

# The Point Mass Concept

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A point-mass concept has been elaborated from the equations of the gravitational field. One application of these deductions results in a black hole configuration of the Schwarzschild type, having no electric charge and no angular momentum. The critical mass of a gravitational collapse with respect to the nuclear binding energy is found to be in the range of 0.4 to 90 solar masses. A second application is connected with the speculation about an extended symmetric law of gravitation, based on the options of positive and negative mass for a particle at given positive energy. This would make masses of equal polarity attract each other, while masses of opposite polarity repel each other. Matter and antimatter are further proposed to be associated with the states of positive and negative mass. Under fully symmetric conditions this could provide a mechanism for the separation of antimatter from matter at an early stage of the universe.

## 1 Introduction

In connection with an earlier elaborated revised quantum-electrodynamic theory, a revised renormalisation procedure has been developed to solve the problem of infinite self-energy of the point-charge-like electron [1, 2]. In the present investigation an analogous procedure is applied to the basic equations of gravitation, to formulate a corresponding point mass concept. Two applications result from such a treatment. The first concerns the special Schwarzschild case of a black hole with its critical limit of gravitational collapse. The second application is represented by the speculation about an extended form of the gravitation law, in which full symmetry is obtained by including both positive and negative mass concepts. This further leads to the question whether such concepts could have their correspondence in matter and antimatter, and in their mutual separation.

## 2 The conventional law of gravitation

In this investigation the analysis is limited to the steady case of spherical symmetry, in a corresponding frame where  $r$  is the only independent variable.

### 2.1 Basic equations

Following Bergmann [3], a steady gravitational field strength

$$\mathbf{g} = -\nabla\phi \quad (1)$$

is considered which originates from the potential  $\phi(r)$ . The source of the field strength is a mass density  $\rho$  related to  $\mathbf{g}$  by

$$-\text{div } \mathbf{g} = 4\pi G\rho = \nabla^2\phi = \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\phi}{dr} \right), \quad (2)$$

where  $G = 6.6726 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$  is the constant of gravitation in SI units. The associated force density becomes

$$\mathbf{f} = \rho \mathbf{g}. \quad (3)$$

In the conventional interpretation there only exists a positive mass density  $\rho \geq 0$ . This makes in a way the gravitational field asymmetric, as compared to the electrostatic field which includes both polarities of electric charge density.

A complete form of the potential  $\phi$  would consist of a series of both positive and negative powers of  $r$ , but the present analysis will be restricted and simplified by studying each power separately, in the form

$$\phi(r) = \phi_0 \left( \frac{r}{r_0} \right)^\alpha. \quad (4)$$

Here  $\phi_0$  is a constant,  $r_0$  represents a characteristic dimension and  $\alpha$  is a positive or negative integer. Equation (2) yields

$$4\pi G\rho = \frac{\phi_0}{r_0^\alpha} \alpha(\alpha+1) r^{\alpha-2} > 0. \quad (5)$$

When limiting the investigations by the condition  $\rho > 0$ , the cases  $\alpha = 0$  and  $\alpha = -1$  have to be excluded, leaving the regimes of positive  $\alpha = (1, 2, \dots)$  and negative  $\alpha = (-2, -3, \dots)$  to be considered for *positive* values of  $\phi_0$ .

### 2.2 Point mass formation

For reasons to become clear from the deductions which follow, we now study a spherical configuration in which the mass density  $\rho$  is zero within an inner hollow region  $0 \leq r \leq r_i$ , and where  $\rho > 0$  in the outer region  $r > r_i$ . From relation (5) the total integrated mass  $P(r)$  inside the radius  $r$  then becomes

$$P(r) = \int_0^r \rho 4\pi r^2 dr = \frac{1}{G} \frac{\phi_0}{r_0^\alpha} \alpha (r^{\alpha+1} - r_i^{\alpha+1}) > 0 \quad (6)$$

with a resulting local field strength  $\mathbf{g} = (g, 0, 0)$  given by

$$g(r) = -G \frac{P(r)}{r^2} = -\frac{\phi_0}{r_0^\alpha} \frac{\alpha}{r^2} (r^{\alpha+1} - r_i^{\alpha+1}) < 0 \quad (7)$$

and a local force density  $\mathbf{f} = (f, 0, 0)$  where

$$f(r) = -\frac{1}{4\pi G} \frac{\phi_0^2}{r_0^\alpha} \alpha^2 (\alpha + 1) r^{\alpha-4} (r^{\alpha+1} - r_i^{\alpha+1}) < 0. \quad (8)$$

Here all  $(P, g, f)$  refer to the range  $r \geq r_i$ , and  $\phi_0 > 0$ .

A distinction is further made between the two regimes of positive and negative  $\alpha$ :

- When  $\alpha = (1, 2, \dots)$  of a convergent potential (4), this hollow configuration has an integrated mass (6) which increases monotonically with  $r$ , from zero at  $r = r_i$  to large values. This behaviour is the same for a vanishing  $r_i$  and does not lead to a point-like mass at small  $r_i$ .
- When  $\alpha = (-2, -3, \dots)$  of a divergent potential (4), the hollow configuration leads to a point-mass-like geometry at small  $r_i$ . This is similar to a point-charge-like geometry earlier treated in a model of the electron [1, 2], and will be considered in the following analysis.

### 2.3 The renormalised point mass

In the range  $\alpha \leq -2$  expressions (6)–(8) are preferably cast into a form with  $\gamma = -\alpha \geq 2$  where

$$P(r) = \frac{1}{G} (\phi_0 r_0^\gamma) \gamma (r_i^{-\gamma+1} - r^{-\gamma+1}) > 0, \quad (9)$$

$$g(r) = -(\phi_0 r_0^\gamma) \frac{\gamma}{r^2} (r_i^{-\gamma+1} - r^{-\gamma+1}) < 0, \quad (10)$$

$$f(r) = -\frac{1}{4\pi G} (\phi_0 r_0^\gamma)^2 \gamma^2 (\gamma-1) r^{-\gamma-4} (r_i^{-\gamma+1} - r^{-\gamma+1}) < 0. \quad (11)$$

Here an erroneous result would be obtained if the terms including  $r_i$  are dropped and the hollow configuration is abandoned. Due to eqs. (9)–(11) this would namely result in a negative mass  $P$ , a positive field strength  $g$ , and a repulsive local gravitational force density  $f$ .

The radius  $r_i$  of the hollow inner region is now made to approach zero. The total integrated mass of eq. (9) is then concentrated to an infinitesimally small layer. Applying a revised renormalisation procedure in analogy with an earlier scheme [1, 2], we “shrink” the combined parameters  $\phi_0 r_0^\gamma$  and  $r_i^{\gamma-1}$  in such a way that

$$\phi_0 r_0^\gamma = c_{\phi r} \cdot \varepsilon \quad r_i^{\gamma-1} = c_i \cdot \varepsilon \quad 0 < \varepsilon \ll 1, \quad (12)$$

where  $\varepsilon$  is a smallness parameter and  $c_{\phi r}$  and  $c_i$  are positive constants. A further introduction of

$$P_0 \equiv \frac{1}{G} \frac{\gamma c_{\phi r}}{c_i} \quad (13)$$

results in  $P(r) = 0$  for  $r \leq r_i$  and

$$P(r) = P_0 \left[ 1 - \left( \frac{r_i}{r} \right)^{\gamma-1} \right] \quad r > r_i. \quad (14)$$

In the limit  $\varepsilon \rightarrow 0$  and  $r_i \rightarrow 0$  there is then a point mass  $P_0$  at the origin. This mass generates a field strength

$$g(r) = -G \frac{P_0}{r^2} \quad (15)$$

at the distance  $r$  according to equations (10), (12) and (13). With another point mass  $P_1$  at the distance  $r$ , there is a mutual attraction force

$$F_{01} = P_1 g(r) = -G \frac{P_0 P_1}{r^2}, \quad (16)$$

which is identical with the gravitation law for two point masses.

To further elucidate the result of eqs. (12)–(16) it is first observed that, in the conventional renormalisation procedure, the divergent behaviour of an infinite self-energy is outbalanced by adding extra infinite ad-hoc *counter-terms* to the Lagrangian, to obtain a finite difference between two “infinities”. Even if such a procedure has been successful, however, it does not appear to be quite acceptable from the logical and physical points of view. The present revised procedure represented by expressions (12) implies on the other hand that the “infinity” of the divergent potential  $\phi_0$  at a shrinking radius  $r_i$  is instead outbalanced by the “zeros” of the inherent shrinking *counter-factors*  $c_{\phi r} \cdot \varepsilon$  and  $c_i \cdot \varepsilon$ .

### 3 A black hole of Schwarzschild type

A star which collapses into a black hole under the compressive action of its own gravitational field is a subject of ever increasing interest. In its most generalized form the physics of the black hole includes both gravitational and electromagnetic fields as well as problems of General Relativity, to account for its mass, net electric charge, and its intrinsic angular momentum. The associated theoretical analysis and related astronomical observations have been extensively described in a review by Misner, Thorne and Wheeler [4] among others. Here the analysis of the previous section will be applied to the far more simplified special case by Schwarzschild, in which there is no electric charge and no angular momentum. Thereby it has also to be noticed that no black hole in the universe has a substantial electric charge [4].

#### 3.1 The inward directed gravitational pressure

From eq. (11) is seen that the inward directed local force density is zero for  $r \leq r_i$ , increases with  $r$  to a maximum within a thin shell, and finally drops to zero at large  $r$ . The integrated inward directed gravitational pressure on this shell thus becomes

$$p = \int_0^\infty f dr = -\frac{G}{8\pi} \frac{(\gamma-1)^2}{(\gamma+1)(\gamma+3)} \frac{P_0^2}{r_i^4} \quad (17)$$

in the limit of small  $\varepsilon$  and  $r_i$ .

When this pressure becomes comparable to the relevant energy density of the compressed matter, a corresponding gravitational collapse is expected to occur.

### 3.2 Gravitational collapse of the nuclear binding forces

Here we consider the limit at which matter is compressed into a body of densely packed nucleons, and when the pressure of eq. (17) tends to exceed the energy density of the nucleon binding energy. The radius of a nucleus is [5]

$$r_N = 1.5 \times 10^{-15} A^{1/2} \quad [\text{m}], \quad (18)$$

where  $A$  is the mass number. A densely packed sphere of  $N$  nuclei has the volume

$$V_N = \frac{4}{3} \pi N r_N^3 = \frac{4}{3} \pi r_{eq}^3, \quad (19)$$

where  $r_{eq}$  is the equivalent radius of the sphere. The total binding energy of a nucleus is further conceived as the work required to completely dissociate it into its component nucleons. This energy is about 8 MeV per nucleon [5, 6]. With  $A$  nucleons per nucleus, the total binding energy of a body of  $N$  nuclei thus becomes

$$W_N = NA w_N, \quad (20)$$

where  $w_N = 8 \text{ MeV} = 1.28 \times 10^{-12} \text{ J}$ . The equivalent binding energy density of the body is then

$$p_N = \frac{W_N}{V_N} = \frac{3}{4} \frac{A w_N}{\pi r_N^3} = 0.907 \times 10^{32} A^{-1/2} \quad [\text{J} \times \text{m}^{-3}]. \quad (21)$$

The shell-like region of gravitational pressure has a force density (11) which reaches its maximum at the radius

$$r_m = r_i \left( \frac{2\gamma + 3}{\gamma + 4} \right)^{1/(\gamma-1)} \quad (22)$$

being only a little larger than  $r_i$ . This implies that the radius  $r_{eq}$  of eq. (19) is roughly equal to  $r_i$  and

$$r_i \cong r_N N^{1/3}. \quad (23)$$

With  $N$  nuclei of the mass  $Am_p$  and  $m_p$  as the proton mass, the total mass becomes

$$P_0 = NAm_p, \quad (24)$$

which yields

$$r_i \cong r_N \left( \frac{P_0}{Am_p} \right)^{1/3} = 1.26 \times 10^{-6} A^{1/6} P_0^{1/3} \quad [\text{m}]. \quad (25)$$

This result finally combines with eq. (17) to an equivalent gravitational pressure

$$p = -1.1 \times 10^{12} \frac{(\gamma-1)^2}{(\gamma+1)(\gamma+3)} A^{-2/3} P_0^{2/3} \quad [\text{J} \times \text{m}^{-3}]. \quad (26)$$

For a gravitational collapse defined by  $-p > p_N$  the point mass  $P_0$  then has to exceed the critical limit

$$P_{0c} \cong 7.5 \times 10^{29} \left[ \frac{(\gamma+1)(\gamma+3)}{(\gamma-1)^2} \right]^{3/2} A^{1/4} \quad [\text{kg}]. \quad (27)$$

For  $\gamma \geq 2$  and  $1 \leq A \leq 250$ , the critical mass would then be found in the range of about  $0.4 \leq P_{0c} \leq 90$  solar masses of about  $1.98 \times 10^{30} \text{ kg}$ .

### 4 Speculations about a generalized law of gravitation

The Coulomb law of interaction between electrically charged bodies is symmetric in the sense that it includes both polarities of charge and attractive as well as repulsive forces. The classical Newtonian law of gravitation includes on the other hand only one polarity of mass and only attractive forces. In fact, this asymmetry does not come out as a necessity from the basic equations (1)–(3) of a curl-free gravitational field strength. The question could therefore be raised whether a more general and symmetric law of gravitation could be deduced from the same equations, and whether this could have a relevant physical interpretation.

#### 4.1 Mass polarity

In relativistic mechanics the momentum  $\mathbf{p}$  of a particle with the velocity  $\mathbf{u}$  and rest mass  $m_0$  becomes [7]

$$\mathbf{p} = m_0 \mathbf{u} \left[ 1 - \left( \frac{u}{c} \right)^2 \right]^{-1/2}. \quad (28)$$

With the energy  $E$  of the particle, the Lorentz invariance further leads to the relation

$$p^2 - \frac{E^2}{c^2} = -m_0^2 c^2, \quad (29)$$

where  $p^2 = \mathbf{p}^2$  and  $u^2 = \mathbf{u}^2$ . Equations (28) and (29) yield

$$E^2 = m_0^2 c^4 \left[ 1 - \left( \frac{u}{c} \right)^2 \right]^{-1} \equiv m^2 c^4, \quad (30)$$

leading in principle to two roots

$$E = \pm mc^2. \quad (31)$$

In this investigation the discussion is limited to a positive energy  $E$ , resulting in positive and negative gravitational masses

$$m = \pm \frac{E}{c^2} \quad E > 0. \quad (32)$$

This interpretation differs from that of the negative energy states of positrons proposed in the ‘‘hole’’ theory by Dirac [8] corresponding to the plus sign in eq. (31) and where both  $E$  and  $m$  are negative.

#### 4.2 An extended law of gravitation

With the possibility of negative gravitational masses in mind, we now return to the potential  $\phi$  of equations (1) and (4) where the amplitude factor  $\phi_0$  can now adopt both positive and negative values, as defined by the notation  $\phi_{0+} > 0$  and  $\phi_{0-} < 0$ , and where corresponding subscripts are introduced for  $(P, g, f, P_0)$  of eqs. (9)–(11) and (13). Then  $P_+ > 0, P_- < 0, P_{0+} > 0, P_{0-} < 0, g_+ < 0, g_- > 0$ , but  $f_+ < 0$  and  $f_- < 0$  always represent an attraction force due to the quadratic dependence on  $\phi_0$  in eq. (11). With  $P_{1+}$  or  $P_{1-}$  as an additional point mass

at the distance  $r$  from  $P_{0+}$  or  $P_{0-}$ , there is then an extended form of the law (16), as represented by the forces

$$P_{1+}g_+ = -G\frac{P_{0+}P_{1+}}{r^2} = P_{1-}g_- = -G\frac{P_{0-}P_{1-}}{r^2}, \quad (33)$$

$$P_{1+}g_- = G\frac{P_{0-}P_{1+}}{r^2} = P_{1-}g_+ = G\frac{P_{0+}P_{1-}}{r^2}. \quad (34)$$

These relations are symmetric in the gravitational force interactions, where masses of equal polarity attract each other, and masses of opposite polarity repel each other. It would imply that the interactions in a universe consisting entirely of negative masses would become the same as those in a universe consisting entirely of positive masses. In this way specific mass polarity could, in fact, become a matter of definition.

### 4.3 A possible rôle of antimatter

At this point the further question may be raised whether the states of positive and negative mass could be associated with those of matter and antimatter, respectively. A number of points become related to such a proposal.

The first point concerns an experimental test of the repulsive behaviour due to eq. (34). If an electrically neutral beam of anti-matter, such as of antihydrogen atoms, could be formed in a horizontal direction, such a beam would be deflected upwards if consisting of negative mass. However, the deflection is expected to be small and difficult to measure.

A model has earlier been elaborated for a particle with elementary charge, being symmetric in its applications to the electron and the positron [1, 2]. The model includes an electric charge  $q_0$ , a rest mass  $m_0$ , and an angular momentum  $s_0$  of the particle. The corresponding relations between included parameters are easily seen to be consistent with electron-positron pair formation in which  $q_0 = -e$ ,  $m_0 = +E/c^2$  and  $s_0 = +h/4\pi$  for the electron and  $q_0 = +e$ ,  $m_0 = -E/c^2$  and  $s_0 = +h/4\pi$  for the positron when the formation is due to a photon of spin  $+h/2\pi$ . The energy of the photon is then at least equal to  $2E$  where the electron and the photon both have positive energies  $E$ .

The energy of photons and their electromagnetic radiation field also have to be regarded as an equivalent mass due to Einstein's mass-energy relation. This raises the additional question whether full symmetry also requires the photon to have a positive or negative gravitational mass, as given by

$$m_\nu = \pm \frac{h\nu}{c^2}. \quad (35)$$

If equal proportions of matter and antimatter would have been formed at an early stage of the universe, the repulsive gravitational force between their positive and negative masses could provide a mechanism which expels antimatter from matter and vice versa, also under fully symmetric conditions. Such a mechanism can become important even if the gravitational forces are much weaker than the electrostatic ones, be-

cause matter and antimatter are expected to appear as electrically quasi-neutral cosmical plasmas. The final result would come out to be separate universes of matter and antimatter.

In a theory on the metagalaxy, Alfvén and Klein [9] have earlier suggested that there should exist limited regions in our universe which contain matter or antimatter, and being separated by thin boundary layers within which annihilation reactions take place. A simplified model of such layers has been established in which the matter-antimatter "ambiplasma" is immersed in a unidirectional magnetic field [10]. The separation of the cells of matter from those of antimatter by a confining magnetic field geometry in three spatial directions is, however, a problem of at least the same complication as that of a magnetically confined fusion reactor.

## 5 Conclusions

From the conventional equations of the gravitational field, the point-mass concept has in this investigation been elaborated in terms of a revised renormalisation procedure. In a first application a black hole configuration of the Schwarzschild type has been studied, in which there is no electric charge and no angular momentum. A gravitational collapse in respect to the nuclear binding energy is then found to occur at a critical point mass in the range of about 0.4 to 90 solar masses. This result becomes modified if the collapse is related to other restrictions such as to the formation of "primordial black holes" growing by the accretion of radiation and matter [4], or to phenomena such as a strong centrifugal force.

A second application is represented by the speculation about an extended law of gravitation, based on the options of positive and negative mass of a particle at a given positive energy, and on the basic equations for a curl-free gravitational fieldstrength. This would lead to a fully symmetric law due to which masses of equal polarity attract each other, and masses of opposite polarity repel each other. A further proposal is made to associate matter and antimatter with the states of positive and negative mass. Even under fully symmetric conditions, this provides a mechanism for separating antimatter from matter at an early stage of development of the universe.

After the completion of this work, the author has been informed of a hypothesis with negative mass by Choi [11], having some points in common with the present paper.

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