Ultracold Fermi and Bose Gases and Spinless Bose Charged Sound Particles

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We propose a novel approach for investigation of the motion of Bose or Fermi liquid (or gas) which consists of decoupled electrons and ions in the uppermost hyperfine state. Hence, we use such a concept as the fluctuation motion of "charged fluid particles" or "charged fluid points" representing a charged longitudinal elastic wave. In turn, this elastic wave is quantized by spinless longitudinal Bose *charged sound particles* with the rest mass m and charge e_0. The existence of spinless Bose *charged sound particles* allows us to present a new model for description of Bose or Fermi liquid via a non-ideal Bose gas of *charged sound particles*. In this respect, we introduce a new postulation for the superfluid component of Bose or Fermi liquid determined by means of *charged sound particles* in the condensate, which may explain the results of experiments connected with ultra-cold Fermi gases of spin-polarized hydrogen, ^{6}Li and ^{40}K, and such a Bose gas as ^{87}Rb in the uppermost hyperfine state, where the Bose-Einstein condensation of *charged sound particles* is realized by tuning the magnetic field.

1 Introduction

The Bose-Einstein condensation (BEC) has a wide application for investigation of superconductivity of metals and superfluidity of liquids. The primary experimental challenge to evaporative cooling of spin-polarized hydrogen was made by a dilution refrigerator, demonstrating that spin-polarized hydrogen can be confined in a statistic magnetic trap and thermally decoupled from the walls [1–3]. At the density $\frac{N}{V} \approx 10^{13}$ cm^{-3} it is observed that the gas consisting of decoupled electrons and ions in the uppermost hyperfine state is evaporatively cooled to a temperature approximately equal to 40 mK.

Here, we remark about BEC that was produced in a vapor of ^{87}Rb bosonic ions confined by magnetic fields and evaporatively cooled [4]. The condensate fraction first appeared near a temperature of 170 nanokelvin at the density $\frac{N}{V} = 2.6 \times 10^{12}$ cm^{-3}. The experiment has shown that the value of temperature 170 nK is reduced to 20 nK. In reality, the strongly interacting spin-$\frac{1}{2}$ ^{6}Li and ^{40}K fermionic gases were realized via tuning the magnetic field [5]. These experimental achievements in the field of ultra-cold Fermi gases are based mainly on the possibility of tuning the scattering length a which becomes much larger in magnitude than the mean interatomic distance by changing the external magnetic field. In this respect, the concept of Fermi surface loses its meaning due to the broadening produced by pairing of fermions, the so-called Feshbach resonances in ultracold atomic Fermi gases. However, in this letter we predict a new method of liquid cooling which is based on the formation of oscillators at every point of liquid by tuning the magnetic field, which in turn leads to vibration of "charged fluid particles". These "charged fluid particles" reproduce charged spinless quasiparticles which determine the superfluidity component of Bose or Fermi liquid by action of the static magnetic field.

In order to investigate the motion of quantum liquid (or quantum gas) in the uppermost hyperfine state, we consider the motion of "charged fluid particles" by means of a charged longitudinal elastic wave [6]. This longitudinal elastic wave is quantized by spinless Bose *charged sound particles* with the mass m and charge e_0. Further, we present a new model for description of charged Bose or Fermi liquid via a non-ideal Bose gas consisting of *charged sound particles*. As opposed to London's postulation about the superfluid component of liquid ^{4}He [7], we introduce a new postulation about the superfluid component of Bose or Fermi liquid via *charged sound particles* in the condensate. On the other hand, we estimate the zero sound speed which leads to the correct explanation of the experimental result connected with the BEC of a gas consisting of spin-polarized hydrogen.

2 Quantization of quantum liquid or quantum gas in the uppermost hyperfine state

Now let us analyze quantization of quantum liquid (or quantum gas) in the uppermost hyperfine state. This quantum liquid (or quantum gas) consists of N Bose or Fermi positive charged ions with the charge e and mass M confined in the volume V where they are in a negative electron background since the entire system of liquid is electro-neutral. Considering quantum liquid as a continuous medium, we investigate the fluctuation motion of the number n of "charged fluid particles" on the basis of hydrodynamics (where a "charged fluid particle" is defined as a very small volume V_0 in regard to the volume V of the liquid ($V_0 \ll V$) with the mass m and charge e_0. The volume V_0 contains the number $N' = \frac{N}{n}$ of liquid ions, therefore the charge e_0 is expressed via the ion charge as $e_0 = \frac{eN}{n}$.

In accordance with the laws of hydrodynamics [6], the mass density ρ and pressure p of liquid are presented as

$$\rho = \rho_0 + \rho^{'}$$

and

$$p = p_0 + p^{'},$$

where $\rho_0 = \frac{MN}{V}$ and p_0 are, respectively, the equilibrium mass density and pressure; $\rho^{'}$ and $p^{'}$ are the relative fluctuations of the mass density and pressure.

As is known, the continuity equation has the form:

$$\frac{\partial \rho^{'}}{\partial t} = -\rho_0 \operatorname{div} \vec{v}, \tag{1}$$

which may present as:

$$\rho^{'} = -\rho_0 \operatorname{div} \vec{u}, \tag{2}$$

where $\vec{v} = \frac{\partial \vec{u}}{\partial t}$ is the speed of a charged fluid particle; $\vec{u} = \vec{u}(\vec{r}, t)$ is the displacement vector of a charged fluid particle which describes a charged longitudinal sound wave.

On the other hand, Euler's equation in the first-order-of-smallness approximation takes the reduced form:

$$\frac{\partial \vec{v}}{\partial t} + \frac{\nabla p^{'}}{\rho_0} = 0. \tag{3}$$

Hence, we consider the fluctuation motion of charged fluid particles as adiabatic, deriving the following equation:

$$p^{'} = \left(\frac{\partial p}{\partial \rho_0}\right)_S \rho^{'} = c_l^2 \rho^{'}, \tag{4}$$

where S is the entropy; $c_l = \sqrt{\left(\frac{\partial p}{\partial \rho_0}\right)_S}$ is the speed of the charged longitudinal elastic wave.

As is known, the fluctuation motion of charged fluid particles represents as a potential one:

$$\operatorname{curl} \vec{v} = \operatorname{curl} \frac{\partial \vec{u}}{\partial t} = 0. \tag{5}$$

Thus, by using the above equation we may get to the wave equation for the vector of displacement $\vec{u} = \vec{u}(\vec{r}, t)$:

$$\nabla^2 \vec{u}(\vec{r}, t) - \frac{1}{c_l^2} \frac{\partial^2 \vec{u}(\vec{r}, t)}{\partial t^2} = 0, \tag{6}$$

which in turn describes the longitudinal charged sound wave.

Now, we state that the longitudinal elastic wave consists of spinless Bose *charged sound particles* with the non-zero rest mass m. Then, the displacement vector $u(\vec{r}, t)$ is expressed via a secondary quantization vector of the wave function of spinless Bose *charged sound particles* directed along the wave vector $\vec{k}$:

$$\vec{u}(\vec{r}, t) = u_l \left(\vec{\phi}(\vec{r}, t) + \vec{\phi}^+(\vec{r}, t)\right), \tag{7}$$

where u_l is the normalization constant which is the amplitude of oscillations; $\vec{\phi}(\vec{r}, t)$ is the secondary quantization of vector wave functions for creation and annihilation of one longitudinal *charged sound particle* with the mass m whose direction $\vec{l}$ is directed towards the wave vector $\vec{k}$:

$$\vec{\phi}(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \vec{a}_{\vec{k}}\, e^{i(\vec{k}\vec{r} - kc_l t)} \tag{8}$$

$$\vec{\phi}^+(\vec{r}, t) = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \vec{a}^+_{\vec{k}}\, e^{-i(\vec{k}\vec{r} - kc_l t)} \tag{9}$$

with the condition

$$\int \vec{\phi}^+(\vec{r}, t)\, \vec{\phi}(\vec{r}, t)\, dV = n_0 + \sum_{\vec{k} \neq 0} \hat{a}^+_{\vec{k}}\, \hat{a}_{\vec{k}} = \hat{n}, \tag{10}$$

where $\vec{a}^+_{\vec{k}}$ and $\vec{a}_{\vec{k}}$ are, respectively, the Bose vector-operators of creation and annihilation for a free *charged sound particle* with the energy $\frac{\hbar^2 k^2}{2m}$, described by the vector $\vec{k}$ whose direction coincides with the direction $\vec{l}$ of a traveling charged longitudinal elastic wave; $\hat{n}$ is the operator of the total number of *charged sound particles*; $\hat{n}_0$ is the total number of *charged sound particles* at the condensate level with the wave vector $\vec{k} = 0$.

Thus, as is seen, the displacement vector $\vec{u}(\vec{r}, t)$ satisfies wave-equation (6) and in turn takes the form:

$$\vec{u}(\vec{r}, t) = \vec{u_0} + \frac{u_l}{\sqrt{V}} \sum_{\vec{k} \neq 0} \left(\vec{a}_{\vec{k}}\, e^{i(\vec{k}\vec{r} - kc_l t)} + \vec{a}^+_{\vec{k}}\, e^{-i(\vec{k}\vec{r} - kc_l t)}\right). \tag{11}$$

While investigating a superfluid liquid, Bogoliubov [8] separated the atoms of helium in the condensate from those atoms filling the states above the condensate. In an analogous manner, we may consider the vector operator $\vec{a}_0 = \vec{l}\sqrt{n_0}$ and $\vec{a}^+_0 = \vec{l}\sqrt{n_0}$ as c-numbers (where $\vec{l}$ is the unit vector in the direction of propagation of the sound wave) within the approximation of a macroscopic number of *sound particles* in the condensate $n_0 \gg 1$. These assumptions lead to a broken Bose-symmetry law for *sound particles* in the condensate. To extend the concept of a broken Bose-symmetry law for *sound particles* in the condensate, we apply the definition of BEC of *sound particles* in the condensate as was postulated by the Penrose-Onsager for the definition of BEC of helium atoms [9]:

$$\lim_{n_0, n \to \infty} \frac{n_0}{n} = const. \tag{12}$$

On the other hand, we may observe that presence of *charged sound particles* filling the condensate level with the wave vector $\vec{k} = 0$ leads to the appearance of the constant displacement $\vec{u}_0 = \frac{2u_l \vec{l}\sqrt{n_0}}{\sqrt{V}}$ of *charged sound particles*.

To find the normalization constant u_l, we introduce the following condition which allows us to suggest that at absolute zero all *sound particles* fill the condensate level $\vec{k} = 0$.

This reasoning implies that at $n_0 = n$ the constant displacement takes the maximal value $2d = \sqrt{|\vec{u}_0|^2}$ which represents the maximal distance between two neighboring *charged sound particles*. On the other hand, this distance is determined by the formula $d = \left(\frac{3V}{4\pi n}\right)^{\frac{1}{3}}$, which is in turn substituted into the expression $2d = \sqrt{|\vec{u}_0|^2}$. Then, consequently, we get to the normalization constant $u_l = 0.65\left(\frac{n}{V}\right)^{-\frac{5}{6}}$.

The condition for conservation of density at each point of liquid stipulates that

$$\rho_0 = \frac{MN}{V} = \frac{mn}{V}, \tag{13}$$

which represents a connection of the mass and density of the *charged sound particles* with the mass and density of the ions. Thus, we argue that liquid (or gas) can be described by the model of an ideal gas of n *charged sound particles* with the mass m and charge e_0 in the volume V. Hence, we remark that the Coulomb scattering between *charged sound particles* is neglected in the considered theory.

3 "Charged fluid particles" in trapped static magnetic field

Now, we consider the Hamiltonian operator $\hat{H}_l$ of liquid [6] in a trapped static magnetic field [10]:

$$\hat{H}_l = \frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV + \frac{1}{2}\int\left(\frac{c_l\rho'}{\sqrt{\rho_0}}\right)^2 dV + \frac{\rho_0}{2}\int(\Omega\vec{u}_l)^2\, dV, \tag{14}$$

where $\Omega = \frac{e_0 H}{mc}$ is the trapping frequency of a "charged fluid particle"; e_0 is the charge of a "fluid particle"; H is the absolute value of the magnetic strain; c is the velocity of light in vacuum. Hence, we note that the charge of a fluid particle equals $e_0 = eN' = \frac{Ne}{n}$, where N' is the number of ions in a small volume V_0 of one charged fluid particle.

Substituting ρ' from (2) into (14), we obtain

$$\hat{H}_l = \frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV + \frac{\rho_0}{2}\int(c_l\,\mathrm{div}\,\vec{u})^2\, dV + \frac{\rho_0}{2}\int(\Omega\vec{u}_l)^2\, dV. \tag{15}$$

Using Dirac's approach in [11] for quantization of the electromagnetic field, we have:

$$\frac{\partial\vec{u}(\vec{r},t)}{\partial t} = -\frac{ic_l\vec{u}_l}{\sqrt{V}}\sum_{\vec{k}} k\left(\vec{a}_{\vec{k}}\, e^{-ikc_lt} - \vec{a}^+_{-\vec{k}}\, e^{ikc_lt}\right)e^{i\vec{k}\vec{r}}, \tag{16}$$

as well as

$$\mathrm{div}\,\vec{u}(\vec{r},t) = \frac{i\vec{u}_l}{\sqrt{V}}\sum_{\vec{k}}\vec{k}\left(\vec{a}_{\vec{k}}\, e^{-ikc_lt} + \vec{a}^+_{-\vec{k}}\, e^{ikc_lt}\right)e^{i\vec{k}\vec{r}}. \tag{17}$$

Now, introducing (16) and (17) into (15) and using

$$\frac{1}{V}\int e^{i(\vec{k}_1+\vec{k}_2)\vec{r}} = \delta^3_{\vec{k}_1+\vec{k}_2},$$

we obtain the terms in the right side of the Hamiltonian of the system presented in (15):

$$\frac{\rho_0}{2}\int\left(\frac{\partial\vec{u}}{\partial t}\right)^2 dV = -\frac{\rho_0 c_l^2 u_l^2}{2}\sum_{\vec{k}} k^2\left(\vec{a}_{\vec{k}} - \vec{a}^+_{-\vec{k}}\right)\left(\vec{a}_{-\vec{k}} - \vec{a}^+_{\vec{k}}\right),$$

$$\frac{\rho_0}{2}\int(\mathrm{div}\,\vec{u})^2\, dV = \frac{\rho_0 c_l^2 u_l^2}{2}\sum_{\vec{k}} k^2\left(\vec{a}_{\vec{k}} + \vec{a}^+_{-\vec{k}}\right)\left(\vec{a}_{-\vec{k}} + \vec{a}^+_{\vec{k}}\right)$$

and

$$\frac{\rho_0}{2}\int(\Omega\vec{u}_l)^2\, dV = \frac{\rho_0\Omega^2 u_l^2}{2}\sum_{\vec{k}}\left(\vec{a}_{\vec{k}} + \vec{a}^+_{-\vec{k}}\right)\left(\vec{a}_{-\vec{k}} + \vec{a}^+_{\vec{k}}\right).$$

These expressions determine the reduced form of the Hamiltonian operator $\hat{H}_l$ by the form:

$$\hat{H}_l = \sum_{\vec{k}}\left(2\rho_0 u_l^2 c_l^2 k^2 + \rho_0\Omega^2 u_l^2\right)\vec{a}^+_{\vec{k}} a_{\vec{k}} + \frac{\rho_0\Omega^2 u_l^2}{2}\sum_{\vec{k}}\left(\vec{a}^+_{-\vec{k}}\vec{a}^+_{\vec{k}} + \vec{a}_{\vec{k}}\vec{a}_{-\vec{k}}\right), \tag{18}$$

where u_l^2 is defined by the first term in the right side of (18) which represents the kinetic energy of a *charged sound particle* $\frac{\hbar^2 k^2}{2m}$, if we suggest:

$$2\rho_0 u_l^2 c_l^2 k^2 = \frac{\hbar^2 k^2}{2m}. \tag{19}$$

Then,

$$u_l^2 = \frac{\hbar^2}{4c_l^2 m\rho_0},$$

which allows one to determine the mass m of a *charged sound particle* using the value of the normalization constant $u_l = 0.65\left(\frac{n}{V}\right)^{-\frac{5}{6}}$ and (13):

$$m = \frac{\hbar}{c_l}\left(\frac{n}{V}\right)^{\frac{1}{3}}. \tag{20}$$

Thus, the main part of the Hamiltonian operator $\hat{H}_l$ takes the form:

$$\hat{H}_l = \sum_{\vec{k}\neq 0}\left(\frac{\hbar^2 k^2}{2m} + mv^2\right)\vec{a}^+_{\vec{k}}\, a_{\vec{k}} + \frac{mv^2}{2}\sum_{\vec{k}\neq 0}\left(\vec{a}^+_{-\vec{k}}\,\vec{a}^+_{\vec{k}} + \vec{a}_{\vec{k}}\,\vec{a}_{-\vec{k}}\right), \tag{21}$$

where we denote $v = \frac{\hbar\Omega}{\sqrt{2}mc_l}$, which in turn is the speed of charged sound in a Bose or Fermi liquid excited by static magnetic field; n_0 is the number of *charged sound particles* in the condensate.

For the evolution of the energy level, it is necessary to diagonalize the Hamiltonian $\hat{H}_l$, which can be accomplished by introducing the vector Bose-operators $\vec{b}^+_{\vec{k}}$ and $\vec{b}_{\vec{k}}$ [12]:

$$\vec{a}_{\vec{k}} = \frac{\vec{b}_{\vec{k}} + L_{\vec{k}} \vec{b}^+_{-\vec{k}}}{\sqrt{1 - L^2_{\vec{k}}}}, \tag{22}$$

where $L_{\vec{k}}$ is the unknown real symmetrical function of the wave vector $\vec{k}$.

By substituting (22) into (21), we obtain

$$\hat{H}_l = \sum_{\vec{k} \neq 0} \varepsilon_{\vec{k}} \vec{b}^+_{\vec{k}} \vec{b}_{\vec{k}}, \tag{23}$$

where $\vec{b}^+_{\vec{k}}$ and $\vec{b}_{\vec{k}}$ are the creation and annihilation operators of charged Bose quasiparticles with the energy:

$$\varepsilon_{\vec{k}} = \left[\left(\frac{\hbar^2 k^2}{2m} \right)^2 + \hbar^2 k^2 v^2 \right]^{1/2}. \tag{24}$$

In this context, the real symmetrical function $L_{\vec{k}}$ of the wave vector $\vec{k}$ is found to be

$$L^2_{\vec{k}} = \frac{\frac{\hbar^2 k^2}{2m} + mv^2 - \varepsilon_{\vec{k}}}{\frac{\hbar^2 k^2}{2m} + mv^2 + \varepsilon_{\vec{k}}}. \tag{25}$$

Thus, the average energy of the system takes the form:

$$\overline{\hat{H}_l} = \sum_{\vec{k} \neq 0} \varepsilon_{\vec{k}} \overline{\vec{b}^+_{\vec{k}} \vec{b}_{\vec{k}}}, \tag{26}$$

where $\overline{\vec{b}^+_{\vec{k}} \vec{b}_{\vec{k}}}$ is the average number of charged Bose quasiparticles with the wave vector $\vec{k}$ at the temperature T:

$$\overline{\vec{b}^+_{\vec{k}} \vec{b}_{\vec{k}}} = \frac{1}{e^{\frac{\varepsilon_{\vec{k}}}{kT}} - 1}. \tag{27}$$

Thus, we have found the spectrum of free charged spinless quasiparticles excited in a Bose or Fermi liquid which is similar to Bogoliubov's one [8]. In fact, the Hamiltonian of system (24) describes an ideal Bose gas consisting of charged spinless phonons at a small wave number $k \ll \frac{2mv}{\hbar}$ but at $k \gg \frac{2mv}{\hbar}$ the Hamiltonian operator describes an ideal gas of *charged sound particles*. This reasoning implies that the tuning magnetic field forms the superfluidity component of a Bose or Fermi liquid which is been in the uppermost hyperfine state.

4 BEC of *charged sound particles*

As opposed to London's postulation concerning BEC of atoms [7], we state that *charged sound particles* in the condensate define the superfluid component of Bose and Fermi liquids. Consequently, statistical equilibrium equation (10) takes the following form:

$$n_{0,T} + \sum_{\vec{k} \neq 0} \overline{\vec{a}^+_{\vec{k}} \vec{a}_{\vec{k}}} = n, \tag{28}$$

where $\overline{\vec{a}^+_{\vec{k}} \vec{a}_{\vec{k}}}$ is the average number of *charged sound particles* with the wave vector $\vec{k}$ at the temperature T.

To find the form $\overline{\vec{a}^+_{\vec{k}} \vec{a}_{\vec{k}}}$, we use the linear transformation presented in (22):

$$\overline{\vec{a}^+_{\vec{k}} \vec{a}_{\vec{k}}} = \frac{1 + L^2_{\vec{p}}}{1 - L^2_{\vec{p}}} \overline{\vec{b}^+_{\vec{k}} \vec{b}_{\vec{k}}} + \frac{L_{\vec{k}}}{1 - L^2_{\vec{k}}} \left(\overline{\vec{b}^+_{\vec{k}} \vec{b}^+_{-\vec{k}}} + \overline{\vec{b}_{\vec{k}} \vec{b}_{-\vec{k}}} \right) + \frac{L^2_{\vec{k}}}{1 - L^2_{\vec{k}}}.$$

According to the Bloch-De-Dominicis theorem, we have

$$\overline{\vec{b}^+_{\vec{k}} \vec{b}^+_{-\vec{k}}} = \overline{\vec{b}_{\vec{k}} \vec{b}_{-\vec{k}}} = 0.$$

In this respect, the equation for the density of *charged sound particles* in the condensate takes the following form:

$$\frac{n_{0,T}}{V} = \frac{n}{V} - \frac{1}{V} \sum_{\vec{k} \neq 0} \frac{L^2_{\vec{k}}}{1 - L^2_{\vec{k}}} - \frac{1}{V} \sum_{\vec{k} \neq 0} \frac{1 + L^2_{\vec{k}}}{1 - L^2_{\vec{k}}} \overline{\vec{b}^+_{\vec{k}} \vec{b}_{\vec{k}}}. \tag{29}$$

Obviously, at the lambda transition $T = T_\lambda$ the density of *charged sound particles* $\frac{n_{0,T_\lambda}}{V} = 0$. Hence, we note that the mass m and density $\frac{n}{V}$ of *charged sound particles* are expressed via the mass of ions M and density of ions $\frac{N}{V}$ when solving a system of two equations presented in (13) and (20):

$$\frac{n}{V} = \left(\frac{M c_l}{\hbar} \frac{N}{V} \right)^{\frac{3}{4}} \tag{30}$$

and

$$m = \left(\frac{\hbar}{c_l} \right)^{\frac{3}{4}} \left(\frac{MN}{V} \right)^{\frac{1}{4}}. \tag{31}$$

In conclusion, it should be noted that the given approach opens up a new direction for investigation of BEC of *charged sound particles* in Fermi gases of spin-polarized hydrogen, ^{6}Li and ^{40}K, and in a Bose gas such as ^{87}Rb, because the model of quantum liquid in the uppermost hyperfine state is considered in the same way as superfluid liquid helium. In this letter, we argue for the first time that the superfluid component of Bose or Fermi liquid in the uppermost hyperfine state is determined by means of *charged sound particles* in the condensate. In fact, we argue that the lambda transition point depends on the strain of static magnetic field due to equation (29) and condition for the density of *charged sound particles* $\frac{n_{0,T_\lambda}}{V} = 0$.

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