Excited Electronic States of Atoms Described by the Model of Oscillations in a Chain System

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We analyzed the numerical values of half-lifes of excited electronic states of the H, He and Li atom, as well as the Li^+ ion. By means of a fractal scaling model originally published by Müller in this journal, we interprete these half-lifes as proton resonance periods. On the logarithmic scale, the half-lifes were expressed by short continued fractions, where all numerators are Euler's number. From this representation it was concluded that the half-lifes are heavily located in nodes or sub-nodes of the spectrum of proton resonance periods.

1 Introduction

The model of a chain of similar harmonic oscillators was proposed by Müller [1–3] as a phenomenological theory describing physical quantities as proton resonance oscillation modes.

In the most general case, the spectrum of eigenfrequencies of a chain system of many proton harmonic oscillators is given by the continuous fraction equation [2]

$$f = f_p \exp S, \tag{1}$$

where f is any natural oscillation frequency of the chain system, f_p the oscillation frequency of one proton and S the continued fraction corresponding to f. S was suggested to be in the canonical form with all partial numerators equal 1 and the partial denominators are positive or negative integer values

$$S = n_0 + \cfrac{1}{n_1 + \cfrac{1}{n_2 + \cfrac{1}{n_3 + \ldots}}}. \tag{2}$$

Particularly interesting properties arise when the numerator equals 2 and all denominators are divisible by 3. Such fractions divide the logarithmic scale in allowed values and empty gaps, i.e. ranges of numbers which cannot be expressed with this type of continued fractions. He showed that these continued fractions generate a self-similar and discrete spectrum of eigenvalues [1], that is also logarithmically invariant. Maximum spectral density areas arise when the free link n_0 and the partial denominators n_i are divisible by 3.

In two previous articles [4, 5] we applied a slightly modified model, where all numerators were substituted by Euler's number. This model was particularly successful describing specific features of the solar system [5].

However, the true physical meaning of the numerator e is not yet clear. It must now be investigated, for which type of data exactly this type of continued fractions can be applied. There might be some data sets, where the numerator is 2, as it was suggested by Müller in a patent [6].

In this article we analyzed a set of very accurately determined half-lifes of excited states of atoms on the logarithmic scale. We show that continued fractions with Euler's number as numerator are adequate to describe these data.

2 Data source and computational details

All atomic spectral data were taken from the web site of the National Institute of Standards and Technology (NIST) [7]. NIST maintains a critical selection of spectral data previously published in regular scientific journals. For the H, He and Li atom, reference was given to a publication by Wiese [8].

Table 1 shows such a data compilation for the Hydrogen atom. We consider here only experimentally observed emission lines (i.e. not Ritz lines), for which the transition probabilities have been determined. We numbered these lines in the order of increasing wavelength and eliminated lines with an already previously listed transition probability. For the Hydrogen atom, this procedure resulted then in a set of 109 lines which have all different transition probabilities. Also, if a transition probability has a numerical error higher than 1% (according to NIST), the corresponding line was ignored.

The transition probability as given by NIST has the unit of frequency $[s^{-1}]$ and is also called the Einstein A coefficient of spontaneous emission. Consider a large number of atoms in an excited state i, decaying to the ground state k (k could also be any lower lying excited state). Equation (3) is then the rate law

$$\frac{\partial N}{\partial t} = -A_{ik} N, \tag{3}$$

which results in

$$N(t) = N_0 \exp(-A_{ik} t), \tag{4}$$

where $N(t)$ is the number of excited atoms at time t, N_0 the number of excited atoms at $t = 0$ and A_{ik} the Einstein A coefficient for the transition $i \to k$. From this exponential law, the half-life $T_{1/2}$ of the transition $i \to k$ can be calculated as

$$T_{1/2} = \frac{\ln(2)}{A_{ik}}. \tag{5}$$

Table 1: Observed emission lines of the Hydrogen atom with corresponding wavelengths and transition probabilities. Obs.: Line no. 18 represents a forbidden transition.

Line no.	Wavelength [Å]	Transition probability [s^{-1}]	Line no.	Wavelength [Å]	Transition probability [s^{-1}]
1	918.125	5.0659×10^4	56	6562.72482	2.2448×10^7
2	919.342	7.8340×10^4	57	6562.77153	2.2449×10^7
3	920.947	1.2631×10^5	58	6562.79	4.4101×10^7
4	923.148	2.1425×10^5	59	6562.85175	6.4651×10^7
5	926.249	3.8694×10^5	60	8392.40	1.5167×10^3
6	930.751	7.5684×10^5	61	8413.32	1.9643×10^3
7	937.801	1.9728×10^7	62	8437.95	2.5804×10^3
8	937.814	1.6440×10^6	63	8467.26	3.4442×10^3
9	949.742	3.4375×10^7	64	8502.49	4.6801×10^3
10	949.742	4.1250×10^6	65	8545.38	6.4901×10^3
11	972.517	1.2785×10^7	66	8598.39	9.2117×10^3
12	972.541	6.8186×10^7	67	8665.02	1.3431×10^4
13	1025.728	1.6725×10^8	68	8750.46	2.0207×10^4
14	1025.728	5.5751×10^7	69	8862.89	3.1558×10^4
15	1215.6699	6.2648×10^8	70	9015.3	5.1558×10^4
16	1215.6699	6.2649×10^8	71	9229.7	8.9050×10^4
17	1215.6701	4.6986×10^8	72	9546.2	1.6506×10^5
18	1215.67312	2.495×10^{-6}	73	10049.8	3.3585×10^5
19	3656.65	9.9657×10^1	74	10938.17	7.7829×10^5
20	3657.25	1.1430×10^2	75	12818.072	2.2008×10^6
21	3658.04	1.3161×10^2	76	15560.46	3.6714×10^3
22	3658.65	1.5216×10^2	77	16411.36	1.6205×10^4
23	3659.41	1.7669×10^2	78	16811.10	2.5565×10^4
24	3660.32	2.0612×10^2	79	17366.885	4.2347×10^4
25	3661.27	2.4162×10^2	80	18179.21	7.4593×10^4
26	3662.22	2.8474×10^2	81	18751.3	8.9860×10^6
27	3663.41	3.3742×10^2	82	21661.178	3.0415×10^5
28	3664.65	4.0224×10^2	83	26258.71	7.7110×10^5
29	3666.08	4.8261×10^2	84	32969.8	6.9078×10^4
30	3667.73	5.8304×10^2	85	37405.76	1.3877×10^5
31	3669.45	7.0963×10^2	86	40522.79	2.6993×10^6
32	3671.32	8.7069×10^2	87	46537.8	3.2528×10^5
33	3673.81	1.0777×10^3	88	51286.5	3.6881×10^4
34	3676.376	1.3467×10^3	89	74599.0	1.0254×10^6
35	3679.370	1.7005×10^3	90	75024.4	1.5609×10^5
36	3682.823	2.1719×10^3	91	81548.4	3.3586×10^3
37	3686.831	2.8093×10^3	92	86644.60	5.0098×10^3
38	3691.551	3.6851×10^3	93	87600.64	3.9049×10^4
39	3697.157	4.9101×10^3	94	93920.3	7.8037×10^3
40	3703.859	6.6583×10^3	95	105035.07	1.2870×10^4
41	3711.978	9.2102×10^3	96	108035.9	2.2679×10^3
42	3721.946	1.3032×10^4	97	113086.81	8.2370×10^4
43	3734.369	1.8927×10^4	98	115395.4	3.3253×10^3
44	3750.151	2.8337×10^4	99	123719.12	4.5608×10^5
45	3770.633	4.3972×10^4	100	123871.53	2.3007×10^4
46	3797.909	7.1225×10^4	101	125870.5	5.0797×10^3
47	3835.397	1.2156×10^5	102	190619.6	2.2720×10^5
48	3889.064	2.2148×10^5	103	278035.0	1.2328×10^5
49	3970.075	4.3889×10^5	104	690717	2.7989×10^4
50	4101.734	9.7320×10^5	105	887610	1.8569×10^4
51	4340.472	2.5304×10^6	106	1118630	1.2709×10^4
52	4861.28694	9.6680×10^6	107	1387500	8.9344×10^3
53	4861.29776	9.6683×10^6	108	1694230	6.4283×10^3
54	4861.35	8.4193×10^6	109	3376000	2.0659×10^3
55	6562.70969	5.3877×10^7			

Finally, the numerical values of continued fractions were always calculated using the the Lenz algorithm as indicated in reference [9].

3 Results and discussion

Half-lifes of exited states of atoms are abundantly available from the NIST web site, however, only for the light atoms such as H, He and Li these data have a very high accuracy. Considering for instance Fe as a heavy element, most of the Einstein A coefficients have uncertainties of 10-18% and are consequently not suitable for a numerical analysis.

Due to results form our previous publications, we suspect that Müller's continued fraction formalism with Euler's number as numerator can still be applied to many data sets, so we set all partial numerators in Müller's continued fractions (given in equation (2)) to Euler's number.

We strictly follow the formalism of previous publications [4–6] and introduce a phase shift p in equation (2). According to [6] the phase shift can only have the values 0 or $\pm\frac{3}{2}$. So we write for the half-lifes of the excited states:

$$\ln \frac{T_{1/2}}{\tau} = p + S, \tag{6}$$

where S is the continued fraction

$$S = n_0 + \cfrac{e}{n_1 + \cfrac{e}{n_2 + \cfrac{e}{n_3 + \dots}}} \tag{7}$$

and $\tau = \frac{\lambda_C}{c}$ is the oscillation period of a hypothetical photon with the reduced Compton wavelength of the proton ($\lambda_C = \frac{h}{2\pi mc} = 2.103089086 \times 10^{-16}$ m) and traveling at light speed (numerical value $7.015150081 \times 10^{-25}$ s).

We abbreviate $p + S$ as $[p; n_0 \mid n_1, n_2, n_3, \ldots]$. The free link n_0 and the partial denominators n_i are integers divisible by 3. For convergence reason, we have to include $|e{+}1|$ as allowed partial denominator. This means the free link n_0 is allowed to be $0, \pm3, \pm6, \pm9 \ldots$ and all partial denominators n_i can take the values $e{+}1$, $-e{-}1$, $\pm6, \pm9, \pm12 \ldots$.

For the calculation of the continued fractions we did not consider any standard deviation of the published data. Practically, we developed the continued fraction and determined only 18 partial denominators. Next we calculated repeatedly the data value from the continued fraction, every time considering one more partial denominator. As soon as considering further denominators did not improve the experimental data value significantly (on the linear scale), we stopped considering further denominators and gave the resulting fraction in Table 2. This means we demonstrate how accurately the calculated half-lifes can be expressed through continued fractions. Additionally we also report the numerical error, which is defined as absolute value of the difference between the half-life calculated from the NIST transition probability and the value calculated from the continued fraction representation.

If this numerical error is higher than 1%, we interpret the result as "no continued fraction found", otherwise the continued fraction representation is in satisfying agreement with the experimental data.

As can be seen from Table 2, with one exception, all half-lifes could be expressed in a satisfactory manner by a continued fraction representation. Only one outlier was found, which underlines the statistical nature of Müller's continued fraction model.

We believe that spectral line number 71 is a true outlier rather than a bad data point, since the Hydrogen spectrum has been thoroughly investigated and is definitely the most easiest one to interpret.

In most cases the numerical errors are several orders of magnitude lower than the data value. This changes when calculating the continued fractions with number 2 as numerator, as it was suggested by Müller in a patent [6]. In this case the number of outliers increases to 12 and the numerical errors of the continued fraction representations are frequently very slightly lower than the 1% limit. So the numerator e is definitely the better choice.

It can be seen that around 25% of the half-lifes could be expressed by two continued fractions, so there is no preferred accumulation of the half-lifes in the neutral zones. The majority of the continued fraction representations terminates with a high partial denominator ($\pm 9, \pm 12, \pm 15 \ldots$). This means there is a general tendency that the half-lifes accumulate in nodes and sub-nodes of the spectrum of the proton resonance periods.

Additionally, in the same manner as here described for the spectral lines of the Hydrogen atom, we analyzed the spectral data of He, Li (neutral atoms) and the Li^+ ion. From the NIST database resulted 142 spectral lines for the He atom, 57 lines for the Li atom and 129 lines for the Li^+ ion.

Again, it was analogously possible to express the half-lifes on the logarithmic scale by continued fraction representations with Euler's number as numerators. Very few outliers were found, 6 in the He data set, only one in the Li data set and 7 in the set of the Li^+ lines (continued fractions not given). Regarding the numerical errors, no significant differences were detected, when comparing with the Hydrogen set.

This result is a contribution to the importance of Euler's number as a possible numerator in the model of oscillations in a chain system. We have now identified the half-lifes of excited states with respect to individual electronic transitions as a further data set where this (still phenomenological) model can be applied. For the half-lifes, apparently it does not matter how many nucleons are in the atom and whether the atom is neutral or charged. It even seems to be that the model applies for both, allowed and forbidden transitions, however, this should be verified with further data; we have here only one forbidden transition in our data set.

4 Conclusions

Numerical investigation of a large data set of 437 half-lifes of electronic transitions from different atoms revealed that Müller's continued fraction model with e as numerator is adequate to express these data on the logarithmic scale. There is a general tendency that half-lifes accumulate in nodes and sub-nodes of the spectrum of proton resonance periods. This accumulation does not seem to be influenced by the atomic charge or the atomic number (chemical element). It can be said that every excited state of an atom (with corresponding transition), has different oscillation properties and goes in resonance with the appropriate proton oscillation. Then, during one proton oscillation period, 50% of the excited atoms become de-excited to a lower-lying state.

This viewpoint has some similarity to the teaching of modern quantum electrodynamics. This theory states that spontaneous emission from atoms is caused by a 50:50 contribution from radiation reaction and vacuum fluctuations [10]. So both models assume an external influence coupled to the atoms, either the proton resonance spectrum or the vacuum fluctuations.

Acknowledgments

The authors greatly acknowledge the financial support from the Brazilian governmental funding agencies FAPESQ and CNPq.

Submitted on June 7, 2011 / Accepted on June 9, 2011

References

1. Müller H. Fractal scaling Models of resonant oscillations in chain systems of harmonic oscillators. *Progress in Physics*, 2009, v. 2 72–76.
2. Müller H. Fractal scaling models of natural oscillations in chain systems and the mass distribution of the celestial bodies in the solar system. *Progress in Physics*, 2010, v. 1 62–66.
3. Müller H. Fractal scaling models of natural oscillations in chain systems and the mass distribution of particles. *Progress in Physics*, 2010, v. 3 61–66.
4. Ries A., Fook M. V. L. Fractal structure of nature's preferred masses: Application of the model of oscillations in a chain system. *Progress in Physics*, 2010, v. 4, 82–89.
5. Ries A., Fook M. V. L. Application of the model of oscillations in a chain system to the solar system. *Progress in Physics*, 2011, v. 1, 103–111.
6. Otte R., Müller H. German patent No. DE102004003753A1, date: 11.08.2005
7. NIST ASD Team (2010). NIST Atomic Spectra Database (ver. 4.0.1), [Online]. Available: http://physics.nist.gov/asd [2011, May 23]. National Institute of Standards and Technology, Gaithersburg, MD.
8. Wiese W. L., Fuhr J. R. Accurate atomic transition probabilities for hydrogen, helium, and lithium. *Journal of Physical and Chemical Reference Data*, 2009, v. 38, 565–726, Erratum: 2009, v. 38, 1129.
9. Press W. H., Teukolsky S. A., Vetterling W. T., Flannery B. P. Numerical recipes in C. Cambridge University Press, Cambridge, 1992.
10. Milonni P. W. The quantum vacuum. An introduction to quantum electrodynamics. Academic Press, San Diego, 1993.

Table 2: Continued fraction representation of half-lifes of excited states of the Hydrogen atom

Line no.	Half-life [s] Continued fraction representation	Numerical error [s]
1	$1.36826068529 \times 10^{-5}$	
	[0; 45 | -e-1, -e-1, e+1, -6, 6, -e-1, 6]	2.5×10^{-11}
	[1.5; 42 | e+1, -e-1, 24, -6, 9]	1.5×10^{-11}
2	$8.84793439571 \times 10^{-6}$	
	[1.5; 42 | 6, -9, e+1]	3.7×10^{-8}
3	$5.48766669749 \times 10^{-6}$	
	[1.5; 42 | 765]	3.0×10^{-11}
4	$3.23522604695 \times 10^{-6}$	
	[0; 42 | e+1, -e-1, e+1, -9, 12]	4.2×10^{-11}
	[1.5; 42 | -6, e+1, -6, -e-1, -9]	1.6×10^{-11}
5	$1.79135571551 \times 10^{-6}$	
	[0; 42 | 6, e+1, -e-1]	1.7×10^{-8}
6	$9.15843745785 \times 10^{-7}$	
	[0; 42 | -9, -6, 9, 6, -21, 117]	1.7×10^{-19}
7	$3.51351977169 \times 10^{-8}$	
	[0; 39 | -6, e+1, -e-1]	4.7×10^{-10}
	[1.5; 36 | e+1, -e-1, e+1, e+1, -e-1]	4.4×10^{-11}
8	$4.21622372603 \times 10^{-7}$	
	[1.5; 39 | 6, 12, e+1, 6]	6.2×10^{-12}
9	$2.01642816163 \times 10^{-8}$	
	[1.5; 36 | 6, e+1, -6, 6, -e-1, 18]	2.0×10^{-15}
10	$1.68035680136 \times 10^{-7}$	
	[1.5; 39 | -6, 6, e+1, -e-1, e+1]	3.4×10^{-10}
11	$5.42156574548 \times 10^{-8}$	
	[0; 39 | -24, 27, -24, -18]	3.6×10^{-17}
12	$1.01655351621 \times 10^{-8}$	
	[1.5; 36 | -9, -6, -42]	4.2×10^{-14}
13	$4.1443777612 \times 10^{-9}$	
	[0; 36 | 9, -6, -e-1]	4.8×10^{-12}
14	$1.24329102717 \times 10^{-8}$	
	[1.5; 36 | -33]	4.9×10^{-11}
15	$1.106415497 \times 10^{-9}$	
	[1.5; 33 | 6, -6, e+1, e+1, -e-1, 12]	1.1×10^{-15}
16	$1.10639783645 \times 10^{-9}$	
	[1.5; 33 | 6, -6, e+1, e+1, -e-1, 6, 15]	4.3×10^{-17}
17	$1.47522066267 \times 10^{-9}$	
	[0; 36 | -e-1, -39, -e-1, e+1, 9, 6]	4.8×10^{-17}
	[1.5; 33 | e+1, -12, e+1, -9, 12, 12]	8.9×10^{-18}
18	277814.501226	
	[0; 69 | -e-1, 6, -e-1, -141]	0.17
	[1.5; 66 | e+1, 6, -81]	0.97
19	0.00695532858264	
	[0; 51 | -9, e+1, -e-1, e+1]	9.2×10^{-5}
20	0.00606427979493	
	[0; 51 | -6, 6, 15]	5.7×10^{-7}
21	0.00526667563681	
	[0; 51 | -e-1, -e-1, -e-1, 6, 12]	4.2×10^{-8}
	[1.5; 48 | e+1, -e-1, -e-1, 6, -e-1, e+1, -30]	6.8×10^{-10}
22	0.00455538367876	
	[0; 51 | -e-1, 12, e+1, -e-1, 12, 9]	8.9×10^{-10}
	[1.5; 48 | e+1, 90, -e-1, e+1, 9]	7.1×10^{-10}
23	0.00392295648062	
	[0; 51 | -e-1, e+1, -12, -12, 54]	1.2×10^{-10}
24	0.00336283320668	
	[1.5; 48 | 6, 6, 39, -6, -18]	2.1×10^{-11}
25	0.00286874919527	
	[1.5; 48 | 9, e+1, -e-1, e+1, -e-1]	2.7×10^{-5}
26	0.00243431615003	
	[1.5; 48 | 27, e+1, e+1, -18]	3.2×10^{-9}
27	0.00205425635872	
	[1.5; 48 | -39, e+1, -e-1, -e-1, e+1, -33]	2.2×10^{-11}
28	0.00172321793099	
	[1.5; 48 | -12, e+1, -e-1, e+1, -e-1, e+1, 12]	2.3×10^{-9}
29	0.00143624703293	
	[1.5; 48 | -6, -9, e+1, -6, e+1]	3.7×10^{-8}
30	0.00118885013131	
	[0; 48 | e+1, -e-1, -e-1, -e-1, e+1]	1.4×10^{-6}
	[1.5; 48 | -e-1, -e-1, -9, -6, 9]	1.6×10^{-9}
31	0.000976772656962	
	[0; 48 | e+1, 12, -e-1, e+1]	5.4×10^{-7}
32	0.000796089515855	
	[0; 48 | 6, -9, e+1, -e-1]	6.6×10^{-7}
33	0.000643172664526	
	[0; 48 | 9, e+1, -e-1]	4.4×10^{-6}
34	0.000514700512779	
	[0; 48 | 60, e+1, -e-1, -60]	2.0×10^{-11}
35	0.000407613749227	
	[0; 48 | -15, e+1, e+1, -e-1, -9]	1.8×10^{-9}
36	0.000319143229688	
	[0; 48 | -6, -9, -e-1, e+1, -6]	4.3×10^{-9}
37	0.000246733058256	
	[0; 48 | -e-1, -12, -6, 6]	3.3×10^{-9}
38	0.000188094537614	
	[0; 48 | -e-1, e+1, -e-1, -9, 6]	6.2×10^{-9}
	[1.5; 45 | 6, -e-1, e+1, -6, e+1, -351]	6.9×10^{-13}
39	0.000141167630101	
	[1.5; 45 | 12, -e-1, e+1]	8.2×10^{-7}
40	0.000104102726005	
	[1.5; 45 | -51, 9]	4.2×10^{-9}
41	$7.525864591 \times 10^{-5}$	
	[1.5; 45 | -6, -e-1, e+1, -e-1]	9.0×10^{-7}
42	$5.31880893616 \times 10^{-5}$	
	[0; 45 | e+1, -12, -e-1, e+1, -9]	5.3×10^{-10}
	[1.5; 45 | -e-1, -90, e+1, 60]	5.1×10^{-13}
43	$3.66221366598 \times 10^{-5}$	
	[0; 45 | 6, e+1, -15, -e-1, -e-1, 18]	5.8×10^{-13}
44	$2.44608526153 \times 10^{-5}$	
	[0; 45 | -1446]	3.6×10^{-11}
45	$1.57633762522 \times 10^{-5}$	
	[0; 45 | -6, -18, e+1]	3.1×10^{-9}
46	$9.73179614686 \times 10^{-6}$	
	[0; 45 | -e-1, e+1, -12, -e-1, e+1, 9]	1.8×10^{-11}
	[1.5; 42 | e+1, e+1, -e-1, e+1, -e-1, e+1, -6, 6]	2.0×10^{-11}
47	$5.70209921487 \times 10^{-6}$	
	[1.5; 42 | 66, -e-1, e+1, -e-1]	4.6×10^{-10}
48	$3.12961522738 \times 10^{-6}$	
	[0; 42 | e+1, -e-1, 6, -31650]	7.5×10^{-17}
49	$1.57931869161 \times 10^{-6}$	
	[0; 42 | 12, -e-1, e+1, -e-1]	1.8×10^{-8}
50	$7.12235080723 \times 10^{-7}$	
	[0; 42 | -6, e+1, -e-1, 6]	6.3×10^{-10}
	[1.5; 39 | e+1, -e-1, e+1, 9]	4.4×10^{-10}
51	$2.73927908852 \times 10^{-7}$	
	[1.5; 39 | 441, e+1, 12]	1.6×10^{-14}
52	$7.16949917832 \times 10^{-8}$	
	[0; 39 | 15, e+1, -e-1]	3.7×10^{-10}
53	$7.16927671421 \times 10^{-8}$	
	[0; 39 | 15, e+1, -e-1]	3.7×10^{-10}
54	$8.23283622819 \times 10^{-8}$	
	[0; 39 | 9, -48]	5.0×10^{-12}
55	$1.2865363338 \times 10^{-8}$	
	[1.5; 36 | -51, -e-1]	4.8×10^{-12}
56	$3.08779036244 \times 10^{-8}$	
	[0; 39 | -e-1, -9, -30]	8.1×10^{-13}
	[1.5; 36 | e+1, -6, -6, e+1, -9]	4.1×10^{-13}

Line no.	Half-life [s] Continued fraction representation	Numerical error [s]
57	$3.08765281554 \times 10^{-8}$	
	[0; 39 | -e-1, -9, -27, e+1, 30]	5.5×10^{-16}
	[1.5; 36 | e+1, -6, -6, e+1, -6, -69]	2.4×10^{-16}
58	$1.57172667413 \times 10^{-8}$	
	[1.5; 36 | 18, 9]	7.5×10^{-12}
59	$1.07213682783 \times 10^{-8}$	
	[1.5; 36 | -12, 6, e+1, 9]	8.2×10^{-14}
60	0.000457010074873	
	[0; 48 | -36, -e-1, -e-1, 72]	8.3×10^{-12}
61	0.000352872361941	
	[0; 48 | -9, e+1, -6, 66]	6.5×10^{-11}
62	0.000268620051372	
	[0; 48 | -e-1, -e-1, 15, -6, 6]	8.3×10^{-10}
	[1.5; 45 | e+1, -e-1, -9, -6, e+1, 6, -135]	3.4×10^{-14}
63	0.000201250560525	
	[0; 48 | -e-1, e+1, 9, 6]	2.9×10^{-8}
	[1.5; 45 | e+1, e+1, -15, e+1, 6, e+1, -72]	4.0×10^{-14}
64	0.000148105207273	
	[1.5; 45 | 9, 30, -6, e+1, -6, e+1, e+1]	7.5×10^{-13}
65	0.00010680069345	
	[1.5; 45 | -96, -e-1, e+1]	6.3×10^{-9}
66	$7.52463910635 \times 10^{-5}$	
	[1.5; 45 | -6, -e-1, e+1, -e-1]	8.9×10^{-7}
67	$5.16080098697 \times 10^{-5}$	
	[0; 45 | e+1, -39, -e-1, 6, -e-1, -9, -6, 18]	1.6×10^{-15}
	[1.5; 45 | -e-1, 24]	6.5×10^{-8}
68	$3.43023299134 \times 10^{-5}$	
	[0; 45 | 9, -e-1, e+1, -27, -6]	3.1×10^{-11}
69	$2.19642303238 \times 10^{-5}$	
	[0; 45 | -24, -e-1, 6, e+1]	4.6×10^{-10}
70	$1.34440277078 \times 10^{-5}$	
	[0; 45 | -e-1, -e-1, 9, -e-1, e+1]	9.5×10^{-9}
	[1.5; 42 | e+1, -e-1, -15, 12, -e-1, -15]	6.3×10^{-13}
71	$7.78379764806 \times 10^{-6}$	
	[1.5; 42 | 9, -e-1, e+1, -e-1, e+1, -e-1, e+1, -e-1]	1.0×10^{-7}
	no continued fraction found	error 1.3%
72	$4.19936496159 \times 10^{-6}$	
	[1.5; 42 | -9, -e-1, e+1, -e-1, e+1, -e-1]	3.3×10^{-8}
73	$2.06385940319 \times 10^{-6}$	
	[0; 42 | 6, -e-1, 6, 12, 6, 12]	1.3×10^{-13}
	[1.5; 42 | -e-1, e+1, -e-1, 9, e+1, 12]	4.2×10^{-12}
74	$8.90602706652 \times 10^{-7}$	
	[0; 42 | -9, 6, e+1, -e-1]	9.1×10^{-10}
75	$3.14952372119 \times 10^{-7}$	
	[1.5; 39 | 18, e+1, 6, 9, -e-1, -6, 9, -9]	2.6×10^{-17}
76	0.000188796421136	
	[0; 48 | -e-1, e+1, -e-1, -e-1, -e-1, e+1]	7.0×10^{-8}
	[1.5; 45 | 6, -e-1, e+1, -e-1, 6]	6.1×10^{-8}
77	$4.27736612502 \times 10^{-5}$	
	[1.5; 45 | -e-1, e+1, -6, 15]	4.4×10^{-9}
78	$2.71131304737 \times 10^{-5}$	
	[0; 45 | 27, -27, e+1, e+1, 9]	2.0×10^{-13}
79	$1.63682712013 \times 10^{-5}$	
	[0; 45 | -6, -e-1, 138]	3.9×10^{-11}
80	$9.29238910568 \times 10^{-6}$	
	[0; 45 | -e-1, e+1, -e-1, 33]	6.2×10^{-10}
	[1.5; 42 | 6, -e-1, e+1, e+1, -15]	1.3×10^{-10}
81	$7.71363432628 \times 10^{-8}$	
	[0; 39 | 12, -e-1, -e-1, -9]	2.2×10^{-12}
82	$2.27896492047 \times 10^{-6}$	
	[0; 42 | e+1, e+1, e+1, e+1, -e-1, e+1, -6, -15]	1.3×10^{-13}
	[1.5; 42 | -e-1, e+1, e+1, 45]	2.9×10^{-11}

Line no.	Half-life [s] Continued fraction representation	Numerical error [s]
83	$8.9890699074 \times 10^{-7}$	
	[0; 42 | -9, 27]	7.5×10^{-11}
84	$1.00342682266 \times 10^{-5}$	
	[0; 45 | -e-1, e+1, 9, -9]	7.6×10^{-10}
	[1.5; 42 | e+1, e+1, -21]	2.9×10^{-9}
85	$4.9949353647 \times 10^{-6}$	
	[1.5; 42 | -30, -105, 6]	3.2×10^{-13}
86	$2.56787752588 \times 10^{-7}$	
	[1.5; 39 | -48, e+1]	2.5×10^{-10}
87	$2.130924682 \times 10^{-6}$	
	[1.5; 42 | -e-1, e+1, -6, 39, -30]	8.3×10^{-14}
88	$1.87941536444 \times 10^{-5}$	
	[0; 45 | -9, -e-1, e+1, -e-1]	1.4×10^{-7}
89	$6.75977355725 \times 10^{-7}$	
	[0; 42 | -e-1, -e-1, e+1, 6, -6, 12]	8.3×10^{-13}
	[1.5; 39 | e+1, -e-1, -312, 24]	5.5×10^{-15}
90	$4.44068922135 \times 10^{-6}$	
	[1.5; 42 | -12, -e-1, e+1, -e-1]	6.5×10^{-9}
91	0.000206379795319	
	[0; 48 | -e-1, e+1, e+1, -e-1, -e-1, -e-1, e+1, 909]	1.2×10^{-14}
	[1.5; 45 | e+1, e+1, e+1, -e-1, -e-1, -e-1, -12]	5.0×10^{-11}
92	0.000138358253934	
	[1.5; 45 | 12, -12, 111]	1.5×10^{-11}
93	$1.77507024651 \times 10^{-5}$	
	[0; 45 | -9, e+1, e+1]	2.7×10^{-8}
94	$8.88228892141 \times 10^{-5}$	
	[1.5; 45 | -12, -e-1, 9, -e-1, e+1, e+1, -6, -6]	4.5×10^{-13}
95	$5.38575897871 \times 10^{-5}$	
	[0; 45 | e+1, -9, -e-1, e+1, -e-1]	8.3×10^{-8}
	[1.5; 45 | -e-1, -27]	4.5×10^{-8}
96	0.000305633925905	
	[0; 48 | -6, 9, 15, 21, -e-1, -12, -e-1, -18]	1.9×10^{-16}
97	$8.41504407624 \times 10^{-6}$	
	[1.5; 42 | 6, 9, -24, 6, -33, -e-1, 12]	7.3×10^{-17}
98	0.000208446510258	
	[0; 48 | -e-1, 6, -e-1, e+1, -e-1]	8.3×10^{-7}
	[1.5; 45 | e+1, 6, -e-1, 6]	5.7×10^{-8}
99	$1.51979297614 \times 10^{-6}$	
	[0; 42 | 12, 6, e+1]	7.1×10^{-10}
100	$3.01276646481 \times 10^{-5}$	
	[0; 45 | 12, e+1, -e-1, e+1]	9.4×10^{-8}
101	0.000136454353714	
	[1.5; 45 | 12, 6, -e-1, 6, e+1, -6, -12]	1.6×10^{-12}
102	$3.0508238581 \times 10^{-6}$	
	[0; 42 | e+1, -e-1, 27]	7.5×10^{-10}
	[1.5; 42 | -e-1, -e-1, e+1, -6, -9]	8.1×10^{-11}
103	$5.62254364504 \times 10^{-6}$	
	[1.5; 42 | 99, -e-1, e+1]	6.6×10^{-10}
104	$2.476498555 \times 10^{-5}$	
	[0; 45 | 261, -e-1]	7.8×10^{-10}
105	$3.73281911013 \times 10^{-5}$	
	[0; 45 | 6, 6, -30, -6, -6]	2.1×10^{-12}
106	$5.45398678543 \times 10^{-5}$	
	[0; 45 | e+1, -9, 6, -e-1, -30]	4.0×10^{-11}
	[1.5; 45 | -e-1, -18, e+1]	7.3×10^{-8}
107	$7.75818387983 \times 10^{-5}$	
	[1.5; 45 | -9, e+1, -e-1, e+1, -e-1]	6.5×10^{-7}
108	0.000107827447468	
	[1.5; 45 | -147, -6, e+1]	4.3×10^{-10}
109	0.000335518263498	
	[0; 48 | -6, -e-1, e+1, -e-1, e+1]	1.9×10^{-6}