Venus, and the Earth are completely located inside the sphere of the singularity breaking of the Sun's space;
- 2. The spheres of the singularity breaking of the internal spaces of all planets intersect among themselves, when being in the state of a "parade of planets";
- 3. The spheres of the singularity breaking of the Earth's space and Mars' space reach the Asteroid strip;
- 4. The sphere of the singularity breaking of Mars' space intersects with the Asteroid strip near the orbit of Phaeton (the hypothetical planet which was orbiting the Sun, according to the Titius–Bode law, at r = 2.8 AU, and whose distraction in the ancient time gave birth to the Asteroid strip).
- 5. Jupiter's singularity breaking surface intersects the Asteroid strip near Phaeton's orbit, r = 2.8 AU, and meets Saturn's singularity breaking from the outer side;
- 6. The singularity breaking surface of Saturn's space is located between those of Jupiter and Uranus;
- 7. The singularity breaking surface of Uranus's space is located between those of Saturn and Neptune;
- The singularity breaking surface of Neptune's space meets, from the outer side, the lower boundary of the Kuiper belt (the strip of the aphelia of the Solar System's comets);
- 9. The singularity breaking surface of Pluto is completely located inside the lower strip of the Kuiper belt.

Just two small notes in addition to these. The intersections of the space breakings of the planets, discussed here, take place for only that case where the planets thenselves are in the state of a "parade of planets". However the conclusions concerning the location of the space breaking spheres, for instance — that the space breaking spheres of the internal planets are located inside the sphere of the Sun's space breaking, while the space breaking spheres of the external planets are located outside it, — are true for any position of the planets.

The fact that the space breaking of the Sun meets the Asteroid strip, near Phaeton's orbit, allows us to say: yes, the space breaking considered in this study has a really physical meaning. As probable the Sun's space breaking did not permit the Asteroids to be joined into a common physical body, Phaeton. Alternatively, if Phaeton was an already existing planet of the Solar System, the common action of the space breaking of the Sun and that of another massive cosmic body, appeared near the Solar System in the ancient ages (for example, another star passing near it), has led to the distraction of Phaeton's body.

Thus the internal constitution of the Solar System was formed by the structure of the Sun's space (space-time) filled with its gravitational field, and according to the laws of the General Theory of Relativity.

These and related results will be published in necessary detail later [5]*.

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^{*}The detailed presentation of the results [5] was already published at the moment when this short paper was accepted.