***LETTERS TO PROGRESS IN PHYSICS***

# On the Exact Solution Explaining the Accelerate Expanding Universe According to General Relativity

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A new method of calculation is applied to the frequency of a photon according to the travelled distance. It consists in solving the scalar geodesic equation (equation of energy) of the photon, and manifests gravitation, non-holonomity, and deformation of space as the intrinsic geometric factors affecting the photon's frequency. The solution obtained in the expanding space of Friedmann's metric manifests the exponential cosmological redshift: its magnitude increases, exponentially, with distance. This explains the accelerate expansion of the Universe registered recently by the astronomers. According to the obtained solution, the redshift reaches the ultimately high value $z = e^{\pi} - 1 = 22.14$ at the event horizon.

During the last three years, commencing in 2009, I published a series of research papers [1–5] wherein I went, step-by-step, in depth of the cosmological redshift problem. I targeted an explanation of the non-linearity of the cosmological redshift law and, hence, the accelerate expansion of the Universe. I suggested that the explanation may be found due to the space-time geometry, i.e. solely with the use of the geometric methods of the General Theory of Relativity.

Naturally, this is the most promising way to proceed in this problem. Consider the following: in 1927, Lemaître's theory [6] already predicted the linear reshift law in an expanding space of Friedmann's metric (a Friedmann universe). As was then shown by Lemaître, this theoretical result matches the linear redshift law registered in distant galaxies*. The anomalously high redshift registered in very distant Ia-type supernovae in the last decade [7–9] manifests the non-linearity of the redshift law. It was then interpreted as the accelerate expansion of our Universe. Thus, once the space-time geometry has already made Lemaître successful in explaining the linear redshift, we should expect a success with the non-linear redshift law when digging more in the theory.

Lemaître deduced the cosmological redshift on the basis of Einstein's field equation. The left-hand side of the equation manifests the space curvature, while the right-hand side describes the substance filling the space. In an expanding space, all objects scatter from each other with the velocity of the space expansion. Lemaître considered the simplest case of deforming spaces — the space of Friedmann's metric. Such a space is free of gravitational fields and rotation, but is curved due to its deformation (expansion or compression). Solving Einstein's equation for Friedmann's metric, Lemaître obtained the curvature radius $R$ of the space and the speed of its change $\dot{R}$. Then he calculated the redshift, assuming that it is a result of the Doppler effect on the scattering objects of the expanding Friedmann universe.

Lemaître's method of deduction would remain quite good, except for three drawbacks, namely —

1) It works only in deforming spaces, i.e. under the assumption that the cosmological redshift is a result of the Doppler effect in an expanding space. In static (non-deforming) spaces, this method does not work. In other words, herein is not a way to calculate how the frequency of a photon will change with the distance of the photon's travel in the space of a static cosmological metric (which is known to be of many kinds);
2) In this old method, the Doppler effect does not follow from the space (space-time) geometry but has the same formula as that of classical physics. Only the speed of change of the curvature radius with time $\dot{R}$ (due to the expansion of space) is used as the velocity of the light source. In other words, the Doppler formula of classical physics is assumed to be the same in an expanding Friedmann universe. This is a very serious simplification, because it is obvious that the Doppler effect should have a formula, which follows from the space geometry (Friedmann's metric in this case);
3) This method gives the linear redshift law — a straight line $z = \frac{\dot{R}}{c}$, which "digs" in the wall $\dot{R} = c$. As a result, the predicted cosmological redshift is limited by the numerical value $z_{max} = 1$. However, we know dozens of much more redshifted galaxies and quasars. In 2011, the highest redshift registered by the astronomers is $z = 10.3$ (the galaxy UDFj-39546284).

So, in his theory, Lemaître calculated the cosmological redshift in a roundabout way: by substituting, into the Doppler formula of classical physics, the speed of change of the curvature radius $\dot{R}$ he obtained his redshift law, i.e., by solving Einstein's equation for Friedmann's metric.

*According to the astronomical observations, spectral lines of distant galaxies and quasars are redshifted as if these objects scatter with the radial velocity $u = H_0 d$, which increases 72 km/sec per each megaparsec of the distance $d$ to the object. $H_0 = 72 \pm 8$ km/sec×Mpc $= (2.3 \pm 0.3) \times 10^{-18}$ sec$^{-1}$ is known as the Hubble constant. 1 parsec = $3.0857 \times 10^{18}$ cm $\simeq 3.1 \times 10^{18}$ cm.

In contrast to Lemaître, I suggested that the cosmological redshift law can be deduced in a more direct and profound way. It is as follows. The generally covariant geodesic equation — the four-dimensional equation of motion of a particle — can be projected onto the time line and the three-dimensional spatial section of an observer. As a result, we obtain the scalar geodesic equation, which is the equation of energy of the particle, and the vectorial geodesic equation (the three-dimensional equation of motion). The in-depth mathematical formalism of the said projection was introduced in 1944 by Zelmanov [10, 11], and is known as the theory of chronometric invariants*. Solving the scalar geodesic equation (equation of energy) of a photon, we shall obtain how the photon's energy and frequency change according to the remoteness of the signal's source to the observer. This is the *frequency shift law*, particular forms of which we can deduce by solving the scalar geodesic equation of a photon in the space of any particular metric.

The same method of deduction may be applied to mass-bearing particles. By solving the scalar geodesic equation for a mass-bearing particle ("stone-like objects"), we shall obtain that the relativistic mass of the object changes according to the remoteness to the observer in the particular space.

First, following this new way of deduction, I showed that the redshift, observed by the astronomers, should be present in a space which rotates at the velocity of light [1, 2]. In this case, the Hubble constant plays a rôle of the frequency of the rotation. The redshift due to the space rotation should be present even if the space is static (non-deforming).

The light-speed rotation is only attributed to the so-called isotropic region of space (home of the trajectories of light). This can be shown by "adapting" the space metric to the isotropic space condition (equality of the metric to zero), which makes a replacement among the components $g_{00}$ and $g_{0i}$ of the fundamental metric tensor $g_{\alpha\beta}$. In Minkowski's space, this replacement means that the isotropic region has a non-diagonal metric, where $g_{00}=0$, $g_{0i}=1$, $g_{11}=g_{22}=g_{33}=-1$. Such isotropic metrics were studied in the 1950's by Petrov: see §25 and the others in his *Einstein Spaces* [12]. More insight into this subject is provided in my third paper on the redshift problem [3].

On the other hand, a regular sublight-speed observer shall observe all events according to the components of the fundamental metric tensor $g_{\alpha\beta}$ of his own (non-isotropic) space — home of "solid objects". Therefore, I then continued the research study with the regular metrics, which are not "adapted" to the isotropic space condition.

In two recent papers [4, 5], I solved the scalar geodesic equation for mass-bearing particles and massless particles (photons), in the most studied particular spaces: in the space of Schwarzschild's mass-point metric, in the space of an electrically charged mass-point (the Reissner-Nordström metric), in the rotating space of Gödel's metric (a homogeneous distribution of ideal liquid and physical vacuum), in the space of a sphere of incompressible liquid (Schwarzschild's metric), in the space of a sphere filled with physical vacuum (de Sitter's metric), and in the deforming space of Friedmann's metric (empty or filled with ideal liquid and physical vacuum).

Herein I shall go into the details of just one of the obtained solutions — that in an expanding Friedmann universe, — wherein I obtained the exponential cosmological redshift, thus giving a theoretical explanation to the accelerate expansion of the Universe registered recently by the astronomers.

The other obtained solutions shall be omitted from this presentation. The readers who are curious about them are directly referred to my two recent publications [4, 5].

So, according to Zelmanov's chronometrically invariant formalism [10, 11], any four-dimensional (generally covariant) quantity is presented with its observable projections onto the line of time and the three-dimensional spatial section of an observer. This is as well true about the generally covariant geodesic equation. As Zelmanov obtained, the projected (chronometrically invariant) geodesic equations of a mass-bearing particle, whose relativistic mass is $m$, are

$$\frac{dm}{d\tau} - \frac{m}{c^2} F_i \mathrm{v}^i + \frac{m}{c^2} D_{ik} \mathrm{v}^i \mathrm{v}^k = 0\,, \tag{1}$$

$$\frac{d(m\mathrm{v}^i)}{d\tau} - mF^i + 2m\left(D^i_k + A^{\cdot i}_{k\cdot}\right)\mathrm{v}^k + m\triangle^i_{nk}\mathrm{v}^n\mathrm{v}^k = 0\,, \tag{2}$$

while the projected geodesic equations of a massless particle-photon, whose relativistic frequency is $\omega$, have the form

$$\frac{d\omega}{d\tau} - \frac{\omega}{c^2} F_i c^i + \frac{\omega}{c^2} D_{ik} c^i c^k = 0\,, \tag{3}$$

$$\frac{d(\omega c^i)}{d\tau} - \omega F^i + 2\omega\left(D^i_k + A^{\cdot i}_{k\cdot}\right)c^k + \omega\triangle^i_{nk}c^n c^k = 0\,. \tag{4}$$

Here $d\tau = \sqrt{g_{00}}\,dt - \frac{1}{c^2}\,v_i\,dx^i$ is the observable time, which depends on the gravitational potential $\mathrm{w} = c^2\,(1-\sqrt{g_{00}}\,)$ and the linear velocity $v_i = -\frac{c\,g_{0i}}{\sqrt{g_{00}}}$ of the rotation of space. Four factors affect the particles: the gravitational inertial force $F_i$, the angular velocity $A_{ik}$ of the rotation of space, the deformation $D_{ik}$ of space, and the Christoffel symbols $\Delta^i_{jk}$ (expressing the space non-uniformity). According to the scalar geodesic equation (equation of energy), two factors, $F_i$ and $D_{ik}$, affect the energy of the particle. They are determined [10, 11] as

$$F_i = \frac{1}{\sqrt{g_{00}}}\left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right), \qquad \sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2}\,, \tag{5}$$

$$D_{ik} = \frac{1}{2\sqrt{g_{00}}}\frac{\partial h_{ik}}{\partial t}\,,\quad D^{ik} = -\frac{1}{2\sqrt{g_{00}}}\frac{\partial h^{ik}}{\partial t}\,,\quad D = \frac{\partial \ln\sqrt{h}}{\sqrt{g_{00}}\,\partial t}\,, \tag{6}$$

where $D = h^{ik}D_{ik}$, while $h_{ik}$ is the chr.inv.-metric tensor

$$h_{ik} = -g_{ik} + \frac{1}{c^2}\,v_i v_k\,, \quad h^{ik} = -g^{ik}, \quad h^i_k = \delta^i_k\,. \tag{7}$$

*The property of chronometric invariance means that the quantity is invariant along the three-dimensional spatial section of the observer.

The geodesic equations of mass-bearing and massless particles have the same form. Only the sublight velocity $v^i$ and the relativistic mass $m$ are used for mass-bearing particles, instead of the observable velocity of light $c^i$ and the frequency $\omega$ of the photon. Therefore, they can be solved in the same way to yield similar solutions.

My suggestion is then self-obvious. By solving the scalar geodesic equation of a mass-bearing particle in each of the so-called cosmological metrics, we should obtain how the observed (relativistic) mass of the particle changes according to the distance from the observer in each of these universes. I will further refer to it as the *cosmological mass-defect*. The scalar geodesic equation of a photon should give the formula of the frequency shift of the photon according to the travelled distance (the *cosmological frequency shift*).

Consider the space of Friedmann's metric

$$ds^2 = c^2 dt^2 - R^2 \left[ \frac{dr^2}{1-\kappa r^2} + r^2 \left( d\theta^2 + \sin^2\theta \, d\varphi^2 \right) \right], \tag{8}$$

wherein Lemaître [6] deduced the linear redshift law. Here $R = R(t)$ is the curvature radius of the space, while $\kappa = 0, \pm 1$ is the curvature factor. If $\kappa = -1$, the three-dimensional subspace possesses hyperbolic (open) geometry. If $\kappa = 0$, its geometry is flat. If $\kappa = +1$, it has elliptic (closed) geometry.

As is seen from the metric, such a space — a Friedmann universe — is free of ($g_{00} = 1$) and rotation ($g_{0i} = 0$), but is deforming, which reveals the functions $g_{ik} = g_{ik}(t)$. It may expand, compress, or oscillate. Such a space can be empty, or filled with a homogeneous and isotropic distribution of ideal (non-viscous) liquid in common with physical vacuum ($\Lambda$-field), or filled with one of the media.

Friedmann's metric is expressed through a "homogeneous" radial coordinate $r$. This is the regular radial coordinate divided by the curvature radius, whose scales change according to the deforming space. As a result, the homogeneous radial coordinate $r$ does not change its scale with time.

The scalar geodesic equation for a photon travelling along the radial direction in a Friedmann universe takes the form

$$\frac{d\omega}{d\tau} + \frac{\omega}{c^2} D_{11} c^1 c^1 = 0\,, \tag{9}$$

where $c^1$ [sec$^{-1}$] is the solely nonzero component of the observable "homogeneous" velocity of the photon. The square of the velocity is $h_{11} c^1 c^1 = c^2$ [cm$^2$/sec$^2$]. We calculate the components of the chr-inv.-metric tensor $h_{ik}$ according to Friedmann's metric. After some algebra, we obtain

$$h_{11} = \frac{R^2}{1-\kappa r^2}\,, \quad h_{22} = R^2 r^2, \quad h_{33} = R^2 r^2 \sin^2\theta\,, \tag{10}$$

$$h = \det\|h_{ik}\| = h_{11} h_{22} h_{33} = \frac{R^6 r^4 \sin^2\theta}{1-\kappa r^2}\,, \tag{11}$$

$$h^{11} = \frac{1-\kappa r^2}{R^2}\,, \quad h^{22} = \frac{1}{R^2 r^2}\,, \quad h^{33} = \frac{1}{R^2 r^2 \sin^2\theta}\,. \tag{12}$$

With these formulae of the components of $h_{ik}$, we obtain the tensor of the space deformation $D_{ik}$ in a Friedmann universe. According to the definition (6), we obtain

$$D = \frac{3\dot{R}}{R}\,, \quad D_{11} = \frac{R\dot{R}}{1-\kappa r^2}\,, \quad D_1^1 = \frac{\dot{R}}{R}\,. \tag{13}$$

The curvature radius as a function of time, $R = R(t)$, can be found by assuming a particular type of the space deformation. The trace of the tensor of the space deformation, $D = h^{ik} D_{ik}$, is by definition the speed of relative deformation of the volume. A volume, which is deforming freely, expands or compresses so that its volume undergoes equal relative changes with time

$$D = const\,, \tag{14}$$

which, in turn, is a world-constant of the space. This is the primary type of space deformation: I suggest referring to it as the *constant* (*homotachydioncotic*) *deformation**.

Consider a constant-deformation (homotachydioncotic) Friedmann universe. With $D = \frac{3\dot{R}}{R}$ according to Friedmann's metric, we have $\frac{\dot{R}}{R} = A = const$ in this case. We thus arrive at the equation $\frac{1}{R}\, dR = A dt$, which is $d \ln R = A dt$. Assuming the curvature radius at the moment of time $t = t_0$ to be $a_0$, we obtain

$$R = a_0 e^{At}, \qquad \dot{R} = a_0 A e^{At}\,, \tag{15}$$

and, therefore,

$$D = 3A\,, \quad D_{11} = \frac{a_0^2 A e^{2At}}{1-\kappa r^2}\,, \quad D_1^1 = A\,. \tag{16}$$

Return now to the scalar geodesic equation of a photon in a Friedmann universe, which is formula (9). Because $g_{00} = 1$ and $g_{0i} = 0$ according to Friedmann's metric, we have $d\tau = dt$. Therefore, because $h_{11} c^1 c^1 = c^2$, the scalar geodesic equation transforms into $h_{11} \frac{d\omega}{dt} + \omega D_{11} = 0$. From here we obtain $h_{11} \frac{d\omega}{\omega} = -D_{11}\, dt$, and, finally, the equation

$$h_{11}\, d \ln \omega = -D_{11}\, dt\,. \tag{17}$$

By substituting $h_{11}$ and $D_{11}$, we obtain

$$d \ln \omega = -A\, dt\,, \tag{18}$$

where $A = \frac{\dot{R}}{R}$ is a world-constant of the Friedmann space.

As is seen, this equation is independent of the curvature factor $\kappa$. Therefore, its solution will be common for the hyperbolic ($\kappa = -1$), flat ($\kappa = 0$), and elliptic ($\kappa = +1$) geometry of the Friedmann space.

This equation solves as $\ln \omega = -At + \ln B$, where $B$ is an integration constant. So forth, we obtain $\omega = B e^{-At}$. We calculate the integration constant $B$ from the condition $\omega = \omega_0$

*I refer to this kind of universe as *homotachydioncotic* (in Greek — ομοταχυδιογκωτικό). This term originates from *homotachydioncosis* — ομοταχυδιόγκωσης — volume expansion with a constant speed, from όμο which is the first part of όμοιος (omeos) — the same, ταχύτητα — speed, διόγκωση — volume expansion, while compression can be considered as negative expansion.

at the initial moment of time $t = t_0 = 0$. We have $B = \omega_0$. Thus, we obtain the final solution $\omega = \omega_0\, e^{-At}$ of the scalar geodesic equation. Expanding the world-constant $A = \frac{\dot{R}}{R}$ and the duration of the photon's travel $t = \frac{d}{c}$, we have

$$\omega = \omega_0\, e^{-\frac{\dot{R}}{R}\frac{d}{c}}, \tag{19}$$

where $d = ct$ [cm] is the distance to the source emitting the photon. At small distances (and durations) of the photon's travel, the obtained solution takes the linearized form

$$\omega \simeq \omega_0 \left(1 - \frac{\dot{R}}{R}\frac{d}{c}\right). \tag{20}$$

The obtained solution manifests that photons travelling in a constant-deformation (homotachydiastolic) Friedmann universe which expands ($A > 0$) should lose energy and frequency with each mile of the travelled distance. The energy and frequency loss law is exponential (19) at large distances of the photon's travel, and is linear (20) at small distances.

Accordingly, the photon's frequency should be redshifted. The magnitude of the redshift increases with the travelled distance. This is a *cosmological redshift*, in other words.

Let a photon have a wavelength $\lambda_0 = \frac{c}{\omega_0}$ being emitted by a distantly located source, while its frequency registered at the arrival is $\lambda = \frac{c}{\omega}$. Then we obtain the magnitude $z = \frac{\lambda-\lambda_0}{\lambda_0} = \frac{\omega_0-\omega}{\omega}$ of the redshift in an expanding constant-deformation (homotachydiastolic) Friedmann universe. It is

$$z = e^{\frac{\dot{R}}{R}\frac{d}{c}} - 1\,, \tag{21}$$

which is an *exponential redshift law*. At small distances of the photon travel, it takes the linearized form

$$z \simeq \frac{\dot{R}}{R}\frac{d}{c}\,. \tag{22}$$

which manifests a *linear redshift law*.

If such a universe compresses ($A < 0$), this effect changes its sign, thus becoming a *cosmological blueshift*.

Our linearized redshift formula (22) is the same as $z = \frac{\dot{R}}{R}\frac{d}{c}$ obtained by Lemaître [6], the "father" of the theory of an expanding universe. He followed, however, another way of deduction which limited him only to the linear formula. He therefore was confined to believing in the linear redshift law alone.

The ultimately high redshift $z_{\max}$, which could be registered in our Universe, is calculated by substituting the ultimately large distance into the redshift law. If following Lemaître's theory [6], $z_{\max}$ should follow from the linear redshift law $z = \frac{\dot{R}}{R}\frac{d}{c} = A\frac{d}{c}$. Because $A = \frac{\dot{R}}{R}$ is the world-constant of the Friedmann space, the ultimately large curvature radius $R_{\max}$ is determined by the ultimately high velocity of the space expansion which is the velocity of light $\dot{R}_{\max} = c$. Hence, $R_{\max} = \frac{c}{A}$. The ultimately large distance $d_{\max}$ (the event horizon) is regularly determined from the linear law for scattering galaxies, which is $u = H_0\, d$: the scattering velocity $u$ should reach the velocity of light ($u = c$) at the event horizon ($d = d_{\max}$).* The law $u = H_0\, d$ is known due to galaxies and quasars whose scattering velocities are much lower than the velocity of light. Despite this fact, the empirical linear law $u = H_0\, d$ is regularly assumed to be valid upto the event horizon. Thus, they obtain $d_{\max} = \frac{c}{H_0} = (1.3 \pm 0.2)\times 10^{28}$ cm. Then they assume the linear coefficient $H_0$ of the empirical law of the scattering galaxies to be the world-constant $A = \frac{\dot{R}}{R}$, which follows from the space geometry. As a result, they obtain $d_{\max} = R_{\max}$ and $z_{\max} = H_0 \frac{d_{\max}}{c} = 1$ due to the linear redshift law. How then to explain the dozens of very distant galaxies and quasars, whose redshift is much higher than $z = 1$?

On the other hand, it is obvious that the ultimately high redshift $z_{\max}$, ensuing from the space (space-time) geometry, should be a result of the laws of relativistic physics. In other words, $z = z_{\max}$ should follow from not a straight line $z = \frac{\dot{R}}{R}\frac{d}{c} = H_0\frac{d}{c} = \frac{u}{c}$, which digs in the vertical "wall" $u = c$, but from a non-linear relativistic function.

In this case, the Hubble constant $H_0$ remains a linear coefficient only in the pseudo-linear beginning of the real redshift law arc, wherein the velocities of scattering are small in comparison with the velocity of light. At velocities of scattering close to the velocity of light (close to the event horizon), the Hubble constant $H_0$ loses the meaning of the linear coefficient and the world-constant $A$ due to the increasing non-linearity of the real redshift law.

Such a non-linear formula has been found in the framework of our theory alluded to here. This is the exponential redshift law (21), which then gives the Lemaître linear redshift law (22) as an approximation at small distances.

We now use the exponential redshift law (21) to calculate the ultimately high redshift $z_{\max}$, which could be conceivable in an expanding Friedmann space of the constant-deformation type. The event horizon $d = d_{\max}$ is determined by the world-constant $A = \frac{\dot{R}}{R}$ of the space. Thus, the ultimately large curvature radius is $R_{\max} = \frac{c}{A}$, while the distance corresponding to $R_{\max}$ on the hypersurface is $d_{\max} = \pi R_{\max} = \frac{\pi c}{A}$. Suppose now that a photon has arrived from a source, which is located at the event horizon. According to the exponential redshift law (21), the photon's redshift at the arrival should be

$$z_{\max} = e^{\frac{\dot{R}}{R}\frac{d_{\max}}{c}} - 1 = e^{\pi} - 1 = 22.14\,, \tag{23}$$

which is the ultimately high redshift in such a universe.

The deduced exponential increase of the redshift implies the accelerate expansion of space. This "key prediction" of our theory was surely registered by the astronomers in the last decade: the very distant Ia-type supernovae manifested the increasing non-linearity of the redshift law and, hence, the accelerate expansion of our Universe [7–9].

*The law for scattering galaxies dictates that distant galaxies and quasars scatter with the radial velocity $u = H_0\, d$, increasing as 72 km/sec per each megaparsec. The linear coefficient of the law, $H_0 = 72 \pm 8$ km/sec×Mpc $= (2.3 \pm 0.3)\times 10^{-18}$ sec$^{-1}$, is known as the Hubble constant.

We therefore can conclude that the observed non-linear redshift law and the accelerate expansion of space have been explained in the constant-deformation (homotachydioncotic) Friedmann universe.

The deduced exponential law points out the ultimately high redshift $z_{\max} = 22.14$ for objects located at the event horizon. The highest redshifted objects, registered by the astronomers, are now the galaxies UDFj-39546284 ($z = 10.3$) and UDFy-38135539 ($z = 8.55$). According to our theory, they are still distantly located from the "world end". We therefore shall expect, with years of further astronomical observation, more "high redshifted surprises" which will approach the upper limit $z_{\max} = 22.14$.

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In analogy to massless particles-photons, we can consider the scalar geodesic equation of a mass-bearing particle. In a Friedmann universe this equation takes the form

$$\frac{dm}{d\tau} + \frac{m}{c^2} D_{11} \mathrm{v}^1 \mathrm{v}^1 = 0 \,, \tag{24}$$

which, alone, is non-solvable. This is because mass-bearing particles can travel at any sub-light velocity, which is therefore an unknown variable of the equation.

This problem vanishes in a constant-deformation Friedmann universe, by the assumption according to which massive bodies travel not arbitrarily, but are only being carried out with the expanding (or compressing) space. In this particular case, particles travel with the velocity of space deformation, $\mathrm{v} = \dot{R}$. Because $\mathrm{v}^2 = h_{ik}\mathrm{v}^i\mathrm{v}^k$, we have $h_{ik}\mathrm{v}^i\mathrm{v}^k = \dot{R}^2$. Thus, and with $d\tau = dt$ according to Friedmann's metric, the scalar geodesic equation of mass-bearing particles transforms into $h_{11} \frac{dm}{dt} + \frac{m}{c^2} D_{11} \dot{R}^2 = 0$, i.e. $h_{11} \frac{dm}{m} = -\frac{\dot{R}^2}{c^2} D_{11}\, dt$. We obtain

$$h_{11}\, d\ln m = -\frac{\dot{R}^2}{c^2} D_{11}\, dt \,. \tag{25}$$

Then, expanding $R$, $\dot{R}$ (15), and $D_{11}$ (16) according to a constant-deformation space, we obtain the scalar geodesic equation in the form

$$d\ln m = -\frac{a_0^2 A^3 e^{2At}}{c^2}\, dt \,, \tag{26}$$

where $A = \frac{\dot{R}}{R} = const$. It solves as $\ln m = -\frac{a_0^2 A^2}{2c^2} e^{2At} + \ln B$, where the integration constant $B$ can be found from the condition $m = m_0$ at the initial moment of time $t = t_0 = 0$. After some algebra, we obtain the final solution of the scalar geodesic equation. It is the double-exponential function

$$m = m_0\, e^{-\frac{a_0^2 A^2}{2c^2}\left(e^{2At} - 1\right)}, \tag{27}$$

which, at a small distance to the object, takes the linearized form

$$m \simeq m_0 \left(1 - \frac{a_0^2 A^3 t}{c^2}\right). \tag{28}$$

The obtained solution manifests the *cosmological mass-defect* in a constant-deformation (homotachydiastolic) Friedmann universe: the more distant an object we observe in an expanding universe is, the less should be its observed mass $m$ to its real mass $m_0$. Contrarily, the more distant an object we observe in a compressing universe, the heavier should be this object according to observation.

Our Universe seems to be expanding. This is due to the cosmological redshift registered in the distant galaxies and quasars. Therefore, according to the cosmological mass-defect deduced here, we should expect distantly located cosmic objects to be much heavier than we estimate on the basis of astronomical observations. The magnitude of the expected mass-defect should be, according to the obtained solution, in the order of the redshift of the objects.

The cosmological mass-defect complies with the cosmological redshift. Both of these effects are deduced in the same way, by solving the scalar geodesic equation for mass-bearing and massless particles, respectively. One effect cannot be in the absence of the other, because the geodesic equations have the same form. This is a basis of the space (space-time) geometry, in other words. Therefore, once the astronomers register the linear redshift law and its non-linearity in very distant cosmic objects, they should also find the corresponding cosmological mass-defect according to the solution presented here. Once the cosmological mass-defect is discovered, we will be able to say, surely, that our Universe is an expanding Friedmann universe of the constant-deformation (homotachydiastolic) type.

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