Five Fallacies Used to Link Black Holes to Einstein's Relativistic Space-Time

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For a particle falling radially toward a compact mass, the Schwarzschild metric maps local time to coordinate time based on radial locations reached by the particle. The mapping shows the particle will not cross a critical radius regardless of the coordinate used to measure time. Herein are discussed five fallacies that have been used to make it appear the particle can cross the critical radius.

1 Introduction

Einstein [1] sets out field equations that describe a matter-free field. A German military officer, Karl Schwarzschild [2], shortly before he died, derived a solution of the field equations for a static gravitational field of spherical symmetry. Schwarzschild's solution is referred to as the Schwarzschild metric.

Einstein [3] showed that matter cannot be compacted below a critical radius defined by the Schwarzschild metric. Weller [4] shows that compacting matter below the critical radius to form a black hole results in a violation of the conservation of momentum and energy.

Why, then, do many believe that black holes exist in Einstein's relativistic space time? The belief appears to have arisen based, at least partly, on an incorrect description of the journey of a particle falling radially towards a hypothetical mass compacted below the critical radius. The description is incorrect in that the particle reaches and crosses the critical radius.

Herein are discussed five fallacies used in the description of the particle's journey. Preliminary to addressing the fallacies, it is shown why the particle will never reach the critical radius.

2 Mapping coordinate time t to local time τ

For a particle falling radially toward a hypothetical mass compacted below a critical radius, a mapping of the coordinate time t of a distant observer to a local time τ of the particle based on a radial distance r is shown in Fig. 1. The data

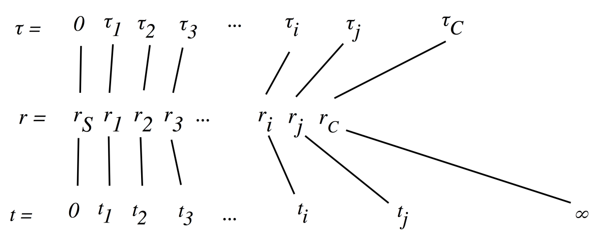

Fig. 1: For a particle falling radially, the Schwarschild Metric maps every value of the coordinate time t of a distant observer — where $0 \leq t \leq \infty$ — into a corresponding value of the local time τ of the particle — where $0 \leq \tau \leq \tau_C$.

shown in Fig. 1 can be obtained using the Schwarzschild metric.

Particularly, for a compact mass M with a Schwarzschild radius R, the Schwarzschild metric can be expressed using reference space coordinates (r, θ, ϕ), a coordinate time t and a local time τ (often referred to as proper time τ), i.e.,

$$c^2 d\tau^2 = c^2 \left(1 - \frac{R}{r}\right) dt^2 - \frac{dr^2}{(1 - R/r)} - r^2 d\theta^2 - (r^2 sin^2\theta) d\phi^2. \quad (1)$$

Reference coordinates (r, θ, ϕ, t) are the space and time coordinates used by the distant observer to make measurements while the particle detects passage of time using local time coordinate τ. For a particle falling radially

$$d\theta = d\phi = 0, \quad (2)$$

so the Schwarzschild metric in (1) reduces to

$$c^2 d\tau^2 = c^2 \left(1 - \frac{R}{r}\right) dt^2 - \frac{dr^2}{(1 - R/r)}, \quad (3)$$

which expresses a relationship between radial location r, local time τ and coordinate time t.

According to the relationship expressed by (3), for every radial location r_i reached from a starting location r_S, the coordinate time t_i to reach radial location r_i can be calculated using an integral

$$t_i = \int_{r_S}^{r_i} dt = \int_{r_S}^{r_i} f_1(r) dr, \quad (4)$$

where $f_1(r)$is a function of r derived from (3) [5, p. 667].

The local time τ_i required to reach the radial location r_i can be calculated using an integral

$$\tau_i = \int_{r_S}^{r_i} d\tau = \int_{r_S}^{r_i} f_2(r) dr, \quad (5)$$

where $f_2(r)$is a function of r derived from (3) [5, p. 663].

When the radial location r_i is set equal to a critical radius r_C, the integrand $f_1(r)$ for the integral in (4) and the integrand $f_2(r)$ for the integral in (5) are undefined; however, the integral in (5) converges while the integral in (4) does not. This indicates that the critical radius r_C is reached in a finite local time τ_C but cannot be reached in finite Schwarzschild coordinate time.

The results of calculations using the integral of (4) and the integral of (5) are summarized in Fig. 1. As shown by Fig. 1, based on the integrals in (4) and (5), any value of coordinate time t, $0 \leq t \leq \infty$, can be mapped into a corresponding value for local time τ, $0 \leq \tau \leq \tau_C$ based on radial location r.

3 A pause to check correctness of Fig. 1

At this point the reader is encouraged to stop, look at Fig. 1, and perform an obviousness check to confirm why the data in Fig. 1 must be correct. The salient points are as follows:

- It takes infinite coordinate time (i.e., $t = \infty$) to reach the critical radius r_C;
- It takes finite local time τ_C to reach the critical radius r_C;
- Both local time τ and coordinate time t monotonically progress with decreasing r;
- To reach each radial location r_i will take a coordinate time t_i to complete and a local time τ_i to complete;
- Based on radial location r_i, a value of coordinate time t_i is mapped to a local time τ_i.

A reader who understands why Fig. 1 must be an accurate description of data derived from the Schwarzschild metric has already made a paradigm shift which if held to provides an intuitive foundation from which to understand the remainder of the paper. There is only one slight modification to Fig. 1 that is necessary to reveal why the critical radius can never be crossed. That is the subject of the next section.

4 Fig. 1 modified to take into account the finite duration of the compact mass

Fig. 1 depicts data from the Schwarzschild metric for a hypothetical compact mass that is presumed to exist forever in coordinate time. But what happens when the compact mass is replaced by an entity that more closely approximates reality in that it has a finite lifetime? For example, replace the compact mass with a theoretical black hole that has a finite lifetime. The result is shown in Fig. 2.

Because of Hawking radiation [6], it is estimated that a black hole will evaporate well within 10^{100} years. Therefore, added to Fig. 2 is finite coordinate time t_E which is the coordinate time required for a hypothetical black hole to completely evaporate [7]. Using the mapping shown in Fig. 1, it is possible to identify a radial location r_E — where $r_E > r_C$ — the particle will have reached simultaneous with the black hole evaporating at coordinate time t_E.

Fig. 2 shows a local time τ_E that represents the local time required for the particle to reach r_E. Local time τ_E corresponds with coordinate time t_E — the coordinate time required for a black hole to completely evaporate. Local time τ_C, as calculated by (5), represents the local time required for the particle to reach critical radius r_C. Because $\tau_E < \tau_C$, the particle will experience in local time τ that the black hole will evaporate before the critical radius can be reached.

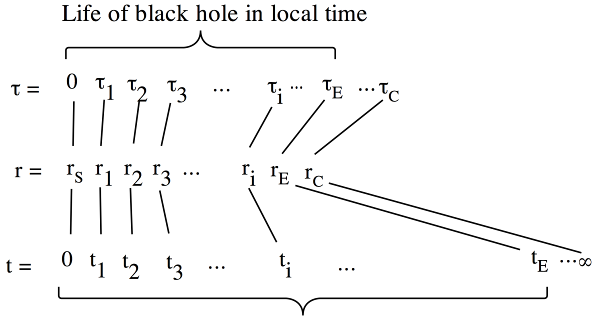

Fig. 2: According to the mapping of coordinate time to local time performed using the Schwarschild metric, the local time required to reach the critical radius of a black hole (τ_C) is longer than the life of the black hole (τ_E).

5 The significance of Fig. 2

Fig. 2, based on the data from the Schwarzschild metric, shows a radially falling particle will never cross the critical radius of the compact mass regardless of what coordinate is used to measure the passage of time. For every radial location reached by the particle (i.e., $r_S \geq r \geq r_E$, there is a corresponding coordinate time t to reach the radial location and a corresponding local time τ to reach the radial location. The final destination of the particle is not dependent upon which measure of time is used to time the journey.

Fig. 2 presents a paradigm that is in conformance with the fundamental requirement of general relativity — and indeed a coherent universe — that there is a single reality with a logical sequence of events. The logical sequence of events does not vary based upon the reference frame from which observations are made.

Fig. 2 is meant to be an anchor from which can be shown how each of the five fallacies discussed below entices a departure from a coherent reality, where the logical sequence of events is consistent for every reference frame, into an incoherent reality where physical events differ based on reference frames from which observations are made.

In the following discussion of fallacies, evaporation of black holes is used as a convenient way to account for the finite lifetime of a hypothetical mass compacted below the critical radius. However, as should be clear from Fig. 2, a particle cannot cross the critical radius and therefore, as pointed out by [3], a mass will never compact below its critical radius. For the implication of this for collapsing stars, see the discussion of fallacy 4 below.

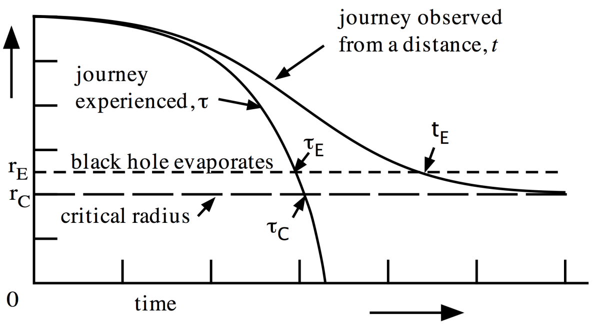

Fig. 3: Fig. 3 arranges the data shown in Fig. 2 in a different format. The trace extending to $r = 0$ incorrectly suggests that it is physically possible to cross the critical radius.

6 Fallacy 1: Showing a particle crosses the critical radius after evaporation of a black hole

For the journey of a particle to a black hole, elapsed time calculated using (4) and (5) is typically not represented as set out in Fig. 2, but rather as set out in Fig. 3 [5, p. 667].

Fig. 3, like Fig. 1 and Fig. 2, is a graphic representation of data obtained from (4) and (5). However, Fig. 3 qualifies as a fallacy because Fig. 3 includes extra data, not shown in Fig. 1 or Fig. 2., that incorrectly portrays the journey of the particle. Particularly, in Fig. 3, the trace representing local time τ extends beyond τ_C, the local time required to reach critical radius r_C.

Ordinary rules of mathematics cannot be used to generate the extra data for local time τ that occur after critical radius r_C is reached. This is because the integrand in (5) is undefined at r_C. Nevertheless, a novel "cycloid principle" [5, See pp. 663–664] has been used to generate this extra data.

But merely showing how the extra data can be mathematically generated does not overcome the logical sequencing problem introduced by adding the extra data to Fig. 3. The extra data suggests r_C can be reached and crossed in local time τ_C. However, this is impossible because as shown in Fig. 2, a black hole will evaporate in local time τ_E, so that critical radius r_C will cease to exist before it can be reached by the particle.

A horizontal line has been included in Fig. 3 to indicate where in Fig. 3 the evaporation of a black hole occurs. As shown by Fig. 3, evaporation of a black hole at radial location r_E, local time τ_E and coordinate time t_E logically occurs before reaching radial location r_C, local time τ_C and coordinate time $t = \infty$.

Fig. 3 should be corrected to show that a physical journey of a particle towards a black hole must end at radial location r_E — short of the critical radius r_C — when the black hole evaporates at local time τ_E and coordinate time t_E. The end of the journey occurs at r_E whether time is measured using coordinate time t or local time τ.

7 Fallacy 2: Declaring coordinates to be "pathological"

Fig. 3 suggests an impossible picture of physical reality. The particle cannot finally arrive at different destinations ($r = 0$ and $r = r_C$) merely based on the coordinate used to measure time.

As discussed in the last section, the logical sequence of events that occurs in all time frames, as out in Fig. 2, makes clear what is wrong with Fig. 3 and how it can be corrected. However, another competing explanation has been put forth.

The infinite coordinate time t required to reach the critical radius has been explained as the result of a "pathology" in the coordinates used to express the Schwarzschild metric. [5, See pp. 820-823].

Declaring coordinates to be pathological is a fallacy because it is a violation of general relativity at its most fundamental level. According to general relativity, all coordinates (reference frames) will observe the same reality. As Einstein [1, p. 117] made clear when setting out the basis for the theory of general relativity: "… all imaginable systems of coordinates, on principle, [are] equally suitable for the description of nature".

If general relativity is true, the events that occur during the journey of the particle occur in the same logical sequence irrespective of the coordinates used to observe the journey. Fig. 2 shows that the logical sequence of events that happens when time is measured using coordinate time t also happens in the same logical order when time is measured using local time τ. The next section shows that even when making observation from specially selected coordinates, the logical sequence of events does not differ from that shown in Fig. 2.

8 Fallacy 3: Use of specially selected coordinates

Fallacy 3 is an attempt to find coordinates that will show the particle can reach and cross the critical radius. The specially selected coordinates achieve this purpose based on a logical fallacy called begging the question in which the thing to be proved is assumed in a premise.

The thing to be proved is that a free falling particle can reach and cross the critical radius. The premise is that the specially selected coordinates can reach and cross the critical radius. When the specially selected coordinates are used as the reference coordinates in the Schwarzschild metric, and it is assumed the specially selected coordinates can cross the critical radius, it is possible to "show" the particle also can cross the critical radius.

But the premise is false. In the Schwarzschild metric, no reference frame can cross its critical radius because to do so would be a violation of the conservation of momentum and energy [4]. Below are considered two classes of specially selected coordinates:

- Coordinates that use the same reference frame as the free falling particle (e.g., the Novikov coordinates);

- Coordinates that use the reference frame of a radially traveling photon, (e.g., ingoing Eddington-Finkelstein coordinates and the Kruskal-Szekeres coordinates).

For each class of specially selected coordinates it is shown that their reference frame cannot cross a critical radius within the time it takes a black hole to evaporate.

Coordinates that use the reference frame of the free falling particle: Coordinates, such as the Novikov coordinates, that share a reference frame with the particle, also share the same time coordinate. Thus the local time coordinate τ measures the passage of time for both the local coordinates and the reference frame of the Novikov coordinates [5, p. 826].

The time required for a black hole to evaporate as measured by the time coordinate τ — which is the time coordinate for the reference frame shared by the Novikov coordinates shared and the local coordinates — has already been shown to be τ_E. See Fig. 2. As discussed above, $\tau_E < \tau_C$, indicating a black hole will evaporate at local time τ_E before the reference frame for the Novikov coordinates and the particle will be able to reach the critical radius at local time τ_C.

Coordinates that use the reference frame of a photon: The reference frame for ingoing Eddington-Finkelstein coordinates and the Kruskal-Szekeres coordinates is a radially traveling photon. [5, See pp. 826–832].

The coordinate time t for the photon to reach its critical radius can be very simply calculated from the Schwarzschild metric in (1). Because the photon is traveling radially, $d\theta = d\phi = 0$. Because local time for a photon does not progress, $d\tau = 0$. Therefore, the form of the Schwarzschild metric used to calculate values for coordinate time t is obtained by setting $d\theta = d\phi = d\tau = 0$ in (1) yielding

$$0 = c^2\left(1 - \frac{R}{r}\right)dt^2 - \frac{dr^2}{(1 - R/r)}. \qquad (6)$$

The integral in (4) can be used to calculate elapsed coordinate time t for the photon based on radial distance. Integrand $f_1(r)$ is obtained by rearranging the terms in (6), i.e.,

$$f_1(r) = \frac{dt}{dr} = \frac{1}{c(1 - R/r)}. \qquad (7)$$

When the photon reaches $r = R$, the integrand in (7) is undefined and the integral in (4) does not converge. Therefore the radially traveling photon will not reach R in finite coordinate time.

A black hole that evaporates in finite coordinate time t_E, will evaporate when the photon reaches a radial location r_L that is outside R. When the photon reaches radial location r_L at coordinate time t_E, the ingoing particle will be at radial location r_E, outside the critical radius r_C, as shown by Fig. 2.

In the reference frame of a photon, the black hole will evaporate when the photon reaches radial location r_L, before the photon reaches its critical radius R. As in all reference frames of the Schwarzschild metric, the reference frame of the photon is not able to reach the critical radius before the black hole evaporates.

9 Fallacy 4: Claiming the existence of surfaces trapped below a surface of last influence

Misner et al. [5, pp. 873–874] makes the argument that once the surface of a collapsing star crosses a critical radius, light reflecting from the surface remains trapped below the critical radius. This is a fallacy because the surface of a collapsing star will never cross the critical radius [3]. The very last particle on the surface to cross the critical radius can be approximately modeled by the radially falling particle of Fig. 2. From the perspective of the distant observer (coordinate time in Fig. 2), the collapsing star evaporates in finite time, before the infinite coordinate time required for the last particle on the surface to cross the critical radius.

From the perspective of a particle on the surface (local time in Fig. 2), the collapsing star evaporates very suddenly as the particle nears the critical radius. It is intriguing to imagine the experience of the particle as the surface of the collapsing star immediately disintegrates into radiation near the critical radius. Such an inferno of unimaginable proportions would tend to be masked from a distant observer by the extreme gravity near the critical radius. But as the surface burns away reducing the mass of the collapsing star — causing the critical radius to retreat farther below the surface of the collapsing star — a less time dilated view of the inferno might be released, perhaps providing an explanation for the sudden appearance of quasars.

Since the surface of a collapsing star cannot cross its critical radius in finite coordinate time t, Misner et al. [5, pp. 873–874] measures time from the reference frame for the ingoing Eddington-Finkelstein coordinates. As discussed in the prior section, use of ingoing Eddington-Finkelstein coordinates to prove the critical radius can be crossed just begs the question. The ingoing Eddington-Finkelstein coordinates will not cross the Schwarzschild metric of the collapsing star before the collapsing star evaporates. This should be especially clear for the example of a collapsing star since the surface, located outside its critical radius, will be an impenetrable barrier that will prevent any photon, serving as a reference frame for the ingoing Eddington-Finkelstein coordinates, from reaching its critical radius at R.

10 Fallacy 5: Claiming the infinite coordinate time to reach the critical radius is an optical illusion

It has been asserted that as measured by proper time, a free-falling traveler quickly reaches the critical radius. To the distant observer it appears to take an infinite amount of coordinate time to reach the critical radius as a result of an optical illusion caused by light propagation introducing a delay in communicating that the critical radius has been reached [5, pp. 874–875]. Fallacy 5 is a departure from general relativity because in general relativity the difference between local time and coordinate time is not merely the result of delay introduced by light propagation. In the theory of general relativity, time progresses at different rates depending on the strength of the gravity field in which measurements are made.

Einstein [8, p. 106] explains: "we must use clocks of unlike constitution, for measuring time at places with differing gravitational potential." This principle of relativity is embodied in the Schwarzschild metric where gravity changes the rate at which time progresses [2]. For a precise description of how in the Schwarzschild metric gravity affects time based on the conservation of momentum and energy, see [4, Eq. 8].

Because fallacy 5 does not properly account for the effect gravity has on time, and is therefore not in accord with general relativity or the Schwarzschild metric, the results predicted by fallacy 5 do not agree with results calculated using the Schwarzschild metric. This is illustrated by a hypothetical in the following section.

11 A hypothetical illustrating the logical contradictions introduced by fallacy 5

According to fallacy 5, as measured by proper time, a radially falling traveler quickly reaches and crosses the critical radius of a black hole. The reality that the traveler quickly reaches the critical radius appears to the distant observer to take an infinite amount of time because of the propagation of light.

Fallacy 5's portrayal of reality is not consistent with calculations made using the Schwarzschild metric.

For example, put a reflector on the back of the traveler and have the distant observer periodically shine a light beam at the traveler. Use the Schwarzschild metric to calculate the radial location at which the faster moving light beam will overtake the slower moving traveler and reflect back to indicate the location of the traveler to the distant observer.

No matter how much of a head start the traveler has before the light is turned on (even trillions of years or longer, as measured using coordinate time), according to the Schwarzschild metric the light will always overtake the traveler before the critical radius is reached. The radial location at which the traveler is overtaken is the same whether local time or coordinate time is used to make the calculations, provided start time and overtake time for each light beam are measured with the same time coordinate. This result is inevitable based on the pattern of the data obtained from the Schwarzschild metric, as shown in Fig. 1.

As shown by Fig. 2, the distant observer can continue to shine light beams at the traveler until the distant observer observes the black hole evaporates. The feedback from the reflected light beams will tell the distant observer that the traveler remains outside the black hole as the black hole evaporates slowly in coordinate time, and quickly in local time. This contradicts the assertion of fallacy 5 that the traveler easily reaches and crosses the critical radius.

The distant observer does not even need to shine a light beam for this experiment as background radiation reflecting from the traveler provides exactly the same information.

Hawking radiation also provides the same information. While the distant observer sees the traveler outside the critical radius, the distant observer will also observe Hawking radiation from the evaporating black hole, which will first have to pass through the radial location of the traveler before reaching the distant observer. This indicates to the distant observer that the traveler will have experienced, before the distant observer, radiation emitted during the disintegration of the black hole. Further, the radiation passing by the traveler will continuously bring information to the distant observer about the location of the traveler confirming the information from the light beams. Each photon of radiation from the evaporating black hole that passes by the traveler is a progress report on the traveler's location that will confirm to the distant observer that the traveler had not yet passed through the critical radius when that photon of radiation passed the traveler. Such progress reports will continue until the black hole completely evaporates.

Light beams from the distant observer, background radiation and Hawking radiation will all intercept the traveler outside the critical radius — according to the Schwarzschild metric — regardless of the coordinates used to make measurements. This result contradicts the assertion of fallacy 5 that the critical radius is quickly crossed and only appears to the distant observer to take infinite time because of light propagation.

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