Sound-Particles and Phonons with Spin 1

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We present a new model for solids which is based on the stimulated vibration of independent neutral Fermi-atoms, representing independent harmonic oscillators with natural frequencies, which are excited by actions of the longitudinal and transverse elastic waves. Due to application of the principle of elastic wave-particle duality, we predict that the lattice of a solid consists of two type Sound Boson-Particles with spin 1 with finite masses. Namely, these lattice Boson-Particles excite the longitudinal and transverse phonons with spin 1. In this letter, we estimate the masses of Sound Boson-Particles which are around 500 times smaller than the atom mass.

1 Introduction

The original theory proposed by Einstein in 1907 was of great historical relevance [1]. In the Einstein model, each atom oscillates relatively to its neighbors in the lattice which execute harmonic motions around fixed positions, the knots of the lattice. He treated the thermal property of the vibration of a lattice of N atoms as a $3N$ harmonic independent oscillator by identical own frequency Ω_0 which was quantized by application of the prescription developed by Plank in connection with the theory of Black Body radiation. The Einstein model could obtain the Dulong and Petit prediction at high temperature but could not reproduce an adequate representation of the the lattice at low temperatures. In 1912, Debye proposed to consider the model of the solid [2], by suggestion that the frequencies of the $3N$ harmonic independent oscillators are not equal as it was suggested by the Einstein model. In addition to his suggestion, the acoustic spectrum of solid may be treated as if the solid represented a homogeneous medium, except that the total number of independent elastic waves is cut off at $3N$, to agree with the number of degrees of freedom of N atoms. In this respect, Debye stated that one longitudinal and two transverse waves are excited in solid. These velocities of sound cannot be observed in a solid at frequencies above the cut-off frequency. Also, he suggested that phonon is a spinless. Thus, the Debye model correctly showed that the heat capacity is proportional to the T^3 law at low temperatures. At high temperatures, he obtained the Dulong-Petit prediction compatible to experimental results.

In this letter, we propose a new model for solids which consists of neutral Fermi-atoms, fixed in the knots of lattice. In turn, within the formalism of Debye, we may predict that lattice represents as the Bose-gas of Sound-Particles with finite masses m_l and m_t, corresponding to a longitudinal and a transverse elastic field. In this sense, the lattice is considered as a new substance of matter consisting of Sound-Particles, which excite the one longitudinal and one transverse elastic waves (this approach is differ from Debye one). These waves act on the Fermi-atoms which are vibrating with the natural frequencies Ω_l and Ω_t. Thus, there are stimulated vibrations of the Fermi-atoms by under action of longitudinal and transverse phonons with spin 1. In this context, we introduce a new principle of elastic wave-particle duality, which allows us to build the lattice model. The given model leads to the same results as presented by Debye's theory.

2 Analysis

As we suggest, the transfer of heat from one part of the body to another occurs through the lattice. This process is very slow. Therefore, we can regard any part of the body as thermally insulated, and there occur adiabatic deformations. In this respect, the equation of motion for an elastic continuum medium [3] represents as

$$\varrho\ddot{\vec{u}}(\vec{r},t) = c_t^2\nabla^2\vec{u}(\vec{r},t) + (c_l^2 - c_t^2)\, grad \cdot div\, \vec{u}(\vec{r},t) \qquad (1)$$

where $\vec{u} = \vec{u}(\vec{r},t)$ is the vector displacement of any particle in solid; c_l and c_t are the velocities of a longitudinal and a transverse ultrasonic wave, respectively.

We shall begin by discussing a plane longitudinal elastic wave with condition $\mathrm{curl}\,\vec{u}(\vec{r},t) = 0$ and a plane transverse elastic wave with condition $\mathrm{div}\,\vec{u}(\vec{r},t) = 0$ in an infinite isotropic medium. In this respect, in direction of vector $\vec{r}$ can be propagated two transverse and one longitudinal elastic waves. The vector displacement $\vec{u}(\vec{r},t)$ is the sum of the vector displacements of a longitudinal $u_l(\vec{r},t)$ and of a transverse ultrasonic wave $u_t(\vec{r},t)$:

$$\vec{u}(\vec{r},t) = \vec{u}_l(\vec{r},t) + \vec{u}_t(\vec{r},t) \qquad (2)$$

where $\vec{u}_l(\vec{r},t)$ and $\vec{u}_t(\vec{r},t)$ are perpendicular with each other or $\vec{u}_l(\vec{r},t)\cdot\vec{u}_t(\vec{r},t) = 0$.

In turn, the equations of motion for a longitudinal and a transverse elastic wave take the form of the wave-equations:

$$\nabla^2\vec{u}_l(\vec{r},t) - \frac{1}{c_l^2}\frac{\partial^2\vec{u}_l(\vec{r},t)}{\partial t^2} = 0 \qquad (3)$$

$$\nabla^2\vec{u}_t(\vec{r},t) - \frac{1}{c_t^2}\frac{\partial^2\vec{u}_l(\vec{r},t)}{\partial t^2} = 0. \qquad (4)$$

It is well known, in quantum mechanics, a matter wave is determined by electromagnetic wave-particle duality or de Broglie wave of matter [4]. We argue that in an analogous manner, we may apply the elastic wave-particle duality. This reasoning allows us to present a model of elastic field as the Bose-gas consisting of the Sound Bose-particles with spin 1 and non-zero rest masses, which are interacting with each other. In this respect, we may express the vector displacement of a longitudinal ultrasonic wave $u_l(\vec{r},t)$ via the second quantization vector wave functions of one Sound Boson of the longitudinal wave. In analogy manner, vector displacement of a transverse ultrasonic waves $u_t(\vec{r},t)$ is expressed via the second quantization vector wave functions of one Sound Boson of the transverse wave:

$$\vec{u}_l(\vec{r},t) = u_l\left(\phi(\vec{r},t) + \phi^+(\vec{r},t)\right) \quad (5)$$

and

$$\vec{u}_t(\vec{r},t) = u_t\left(\psi(\vec{r},t) + \psi^+(\vec{r},t)\right) \quad (6)$$

where u_l and u_t are, respectively, the norm coefficients for longitudinal and transverse waves; $\vec{\phi}(\vec{r},t)$ and $\vec{\phi}^+(\vec{r},t)$ are, respectively, the second quantization wave vector functions for "creation" and "annihilation" of one Sound-Particle of the longitudinal wave, in point of coordinate $\vec{r}$ and time t whose direction $\vec{l}$ is directed toward to wave vector $\vec{k}$; $\vec{\psi}(\vec{r},t)$ and $\vec{\psi}^+(\vec{r},t)$ are, respectively, the second quantization wave vector functions for "creation" and "annihilation" of one Sound-Particle of the transverse wave, in point of coordinate $\vec{r}$ and time t, whose direction $\vec{t}$ is perpendicular to the wave vector $\vec{k}$:

$$\vec{\phi}(\vec{r},t) = \frac{1}{\sqrt{V}}\sum_{\vec{k},\sigma}\vec{a}_{\vec{k},\sigma}e^{i(\vec{k}\vec{r}-kc_lt)} \quad (7)$$

$$\vec{\phi}^+(\vec{r},t) = \frac{1}{\sqrt{V}}\sum_{\vec{k},\sigma}\vec{a}^+_{\vec{k},\sigma}e^{-i(\vec{k}\vec{r}-kc_lt)} \quad (8)$$

and

$$\vec{\psi}(\vec{r},t) = \frac{1}{\sqrt{V}}\sum_{\vec{k},\sigma}\vec{b}_{\vec{k},\sigma}e^{i(\vec{k}\vec{r}+-kc_tt)} \quad (9)$$

$$\vec{\psi}^+(\vec{r},t) = \frac{1}{\sqrt{V}}\sum_{\vec{k},\sigma}\vec{b}^+_{\vec{k},\sigma}e^{-i(\vec{k}\vec{r}-kc_tt)} \quad (10)$$

with condition

$$\int\phi^+(\vec{r},\sigma)\phi(\vec{r},\sigma)dV + \int\psi^+(\vec{r},\sigma)\psi(\vec{r},\sigma)dV = \\ = n_o + \sum_{\vec{k}\neq 0,\sigma}\hat{a}^+_{\vec{k},\sigma}\hat{a}_{\vec{k},\sigma} + \sum_{\vec{k}\neq 0,\sigma}\hat{b}^+_{\vec{k},\sigma}\hat{b}_{\vec{k},\sigma} = \hat{n} \quad (11)$$

where $\vec{a}^+_{\vec{k},\sigma}$ and $\vec{a}_{\vec{k},\sigma}$ are, respectively, the Bose vector-operators of creation and annihilation for one free longitudinal Sound Particle with spin 1, described by a vector $\vec{k}$ whose direction coincides with the direction $\vec{l}$ of the longitudinal wave; $\vec{b}^+_{\vec{k},\sigma}$ and $\vec{b}_{\vec{k},\sigma}$ are, respectively, the Bose vector-operators of creation and annihilation for one free transverse Sound Particles with spin 1, described by a vector $\vec{k}$ which is perpendicular to the direction $\vec{t}$ of the transverse wave; $\hat{n}$ is the operator of total number of the Sound Particles; $\hat{n}_0 = n_{0,l} + n_{0,t}$ is the total number of Sound Particles in the condensate level with wave vector $\vec{k} = 0$ which consists of a number of Sound Particles $n_{0,l}$ of the longitudinal wave and a number of Sound Particles $n_{0,t}$ of the transverse wave.

The energies of longitudinal $\frac{\hbar^2k^2}{2m_l}$ and transverse $\frac{\hbar^2k^2}{2m_t}$ free Sound Particles have the masses of Sound Particles m_l and m_t and the value of its spin z-component $\sigma = 0; \pm 1$. In this respect, the vector-operators $\vec{a}^+_{\vec{k},\sigma}$, $\vec{a}_{\vec{k},\sigma}$ and $\vec{b}^+_{\vec{k},\sigma}$, $\vec{b}_{\vec{k},\sigma}$ satisfy the Bose commutation relations as:

$$\left[\hat{a}_{\vec{k},\sigma}, \hat{a}^+_{\vec{k}',\sigma'}\right] = \delta_{\vec{k},\vec{k}'}\cdot\delta_{\sigma,\sigma'}$$

$$[\hat{a}_{\vec{k},\sigma}, \hat{a}_{\vec{k}',\sigma'}] = 0$$

$$[\hat{a}^+_{\vec{k},\sigma}, \hat{a}^+_{\vec{k}',\sigma'}] = 0$$

and

$$\left[\hat{b}_{\vec{k},\sigma}, \hat{b}^+_{\vec{k}',\sigma'}\right] = \delta_{\vec{k},\vec{k}'}\cdot\delta_{\sigma,\sigma'}$$

$$[\hat{b}_{\vec{k},\sigma}, \hat{b}_{\vec{k}',\sigma'}] = 0$$

$$[\hat{b}^+_{\vec{k},\sigma}, \hat{b}^+_{\vec{k}',\sigma'}] = 0$$

Thus, as we see, the vector displacements of a longitudinal $\vec{u}_l$ and of a transverse $\vec{u}_t$ ultrasonic wave satisfy the wave-equations of (3) and (4) and have the forms:

$$\vec{u}_l(\vec{r},t) = u_{\vec{0},l} + \frac{u_l}{\sqrt{V}}\sum_{\vec{k}\neq 0,\sigma}\left(\vec{a}_{\vec{k},\sigma}e^{i(\vec{k}\vec{r}-kc_lt)} + \vec{a}^+_{\vec{k},\sigma}e^{-i(\vec{k}\vec{r}-kc_lt)}\right) \quad (12)$$

and

$$\vec{u}_t(\vec{r},t) = u_{\vec{0},t} + \frac{u_t}{\sqrt{V}}\sum_{\vec{k}\neq 0,\sigma}\left(\vec{b}_{\vec{k},\sigma}e^{i(\vec{k}\vec{r}-kc_tt)} + \vec{b}^+_{\vec{k},\sigma}e^{-i(\vec{k}\vec{r}-kc_tt)}\right). \quad (13)$$

While investigating superfluid liquid, Bogoliubov [5] separated the atoms of helium in the condensate from those atoms, filling states above the condensate. In an analogous manner, we may consider the vector operators $\hat{a}_0 = \vec{l}\sqrt{n_{0,l}}$, $\hat{b}_0 = \vec{t}\sqrt{n_{0,t}}$ and $\hat{a}^+_0 = \vec{l}\sqrt{n_{0,l}}$, $\hat{b}^+_0 = \vec{t}\sqrt{n_{0,t}}$ as c-numbers (where $\vec{l}$ and $\vec{t}$ are the unit vectors in the direction of the longitudinal and transverse elastic fields, respectively, and also $\vec{l}\cdot\vec{t} = 0$) within the approximation of a macroscopic number of Sound Particles in the condensate $n_{0,l} \gg 1$ and $n_{0,t} \gg 1$. This assumptions lead to a broken Bose-symmetry law for Sound Particles of longitudinal and transverse waves in the condensate. In fact, we may state that if a number of Sound Particles of longitudinal and transverse waves fills a condensate level with the wave vector $\vec{k} = 0$, then they reproduce the constant displacements $\vec{u}_{0,l} = \frac{2u_l\vec{e}\sqrt{n_{0,l}}}{\sqrt{V}}$ and $\vec{u}_{0,t} = \frac{2u_t\vec{e}\sqrt{n_{0,t}}}{\sqrt{V}}$.

In this context, we may emphasize that the Bose vector operators $\vec{a}^+_{\vec{k},\sigma}$, $\vec{a}_{\vec{k},\sigma}$ and $\vec{b}^+_{\vec{k},\sigma}$ and $\vec{b}_{\vec{k},\sigma}$ communicate with each other because the vector displacements of a longitudinal $\vec{u}_l(\vec{r},t)$ and a transverse ultrasonic wave $\vec{u}_t(\vec{r},t)$ are independent, and in turn, satisfy to the Bose commutation relation $[\vec{u}_l(\vec{r},t), \vec{u}_t(\vec{r},t)] = 0$.

Now, we note that quantization of elastic field means that this field operator does not commute with its momentum density. Taking the commutators gives

$$\left[\vec{u}_l(\vec{r},t), \vec{p}_l(\vec{r}\,',t)\right] = i\hbar\delta^3_{\vec{r}-\vec{r}\,'} \tag{14}$$

and

$$\left[\vec{u}_t(\vec{r},t), \vec{p}_t(\vec{r}\,',t)\right] = i\hbar\delta^3_{\vec{r}-\vec{r}\,'} \tag{15}$$

where the momentums of the longitudinal and transverse waves are defined as

$$\vec{p}_l(\vec{r},t) = \rho_l(\vec{r})\frac{\partial\vec{u}_l(\vec{r},t)}{\partial t} \tag{16}$$

and

$$\vec{p}_t(\vec{r},t) = \rho_t(\vec{r})\frac{\partial\vec{u}_t(\vec{r},t)}{\partial t} \tag{17}$$

where $\rho_l(\vec{r})$ and $\rho_t(\vec{r})$ are, respectively, the mass densities of longitudinal and transverse Sound Particles in the coordinate space, which are presented by the equations

$$\rho_l(\vec{r}) = \rho_{0,l} + \sum_{\vec{k}\neq 0}\rho_l(\vec{k})e^{i\vec{k}\vec{r}} \tag{18}$$

and

$$\rho_t(\vec{r}) = \rho_{0,t} + \sum_{\vec{k}\neq 0}\rho_t(\vec{k})e^{i\vec{k}\vec{r}}. \tag{19}$$

The total mass density $\rho(\vec{r})$ is

$$\rho(\vec{r}) = \rho_0 + \sum_{\vec{k}\neq 0}\rho_l(\vec{k})e^{i\vec{k}\vec{r}} + \sum_{\vec{k}\neq 0}\rho_t(\vec{k})e^{i\vec{k}\vec{r}} \tag{20}$$

where $\rho_l(\vec{k})$ and $\rho_t(\vec{k})$ are, respectively, the fluctuations of the mass densities of the longitudinal and transverse Sound Particles which represent as the symmetrical function from wave vector $\vec{k}$ or $\rho_l(\vec{k}) = \rho_l(-\vec{k})$; $\rho_t(\vec{k}) = \rho_t(-\vec{k})$; $\rho_0 = \rho_{0,l} + \rho_{0,t}$ is the equilibrium density of Sound Particles.

Applying (12) and (13) to (16) and (17), and taking (18) and (19), we get

$$\vec{p}_l(\vec{r},t) = -\frac{ic_lu_l}{\sqrt{V}}\sum_{\vec{k}'}\sum_{\vec{k},\sigma}k\rho_l(\vec{k}')\left(\vec{a}_{\vec{k},\sigma}e^{-ikc_lt} - \vec{a}^+_{-\vec{k},\sigma}e^{ikc_lt}\right)e^{i(\vec{k}+\vec{k}')\vec{r}} \tag{21}$$

$$\vec{p}_t(\vec{r},t) = -\frac{ic_lu_t}{\sqrt{V}}\sum_{\vec{k}'}\sum_{\vec{k},\sigma}\rho_t(\vec{k}')k\left(\vec{b}_{\vec{k},\sigma}e^{-ikc_lt} - \vec{b}^+_{-\vec{k},\sigma}e^{ikc_lt}\right)e^{i(\vec{k}+\vec{k}')\vec{r}} \tag{22}$$

Application of (12), (21) and (13), (22) to (14) and (15), and taking the Bose commutation relations presented above, we obtain

$$\left[\vec{u}_l(\vec{r},t), \vec{p}_l(\vec{r}\,',t)\right] = \frac{2iu_l^2c_l}{V}\sum_{\vec{k}}k\rho_l(\vec{k})e^{i\vec{k}(\vec{r}-\vec{r}\,')} \tag{23}$$

and

$$\left[\vec{u}_t(\vec{r},t), \vec{p}_t(\vec{r}\,',t)\right] = \frac{2iu_t^2c_t}{V}\sum_{\vec{k}}k\rho_t(\vec{k})e^{i\vec{k}(\vec{r}-\vec{r}\,')} \tag{24}$$

The right sides of Eqs. (14) and (23) as well as Eqs. (15) and (24) coincide when

$$\rho_l(\vec{k}) = \frac{\hbar}{2ku_l^2c_l} \tag{25}$$

and

$$\rho_t(\vec{k}) = \frac{\hbar}{2ku_t^2c_t} \tag{26}$$

by using

$$\frac{1}{V}\sum_{\vec{k}}e^{i\vec{k}(\vec{r}-\vec{r}\,')} = \delta^3_{\vec{r}-\vec{r}\,'}$$

3 Sound-Particles and Phonons

The Hamiltonian operator $\hat{H}$ of the system, consisting of the vibrated Fermi-atoms with mass M, is represented by the following form

$$\hat{H} = \hat{H}_l + \hat{H}_t \tag{27}$$

where

$$\hat{H}_l = \frac{MN}{2V}\int\left(\frac{\partial\vec{u}_l}{\partial t}\right)^2 dV + \frac{NM\Omega_l^2}{2V}\int(\vec{u}_l)^2 dV \tag{28}$$

and

$$\hat{H}_t = \frac{MN}{2V}\int\left(\frac{\partial\vec{u}_t}{\partial t}\right)^2 dV + \frac{NM\Omega_t^2}{2V}\int(\vec{u}_t)^2 dV \tag{29}$$

with Ω_l and Ω_t which are, respectively, the natural frequencies of the atom by action of longitudinal and transverse elastic waves.

To find the Hamiltonian operator $\hat{H}$ of the system, we use the framework of Dirac [6] for the quantization of electromagnetic field:

$$\frac{\partial \vec{u}_l(\vec{r},t)}{\partial t} = -\frac{ic_l u_l}{\sqrt{V}}\sum_{\vec{k},\sigma} k\left(\vec{a}_{\vec{k},\sigma}e^{-ikc_l t} - \vec{a}^+_{-\vec{k},\sigma}e^{ikc_l t}\right)e^{i\vec{k}\vec{r}} \quad (30)$$

and

$$\frac{\partial \vec{u}_t(\vec{r},t)}{\partial t} = -\frac{ic_t u_t}{\sqrt{V}}\sum_{\vec{k},\sigma} k\left(\vec{b}_{\vec{k},\sigma}e^{-ikc_t t} - \vec{b}^+_{-\vec{k},\sigma}e^{ikc_t t}\right)e^{i\vec{k}\vec{r}} \quad (31)$$

which by substituting into (28) and (29) using (12) and (13), we obtain the reduced form for the Hamiltonian operators $\hat{H}_l$ and $\hat{H}_t$:

$$\hat{H}_l = \sum_{\vec{k},\sigma}\left[\left(\frac{MNu_l^2c_l^2k^2}{V} + \frac{MNu_l^2\Omega_l^2}{V}\right)\vec{a}^+_{\vec{k},\sigma}a_{\vec{k},\sigma} - \left(\frac{MNu_l^2c_l^2k^2}{V} - \frac{MNu_l^2\Omega_l^2}{V}\right)\left(\vec{a}_{-\vec{k},\sigma}\vec{a}_{\vec{k},\sigma} + \vec{a}^+_{\vec{k},\sigma}\vec{a}^+_{-\vec{k},\sigma}\right)\right] \quad (32)$$

and

$$\hat{H}_t = \sum_{\vec{k},\sigma}\left[\left(\frac{MNu_t^2c_t^2k^2}{V} + \frac{MNu_t^2\Omega_t^2}{V}\right)\vec{a}^+_{\vec{k},\sigma}a_{\vec{k},\sigma} - \left(\frac{MNu_t^2c_t^2k^2}{V} - \frac{MNu_l^2\Omega_t^2}{V}\right)\left(\vec{a}_{-\vec{k},\sigma}\vec{a}_{\vec{k},\sigma} + \vec{a}^+_{\vec{k},\sigma}\vec{a}^+_{-\vec{k},\sigma}\right)\right] \quad (33)$$

where u_l and u_t are defined by the first term in right side of (32) and (33) which represent as the kinetic energies of longitudinal Sound Particle $\frac{\hbar^2k^2}{2m_l}$ and transverse Sound Particles $\frac{\hbar^2k^2}{2m_t}$. Therefore, u_l and u_l are found, if we suggest:

$$\frac{MNu_l^2c_l^2k^2}{V} = \frac{\hbar^2k^2}{2m_l} \quad (34)$$

and

$$\frac{MNu_t^2c_t^2k^2}{V} = \frac{\hbar^2k^2}{2m_t} \quad (35)$$

which in turn determine

$$u_l = \frac{\hbar}{c_l\sqrt{2m_l\rho}}$$

and

$$u_t = \frac{\hbar}{c_t\sqrt{2m_t\rho}}$$

where $\rho = \frac{MN}{V}$ is the density of solid.

$$\hat{H}_l = \sum_{\vec{k},\sigma}\left[\left(\frac{\hbar^2k^2}{2m_l} + \frac{\hbar^2\Omega_l^2}{2m_lc_l^2}\right)\vec{a}^+_{\vec{k},\sigma}a_{\vec{k},\sigma} + \frac{U_{\vec{k},l}}{2}\left(\vec{a}_{-\vec{k},\sigma}\vec{a}_{\vec{k},\sigma} + \vec{a}^+_{\vec{k},\sigma}\vec{a}^+_{-\vec{k},\sigma}\right)\right] \quad (36)$$

and

$$\hat{H}_t = \sum_{\vec{k}}\left[\left(\frac{\hbar^2k^2}{2m} + \frac{\hbar^2\Omega_t^2}{2m_tc_t^2}\right)\vec{b}^+_{\vec{k},\sigma}b_{\vec{k},\sigma} + \frac{U_{\vec{k},t}}{2}\left(\vec{b}_{-\vec{k},\sigma}\vec{b}_{\vec{k},\sigma} + \vec{b}^+_{\vec{k},\sigma}\vec{b}^+_{-\vec{k},\sigma}\right)\right] \quad (37)$$

$U_{\vec{k},l}$ and $U_{\vec{k},t}$ are the interaction potentials between identical Sound Particles.

In analogous manner, as it was done in letter [7] regarding the quantization of the electromagnetic field, the boundary wave numbers $k_l = \frac{\Omega_l}{c_l}$ for the longitudinal elastic field and $k_t = \frac{\Omega_t}{c_t}$ for the transverse one are determined by suggestion that identical Sound Particles interact with each other by the repulsive potentials $U_{\vec{k},l}$ and $U_{\vec{k},t}$ in wave vector space

$$U_{\vec{k},l} = -\frac{\hbar^2k^2}{2m_l} + \frac{\hbar^2\Omega_l^2}{2m_lc_l^2} > 0$$

and

$$U_{\vec{k},t} = -\frac{\hbar^2k^2}{2m_t} + \frac{\hbar^2\Omega_t^2}{2m_tc_t^2} > 0$$

As results, there are two conditions for wave numbers of longitudinal $k < k_l$ and transverse $k < k_t$ Sound Particles which are provided by property of the model of hard spheres [8]. Indeed, there is a request of presence of repulsive potential interaction between identical kind of particles (recall S-wave repulsive pseudopotential interaction between atoms in the superfluid liquid ^{4}He in the model of hard spheres [8]).

On the other hand, it is well known that at absolute zero $T = 0$, the Fermi atoms fill the Fermi sphere in momentum space. As it is known, the total numbers of the Fermi atoms with opposite spins are the same, therefore, the Fermi wave number k_f is determined by a condition:

$$\frac{V}{2\pi^2}\int_0^{k_f} k^2dk = \frac{N}{2} \quad (38)$$

where N is the total number of Fermi-atoms in the solid. This reasoning together with the model of hard spheres claims the important condition as introduction the boundary wave number $k_f = \left(\frac{3\pi^2N}{V}\right)^{\frac{1}{3}}$ coinciding with k_l and k_t. Thus we claim that all Fermi atoms had one natural wavelength

$$\lambda_0 = \frac{2\pi}{k_f} = \frac{2\pi}{k_l} = \frac{2\pi}{k_t} \quad (39)$$

This approach is a similar to the Einstein model of solid where he suggested that all atoms have the same natural frequencies.

Now, to evaluate of the energy levels of the operator $\hat{H}_l$ (36) and $\hat{H}_t$ (37) in diagonal form, we use a new transformation of the vector-Bose-operators presented in [6]:

$$\vec{a}_{\vec{k},\sigma} = \frac{\vec{c}_{\vec{k},\sigma} + L_{\vec{k}}\vec{c}^+_{-\vec{k},\sigma}}{\sqrt{1 - L^2_{\vec{k}}}} \quad (40)$$

and

$$\vec{b}_{\vec{k},\sigma} = \frac{\vec{d}_{\vec{k},\sigma} + M_{\vec{k}}\vec{d}^{+}_{-\vec{k},\sigma}}{\sqrt{1 - M^2_{\vec{k}}}} \tag{41}$$

where $L_{\vec{k}}$ and $M_{\vec{k}}$ are, respectively, the real symmetrical functions of a wave vector $\vec{k}$. Consequently:

$$\hat{H}_l = \sum_{k<k_f,\sigma} \varepsilon_{\vec{k},l}\vec{c}^{+}_{\vec{k},\sigma}\vec{c}_{\vec{k},\sigma} \tag{42}$$

and

$$\hat{H}_t = \sum_{k<k_f,\sigma} \varepsilon_{\vec{k},t}\vec{d}^{+}_{\vec{k},\sigma}\vec{d}_{\vec{k},\sigma}. \tag{43}$$

Hence, we infer that the Bose-operators $\vec{c}^{+}_{\vec{k},\sigma}$, $\vec{c}_{\vec{k},\sigma}$ and $\vec{d}^{+}_{\vec{k},\sigma}$, $\vec{d}_{\vec{k},\sigma}$ are, respectively, the vector of "creation" and the vector of "annihilation" operators of longitudinal and transverse phonons with spin 1 and having the energies:

$$\varepsilon_{\vec{k},l} = \sqrt{\left(\frac{\hbar^2k^2}{2m_l} + \frac{\hbar^2\Omega^2_l}{2m_lc^2_l}\right)^2 - \left(\frac{\hbar^2k^2}{2m_l} - \frac{\hbar^2\Omega^2_l}{2m_lc^2_l}\right)^2} = \hbar k c_l \tag{44}$$

and

$$\varepsilon_{\vec{k},t} = \sqrt{\left(\frac{\hbar^2k^2}{2m_t} + \frac{\hbar^2\Omega^2_t}{2m_tc^2_t}\right)^2 - \left(\frac{\hbar^2k^2}{2m_t} - \frac{\hbar^2\Omega^2_l}{2m_tc^2_t}\right)^2} = \hbar k c_t \tag{45}$$

where the mass of longitudinal Sound Particle equals to

$$m_l = \frac{\hbar\Omega_l}{c^2_l} \tag{46}$$

but the mass of transverse Sound Particle is

$$m_t = \frac{\hbar\Omega_t}{c^2_t}. \tag{47}$$

Thus, we may state that there are two different Sound Particles with masses m_l and m_t which correspond to the longitudinal and transverse waves.

4 Thermodynamic property of solid

Now, we demonstrate that the herein presented theory leads to same results which were obtained by Debye in his theory investigating the thermodynamic properties of solids. So that, at the statistical equilibrium, the average energy of solid equals to

$$\overline{H} = \sum_{k<k_f,\sigma} \varepsilon_{\vec{k},l}\overline{\vec{c}^{+}_{\vec{k},\sigma}\vec{c}_{\vec{k},\sigma}} + \sum_{k<k_f,\sigma} \varepsilon_{\vec{k},t}\overline{\vec{d}^{+}_{\vec{k},\sigma}\vec{d}_{\vec{k},\sigma}} \tag{48}$$

where $\overline{\hat{c}^{+}_{\vec{k},\sigma}\hat{c}_{\vec{k},\sigma}}$ and $\overline{\hat{d}^{+}_{\vec{k},\sigma}\hat{d}_{\vec{k},\sigma}}$ are, respectively, the average number of phonons with the wave vector $\vec{k}$ corresponding to the longitudinal and transverse fields at temperature T:

$$\overline{\hat{c}^{+}_{\vec{k},\sigma}\hat{c}_{\vec{k},\sigma}} = \frac{1}{e^{\frac{\varepsilon_{\vec{k},l}}{kT}} - 1}$$

and

$$\overline{\hat{d}^{+}_{\vec{k},\sigma}\hat{d}_{\vec{k},\sigma}} = \frac{1}{e^{\frac{\varepsilon_{\vec{k},t}}{kT}} - 1}.$$

Thus, at thermodynamic limit, the average energy of solid may rewritten down as

$$\overline{H} = \frac{3Vk^4T^4}{2\pi^2\hbar^3c^3_l}\int_0^{\frac{\Theta_l}{T}} \frac{x^3dx}{e^x - 1} + \frac{3Vk^4T^4}{2\pi^2\hbar^3c^3_t}\int_0^{\frac{\Theta_t}{T}} \frac{x^3dx}{e^x - 1} \tag{49}$$

where $\Theta_l = \frac{\hbar k_f c_l}{k}$ and $\Theta_t = \frac{\hbar k_f c_t}{k}$ are, respectively, the characteristic temperatures for solid corresponding to longitudinal and transverse waves; k is the Boltzmann constant. In our theory we denote

$$\frac{1}{c^3_l} + \frac{1}{c^3_t} = \frac{2}{c^3}$$

where c is the average velocity of phonons with spin 1 in the given theory; $\Theta_B = \frac{\hbar k_f c}{k}$ is the new characteristic temperature.

Hence, we may note that the coefficient with number 3 must be appear before both integrals on the right side of equation (49) because it reflects that phonons of longitudinal and transverse waves have number 3 quantities of the value of spin z-component $\sigma = 0; \pm 1$. At $T \ll \Theta_l$ and $T \ll \Theta_t$, the equation (49) takes the form:

$$\overline{H} = \frac{3\pi^4}{5}\frac{NkT^4}{2}\left(\frac{1}{\Theta^3_l} + \frac{1}{\Theta^3_t}\right) \tag{50}$$

where $\int_0^\infty \frac{x^3dk}{e^x-1} = \frac{\pi^4}{15}$.

Thus, Eq.(50) may be rewritten as

$$\overline{H} \approx \frac{3\pi^4}{5}\frac{RT^4}{\Theta^3_B} \tag{51}$$

where $R = Nk$ is the gas constant. Hence, we may note that at $T \gg \Theta_l$ and $T \gg \Theta_t$, the equation (49) takes the form:

$$\overline{H} = 3RT. \tag{52}$$

In this context, the heat capacity is determined as

$$C_V = \left(\frac{d\overline{H}}{dT}\right)_V \tag{53}$$

which obviously, at $T \ll \Theta_l$ and $T \ll \Theta_t$, the equation (53) with (51) reflects the Debye law T^3 at low temperatures:

$$C_V \approx \frac{12\pi^4}{5}\frac{RT^3}{\Theta^3_B}. \tag{54}$$

But at high temperatures $T \gg \Theta_l$ and $T \gg \Theta_t$, the equation (53) with (52) recovers the Dulong-Petit law:

$$C_V \approx 3R. \tag{55}$$

Obviously, the average velocity of phonon c and new characteristic temperature Θ_B are differ from their definition in Debye theory because the average energy of solid in Debye theory is presented as

$$\overline{H}_D = \frac{3Vk^4T^4}{2\pi^2\hbar^3 c_l^3}\int_0^{\frac{\Theta_l}{T}} \frac{x^3 dx}{e^x - 1} + \frac{3Vk^4T^4}{\pi^2\hbar^3 c_t^3}\int_0^{\frac{\Theta_t}{T}} \frac{x^3 dx}{e^x - 1} \quad (56)$$

where $\Theta_l = \frac{\hbar k_D c_l}{k}$ and $\Theta_t = \frac{\hbar k_D c_t}{k}$ are, respectively, the characteristic temperatures for solid corresponding to one longitudinal and two transverse waves:

$$\frac{1}{c_l^3} + \frac{2}{c_t^3} = \frac{3}{\upsilon_0^3} \quad (57)$$

where υ_0 is the average velocity of spinless phonons in Debye theory; $k_D = \left(\frac{6\pi^2 N}{V}\right)^{\frac{1}{3}}$ is the Debye wave number; $\Theta_D = \frac{\hbar k_D \upsilon_0}{k}$ is the Debye characteristic temperature which is

$$\frac{1}{\Theta_l^3} + \frac{2}{\Theta_t^3} = \frac{3}{\Theta_D^3} \quad (58)$$

As we see the average energy of solid $\overline{H}_D$ in (56) is differ from one in (49) by coefficient 2 in ahead of second term in right side of Eq.(56) (which is connected with assumption of presence two transverse waves), as well as introduction of Debye wave number k_D. So that due to definition of the average velocity υ_0 of spinless phonons by (57), Debye may accept a phonon as spinless quasiiparticle.

5 Concussion

Thus, in this letter, we propose new model for solids which is different from the well-known models of Einstein and Debye because: 1), we suggest that the atoms are the Fermi particles which are absent in the Einstein and Debye models; 2), we consider the stimulated oscillation of atoms by action of longitudinal and transverse waves in the solid. The elastic waves stimulate the vibration of the fermion-atoms with one natural wavelength, we suggested that atoms have two independent natural frequencies corresponding to a longitudinal and a transverse wave, due to application of the principle of the elastic wave-particle duality, the model of hard spheres and considering the atoms as the Fermi particles. In accordance to this reasoning, there is an appearance of a cut off in the energy spectrum of phonons; 3), In our model, we argue that the photons have spin 1 which is different from models presented by Einstein and Debye. On the other hand, we suggest that only one longitudinal and one transverse wave may be excited in the lattice of the solid which is different from Debye who suggested a presence of two sorts of transverse waves.

The quantization of the elastic wave by our theory leads to a view of the lattice as the diffraction picture. Within our theory, the mass density $\rho(\vec{r})$ in coordinate space, due to substituting $\rho_l(\vec{r}, t)$ and $\rho_l(\vec{r}, t)$ from (25) and (26) into (20), represents as

$$\rho(\vec{r}) = \rho_0 + \frac{8\pi\hbar k_f^2}{u_l^2 c_l}\left(\frac{\sin k_f r}{k_f r}\right)^2 + \frac{8\pi\hbar k_f^2}{u_t^2 c_t}\left(\frac{\sin k_f r}{k_f r}\right)^2 \quad (59)$$

which implies that the lattice has the diffraction picture.

Now, we try to estimate the masses of the Sound Particles in substance as Aluminium Al. In this respect, we use of (46) and (47) with introducing of the Fermi momentum $p_f = \hbar k_f = \frac{\hbar\Omega_l}{c_l} = \frac{\hbar\Omega_t}{c_t}$, for instance, for such material as Al with $c_l = 6.26 \cdot 10^3 \frac{m}{sec}$ and $c_t = 3.08 \cdot 10^3 \frac{m}{sec}$ at room temperature [9], and $p_f = 1.27 \cdot 10^{-24} \frac{kg \cdot m}{sec}$ we may estimate $m_l = \frac{p_f}{c_l} = 2 \cdot 10^{-28} kg$ and $m_t = \frac{p_f}{c_t} = 4 \cdot 10^{-28} kg$.

It is well known that the mass of atom Al is $M = 10^{-25} kg$ which is around 500 time more in regard to the masses of Sound Particles.

In this context, we remark that the new characteristic temperature Θ_B almost coincide with the Debye temperature Θ_D. Indeed, by our theory for Al:

$$\Theta_B = \frac{2^{\frac{1}{3}} p_f c_l}{k\left(1 + \frac{c_l^3}{c_t^3}\right)^{\frac{1}{3}}} \approx 400K$$

but Debye temperature equals to $\Theta_D = 418K$.

Acknowledgements

We are particularly grateful to Dr. Andreas Ries for valuable scientific support and for help with the English.

Submitted on October 20, 2010 / Accepted on November 9, 2010

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