

Fermion-Antifermion Asymmetry

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An event with positive energy transfers this energy photons which carries it on recorders observers. Observers know that this event occurs, not before it happens. But events with negative energy should absorb this energy from observers. Consequently, observers know that this event happens before it happens. Since time is irreversible then only the events with positive energy can occur. In single-particle states, events with a fermion have positive energy and occurrences with an antifermion have negative energy. In double-particle states, events with pair of antifermions have negative energy and events with pair of fermions and with fermion-antifermion pair have positive energy.

1 Introduction

Let t, x_1, x_2, x_3 be real numbers, and let $\mathbf{x} := \langle x_1, x_2, x_3 \rangle$.

Let \mathcal{A} be some pointlike event.

Let $\varphi(t, \mathbf{x})$ be a 4×1 -complex matrix such that

$$\varphi^\dagger(t, \mathbf{x})\varphi(t, \mathbf{x}) = \rho(t, \mathbf{x}) \quad (1)$$

where $\rho(t, \mathbf{x})$ is the probability density of \mathcal{A} .

Let* $\rho(t, \mathbf{x}) = 0$ if $t > \frac{\pi c}{h}$ and/or $|\mathbf{x}| > \frac{\pi c}{h}$.

In that case $\varphi(t, \mathbf{x})$ obeys some generalization of the Dirac equation [1]. The Dirac equation for free fermion does have the following form:

$$\left(\frac{1}{c} \frac{\partial}{\partial t} - \sum_{s=1}^3 \beta^{[s]} \frac{\partial}{\partial x_s} - i \frac{h}{c} n \gamma^{[0]} \right) \varphi(t, \mathbf{x}) = 0.$$

Here n is a natural number and

$$\beta^{[1]} := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \beta^{[2]} := \begin{bmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{bmatrix},$$

$$\beta^{[3]} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \gamma^{[0]} := \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

In this case operator \widehat{H}_0 is the free Dirac Hamiltonian if

$$\widehat{H}_0 := c \left(\sum_{s=1}^3 \beta^{[s]} \frac{\partial}{\partial x_s} + \frac{h}{c} n \gamma^{[0]} \right).$$

Let \mathbf{k} be a vector $\langle k_1, k_2, k_3 \rangle$ where k_s are integer numbers and let

$$\omega(\mathbf{k}) := \sqrt{k_1^2 + k_2^2 + k_3^2 + n^2}$$

where n is a natural number.

*c := 299792458, h := 6.6260755⁻³⁴

Let

$$e_1(\mathbf{k}) := \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}} \begin{bmatrix} \omega(\mathbf{k}) + n + k_3 \\ k_1 + ik_2 \\ \omega(\mathbf{k}) + n - k_3 \\ -k_1 - ik_2 \end{bmatrix},$$

$$e_2(\mathbf{k}) := \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}} \begin{bmatrix} k_1 - ik_2 \\ \omega(\mathbf{k}) + n - k_3 \\ -k_1 - ik_2 \\ \omega(\mathbf{k}) + n + k_3 \end{bmatrix},$$

$$e_3(\mathbf{k}) := \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}} \begin{bmatrix} -\omega(\mathbf{k}) - n + k_3 \\ k_1 + ik_2 \\ \omega(\mathbf{k}) + n + k_3 \\ k_1 + ik_2 \end{bmatrix},$$

$$e_4(\mathbf{k}) := \frac{1}{2\sqrt{\omega(\mathbf{k})(\omega(\mathbf{k})+n)}} \begin{bmatrix} k_1 - ik_2 \\ -\omega(\mathbf{k}) - n - k_3 \\ k_1 - ik_2 \\ \omega(\mathbf{k}) + n - k_3 \end{bmatrix}.$$

In that case, functions $e_1(\mathbf{k})(2c/h)^{3/2} \exp(-i(h/c)\mathbf{kx})$ and $e_2(\mathbf{k})(2c/h)^{3/2} \exp(-i(h/c)\mathbf{kx})$ are eigenvectors of \widehat{H}_0 with eigenvalues $(+\hbar\omega(\mathbf{k}))$, and functions $e_3(\mathbf{k})(2c/h)^{3/2} \exp(-i(h/c)\mathbf{kx})$ and $e_4(\mathbf{k})(2c/h)^{3/2} \exp(-i(h/c)\mathbf{kx})$ are eigenvectors of \widehat{H}_0 with eigenvalues $(-\hbar\omega(\mathbf{k}))$.

2 Single-Particle States

Let \mathfrak{H} be some unitary space. Let $\widetilde{0}$ be the zero element of \mathfrak{H} . That is any element \widetilde{F} of \mathfrak{H} obeys to the following conditions:

$$0\widetilde{F} = \widetilde{0}, \widetilde{0} + \widetilde{F} = \widetilde{F}, \widetilde{0}^\dagger \widetilde{F} = \widetilde{F}, \widetilde{0}^\dagger = \widetilde{0}.$$

Let $\widetilde{0}$ be the zero operator on \mathfrak{H} . That is any element \widetilde{F} of

\mathfrak{H} obeys to the following condition:

$$\widehat{0F} = 0\widehat{F}, \text{ and if } \widehat{b} \text{ is any operator on } \mathfrak{H} \text{ then}$$

$$\widehat{0} + \widehat{b} = \widehat{b} + \widehat{0} = \widehat{b}, \widehat{0b} = \widehat{b0} = \widehat{0}.$$

Let $\widehat{1}$ be the identity operator on \mathfrak{H} . That is any element \widehat{F} of \mathfrak{H} obeys to the following condition:

$$\widehat{1F} = 1\widehat{F} = \widehat{F}, \text{ and if } \widehat{b} \text{ is any operator on } \mathfrak{H} \text{ then}$$

$$\widehat{1b} = \widehat{b1} = \widehat{b}.$$

Let linear operators $b_{s,\mathbf{k}}$ ($s \in \{1, 2, 3, 4\}$) act on all elements of this space. And let these operators fulfill the following conditions:

$$\{b_{s,\mathbf{k}}^\dagger, b_{s',\mathbf{k}'}\} := b_{s,\mathbf{k}}^\dagger b_{s',\mathbf{k}'} + b_{s',\mathbf{k}'} b_{s,\mathbf{k}}^\dagger = \left(\frac{\hbar}{2\pi c}\right)^3 \delta_{\mathbf{k},\mathbf{k}'} \delta_{s,s'} \widehat{1},$$

$$\{b_{s,\mathbf{k}}, b_{s',\mathbf{k}'}\} = b_{s,\mathbf{k}} b_{s',\mathbf{k}'} + b_{s',\mathbf{k}'} b_{s,\mathbf{k}} = \{b_{s,\mathbf{k}}^\dagger, b_{s',\mathbf{k}'}^\dagger\} = \widehat{0}.$$

Hence,

$$b_{s,\mathbf{k}} b_{s,\mathbf{k}} = b_{s,\mathbf{k}}^\dagger b_{s,\mathbf{k}}^\dagger = \widehat{0}.$$

There exists element \widehat{F}_0 of \mathfrak{H} such that $\widehat{F}_0^\dagger \widehat{F}_0 = 1$ and for any $b_{s,\mathbf{k}}$: $b_{s,\mathbf{k}} \widehat{F}_0 = \widehat{0}$. Hence, $\widehat{F}_0^\dagger b_{s,\mathbf{k}}^\dagger = \widehat{0}$.

Let

$$\psi_s(\mathbf{x}) := \sum_{\mathbf{k}} \sum_{r=1}^4 b_{r,\mathbf{k}} e_{r,s}(\mathbf{k}) \exp\left(-i\frac{\hbar}{c}\mathbf{k}\mathbf{x}\right).$$

Because

$$\sum_{r=1}^4 e_{r,s}(\mathbf{k}) e_{r,s'}(\mathbf{k}) = \delta_{s,s'}$$

and

$$\sum_{\mathbf{k}} \exp\left(-i\frac{\hbar}{c}\mathbf{k}(\mathbf{x} - \mathbf{x}')\right) = \left(\frac{2\pi c}{\hbar}\right)^3 \delta(\mathbf{x} - \mathbf{x}')$$

then

$$\{\psi_s^\dagger(\mathbf{x}), \psi_{s'}(\mathbf{x}')\} := \psi_s^\dagger(\mathbf{x}) \psi_{s'}(\mathbf{x}') + \psi_{s'}(\mathbf{x}') \psi_s^\dagger(\mathbf{x})$$

$$= \delta(\mathbf{x} - \mathbf{x}') \delta_{s,s'} \widehat{1}.$$

And these operators obey the following conditions:

$$\psi_s(\mathbf{x}) \widehat{F}_0 = \widehat{0}, \{\psi_s(\mathbf{x}), \psi_{s'}(\mathbf{x}')\} = \{\psi_s^\dagger(\mathbf{x}), \psi_{s'}^\dagger(\mathbf{x}')\} = \widehat{0}.$$

Hence,

$$\psi_s(\mathbf{x}) \psi_{s'}(\mathbf{x}') = \psi_s^\dagger(\mathbf{x}) \psi_{s'}^\dagger(\mathbf{x}') = \widehat{0}.$$

Let

$$\Psi(t, \mathbf{x}) := \sum_{s=1}^4 \varphi_s(t, \mathbf{x}) \psi_s^\dagger(\mathbf{x}) \widehat{F}_0.$$

These functions obey the following condition:

$$\Psi^\dagger(t, \mathbf{x}') \Psi(t, \mathbf{x}) = \varphi^\dagger(t, \mathbf{x}') \varphi(t, \mathbf{x}) \delta(\mathbf{x} - \mathbf{x}').$$

Hence,

$$\int d\mathbf{x}' \cdot \Psi^\dagger(t, \mathbf{x}') \Psi(t, \mathbf{x}) = \rho(t, \mathbf{x}).$$

Let a Fourier series of $\varphi_s(t, \mathbf{x})$ has the following form:

$$\varphi_s(t, \mathbf{x}) = \sum_{\mathbf{p}} \sum_{r=1}^4 c_r(t, \mathbf{p}) e_{r,s}(\mathbf{p}) \exp\left(-i\frac{\hbar}{c}\mathbf{p}\mathbf{x}\right).$$

In that case:

$$\underline{\Psi}(t, \mathbf{p}) := \left(\frac{2\pi c}{\hbar}\right)^3 \sum_{r=1}^4 c_r(t, \mathbf{p}) b_{r,\mathbf{p}}^\dagger \widehat{F}_0.$$

If

$$\mathcal{H}_0(\mathbf{x}) := \psi^\dagger(\mathbf{x}) \widehat{H}_0 \psi(\mathbf{x}) \quad (2)$$

then $\mathcal{H}_0(\mathbf{x})$ is called a Hamiltonian \widehat{H}_0 density.

Because

$$\widehat{H}_0 \varphi(t, \mathbf{x}) = i \frac{\partial}{\partial t} \varphi(t, \mathbf{x})$$

then

$$\int d\mathbf{x}' \cdot \mathcal{H}_0(\mathbf{x}') \Psi(t, \mathbf{x}) = i \frac{\partial}{\partial t} \Psi(t, \mathbf{x}). \quad (3)$$

Therefore, if

$$\widehat{\mathbb{H}} := \int d\mathbf{x}' \cdot \mathcal{H}_0(\mathbf{x}')$$

then $\widehat{\mathbb{H}}$ acts similar to the Hamiltonian on space \mathfrak{H} .

And if

$$E_\Psi(\widehat{F}_0) := \sum_{\mathbf{p}} \underline{\Psi}^\dagger(t, \mathbf{p}) \widehat{\mathbb{H}} \underline{\Psi}(t, \mathbf{p})$$

then $E_\Psi(\widehat{F}_0)$ is an energy of Ψ on vacuum \widehat{F}_0 .

Operator $\widehat{\mathbb{H}}$ obeys the following condition:

$$\widehat{\mathbb{H}} = \left(\frac{2\pi c}{\hbar}\right)^3 \sum_{\mathbf{k}} \hbar \omega(\mathbf{k}) \left(\sum_{r=1}^2 b_{r,\mathbf{k}}^\dagger b_{r,\mathbf{k}} - \sum_{r=3}^4 b_{r,\mathbf{k}}^\dagger b_{r,\mathbf{k}} \right).$$

This operator is not positive defined and in this case

$$E_\Psi(\widehat{F}_0) = \left(\frac{2\pi c}{\hbar}\right)^3 \sum_{\mathbf{p}} \hbar \omega(\mathbf{p}) \left(\sum_{r=1}^2 |c_r(t, \mathbf{p})|^2 - \sum_{r=3}^4 |c_r(t, \mathbf{p})|^2 \right).$$

This problem is usually solved in the following way [2, p. 54]:

Let:

$$v_1(\mathbf{k}) : = \gamma^{[01]} e_3(\mathbf{k}),$$

$$v_2(\mathbf{k}) : = \gamma^{[01]} e_4(\mathbf{k}),$$

$$d_{1,\mathbf{k}} : = -b_{3,-\mathbf{k}}^\dagger,$$

$$d_{2,\mathbf{k}} : = -b_{4,-\mathbf{k}}^\dagger.$$

In that case:

$$\begin{aligned} e_3(\mathbf{k}) &= -v_1(-\mathbf{k}), \\ e_4(\mathbf{k}) &= -v_2(-\mathbf{k}), \\ b_{3,\mathbf{k}} &= -d_{1,-\mathbf{k}}^\dagger, \\ b_{4,\mathbf{k}} &= -d_{2,-\mathbf{k}}^\dagger. \end{aligned}$$

Therefore,

$$\begin{aligned} \psi_s(\mathbf{x}) : &= \sum_{\mathbf{k}} \sum_{r=1}^2 \left(b_{r,\mathbf{k}} e_{r,s}(\mathbf{k}) \exp\left(-i\frac{\hbar}{c}\mathbf{k}\mathbf{x}\right) + \right. \\ &\left. + d_{r,\mathbf{k}}^\dagger v_{r,s}(\mathbf{k}) \exp\left(i\frac{\hbar}{c}\mathbf{k}\mathbf{x}\right) \right) \end{aligned}$$

$$\begin{aligned} \widehat{\mathbb{H}} &= \left(\frac{2\pi c}{\hbar}\right)^3 \sum_{\mathbf{k}} \hbar\omega(\mathbf{k}) \sum_{r=1}^2 \left(b_{r,\mathbf{k}}^\dagger b_{r,\mathbf{k}} + d_{r,\mathbf{k}}^\dagger d_{r,\mathbf{k}} \right) \\ &- 2 \sum_{\mathbf{k}} \hbar\omega(\mathbf{k}) \widehat{1}. \end{aligned}$$

The first term on the right side of this equality is positive defined. This term is taken as the desired Hamiltonian. The second term of this equality is infinity constant. And this infinity is deleted (?) [2, p. 58]

But in this case $d_{r,\mathbf{k}} \widetilde{F}_0 \neq \widetilde{0}$. In order to satisfy such condition, the vacuum element \widetilde{F}_0 must be replaced by the following:

$$\widetilde{F}_0 \rightarrow \widetilde{\Phi}_0 := \prod_{\mathbf{k}} \prod_{r=3}^4 \left(\frac{2\pi c}{\hbar}\right)^3 b_{r,\mathbf{k}}^\dagger \widetilde{F}_0.$$

But in this case:

$$\psi_s(\mathbf{x}) \widetilde{\Phi}_0 \neq \widetilde{0}.$$

And condition (3) isn't carried out.

In order to satisfy such condition, operators $\psi_s(\mathbf{x})$ must be replaced by the following:

$$\begin{aligned} \psi_s(\mathbf{x}) \rightarrow \phi_s(\mathbf{x}) := \\ := \sum_{\mathbf{k}} \sum_{r=1}^2 \left(b_{r,\mathbf{k}} e_{r,s}(\mathbf{k}) \exp\left(-i\frac{\hbar}{c}\mathbf{k}\mathbf{x}\right) + d_{r,\mathbf{k}} v_r(\mathbf{k}) \exp\left(i\frac{\hbar}{c}\mathbf{k}\mathbf{x}\right) \right). \end{aligned}$$

Hence,

$$\begin{aligned} \widehat{\mathbb{H}} &= \int d\mathbf{x} \cdot \mathcal{H}(\mathbf{x}) = \int d\mathbf{x} \cdot \phi^\dagger(\mathbf{x}) \widehat{H}_0 \phi(\mathbf{x}) = \\ &= \left(\frac{2\pi c}{\hbar}\right)^3 \sum_{\mathbf{k}} \hbar\omega(\mathbf{k}) \sum_{r=1}^2 \left(b_{r,\mathbf{k}}^\dagger b_{r,\mathbf{k}} - d_{r,\mathbf{k}}^\dagger d_{r,\mathbf{k}} \right). \end{aligned}$$

And again we get negative energy.

Let's consider the meaning of such energy: An event with positive energy transfers this energy photons which carries it

on recorders observers. Observers know that this event occurs, not before it happens. But event with negative energy should absorb this energy from observers. Consequently, observers know that this event happens before it happens. This contradicts Theorem 3.4.2 [3]. Therefore, events with negative energy do not occur.

Hence, over vacuum $\widetilde{\Phi}_0$ single fermions can exist, but there are no single antifermions.

3 Two-Particle States

A two-particle state is defined the following field operator [4]:

$$\psi_{s_1, s_2}(\mathbf{x}, \mathbf{y}) := \begin{vmatrix} \phi_{s_1}(\mathbf{x}) & \phi_{s_2}(\mathbf{x}) \\ \phi_{s_1}(\mathbf{y}) & \phi_{s_2}(\mathbf{y}) \end{vmatrix}.$$

In that case:

$$\widehat{\mathbb{H}} = 2\hbar \left(\frac{2\pi c}{\hbar}\right)^6 \left(\widehat{\mathbb{H}}_a + \widehat{\mathbb{H}}_b\right)$$

where

$$\begin{aligned} \widehat{\mathbb{H}}_a : &= \sum_{\mathbf{k}} \sum_{\mathbf{p}} (\omega(\mathbf{k}) - \omega(\mathbf{p})) \sum_{r=1}^2 \sum_{j=1}^2 \times \\ &\times \left\{ v_j^\dagger(-\mathbf{k}) v_j(-\mathbf{p}) e_r^\dagger(\mathbf{p}) e_r(\mathbf{k}) \times \right. \\ &\times \left(+b_{r,\mathbf{p}}^\dagger d_{j,-\mathbf{k}}^\dagger d_{j,-\mathbf{p}} b_{r,\mathbf{k}} \right) + \\ &+ \left(+d_{r,-\mathbf{p}}^\dagger b_{j,\mathbf{k}}^\dagger b_{j,\mathbf{k}} d_{r,-\mathbf{p}} \right) + \\ &+ v_j^\dagger(-\mathbf{p}) v_j(-\mathbf{k}) e_r^\dagger(\mathbf{k}) e_r(\mathbf{p}) \times \\ &\times \left(-b_{r,\mathbf{k}}^\dagger d_{j,-\mathbf{p}}^\dagger d_{j,-\mathbf{k}} b_{r,\mathbf{p}} \right) + \\ &\left. + \left(-b_{r,\mathbf{p}}^\dagger d_{j,-\mathbf{k}}^\dagger d_{j,-\mathbf{k}} b_{r,\mathbf{p}} \right) \right\} \end{aligned}$$

and

$$\begin{aligned} \widehat{\mathbb{H}}_b : &= \sum_{\mathbf{k}} \sum_{\mathbf{p}} (\omega(\mathbf{k}) + \omega(\mathbf{p})) \sum_{r=1}^2 \sum_{j=1}^2 \times \\ &\times \left\{ v_j^\dagger(-\mathbf{p}) v_j(-\mathbf{k}) v_r^\dagger(-\mathbf{k}) v_r(-\mathbf{p}) \times \right. \\ &\times \left(-d_{r,-\mathbf{k}}^\dagger d_{j,-\mathbf{p}}^\dagger d_{j,-\mathbf{k}} d_{r,-\mathbf{p}} \right) + \\ &+ \left(-d_{r,-\mathbf{p}}^\dagger d_{j,-\mathbf{k}}^\dagger d_{j,-\mathbf{k}} d_{r,-\mathbf{p}} \right) \\ &+ e_r^\dagger(\mathbf{k}) e_r(\mathbf{p}) e_j^\dagger(\mathbf{p}) e_j(\mathbf{k}) \times \\ &\times \left(+b_{r,\mathbf{k}}^\dagger b_{j,\mathbf{p}}^\dagger b_{j,\mathbf{k}} b_{r,\mathbf{p}} \right) + \\ &\left. + \left(+b_{r,\mathbf{p}}^\dagger b_{j,\mathbf{k}}^\dagger b_{j,\mathbf{k}} b_{r,\mathbf{p}} \right) \right\}. \end{aligned}$$

If velocities are small then the following formula is fair.

$$\widehat{\mathbb{H}} = 4\hbar \left(\frac{2\pi c}{\hbar}\right)^6 \left(\widehat{\mathbb{H}}_a + \widehat{\mathbb{H}}_b\right)$$

where

$$\begin{aligned} \widehat{\mathbb{H}}_a & : = \sum_{\mathbf{k}} \sum_{\mathbf{p}} (\omega(\mathbf{k}) - \omega(\mathbf{p})) \times \\ & \times \sum_{r=1}^2 \sum_{j=1}^2 (d_{j,-\mathbf{p}}^\dagger b_{r,\mathbf{k}}^\dagger b_{r,\mathbf{k}} d_{j,-\mathbf{p}} - b_{j,\mathbf{p}}^\dagger d_{r,-\mathbf{k}}^\dagger d_{r,-\mathbf{k}} b_{j,\mathbf{p}}) \end{aligned}$$

and

$$\begin{aligned} \widehat{\mathbb{H}}_b & : = \sum_{\mathbf{k}} \sum_{\mathbf{p}} (\omega(\mathbf{k}) + \omega(\mathbf{p})) \times \\ & \times \sum_{j=1}^2 \sum_{r=1}^2 (b_{j,\mathbf{p}}^\dagger b_{r,\mathbf{k}}^\dagger b_{r,\mathbf{k}} b_{j,\mathbf{p}} - d_{j,-\mathbf{p}}^\dagger d_{r,-\mathbf{k}}^\dagger d_{r,-\mathbf{k}} d_{j,-\mathbf{p}}). \end{aligned}$$

Therefore, in any case events with pairs of fermions and events with fermion-antifermion pairs can occur, but events with pairs of antifermions can not happen.

4 Conclusion

Therefore, an antifermion can exist only with a fermion.

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