Relativistic Dynamics in the Vicinity of a Uniformly Charged Sphere

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The motion of test and photons in the vicinity of a uniformly charged spherically symmetric mass distribution is studied using a newly developed relativistic dynamical approach. The derived expressions for the mechanical energy and acceleration vector of test particles have correction terms of all orders of c^{-2}. The expression for the gravitational spectral shift also has additional terms which are functions of the electric potential on the sphere.

1 Introduction

In a recent article [1], the relativistic dynamical approach to the study of classical mechanics in homogeneous spherical distributions of mass (Schwarzschild's gravitational field) was introduced. Here, the relativistic dynamical theory of a combined gravitational and electric field within homogeneous spherical distributions of mass is developed.

2 Motion of test particles

According to Maxwell's theory of electromagnetism, the electric potential energy for a particle of non-zero rest mass in an electric field V_e is given by

$$V_e = q\,\Phi_e\,, \tag{1}$$

where q is the electric charge of the particle and Φ_e is the electric scalar potential. Also, from Newton's dynamical theory, it is postulated [2] that the instantaneous mechanical energy for test particles in combined gravitational and electric fields is defined by

$$E = T + V_g + V_e\,, \tag{2}$$

where T is the total relativistic kinetic energy and V_g is the gravitational potential. From [1], T and V_g in Schwarzschild's gravitational field are given by

$$T = \left[\left(1-\frac{u^2}{c^2}\right)^{-1/2} - 1\right] m_0 c^2 \tag{3}$$

and the instantaneous relativistic gravitational potential energy (V_g) for a particle of nonzero rest mass is

$$V_g = m_p \Phi_e = -\left(1-\frac{u^2}{c^2}\right)^{-1/2} \frac{GMm_0}{r}\,, \tag{4}$$

where $\Phi_e = \frac{-GM}{r}$ is the gravitational scalar potential in a spherically symmetric gravitational field, $r > R$, the radius of the homogeneous sphere, G is the universal gravitational constant, c is the speed of light in vacuum, m_p is the passive mass of the test particle, M is the mass of the static homogeneous spherical mass, m_0 is the rest mass of the test particle and u is the instantaneous velocity of the test particle. Also, for a uniformly charged spherically symmetric mass the electric potential energy is given as

$$V_e = \frac{qQ}{4\pi\epsilon_0 r}\,, \tag{5}$$

where Q is the total charge on the sphere and q is the charge on the test particle. Thus, the instantaneous mechanical energy for the test particle can be written more explicitly as

$$E = m_0 c^2 \left[\left(1-\frac{GM}{c^2 r}\right)\left(1-\frac{u^2}{c^2}\right)^{-1/2} - 1\right] + \frac{qQ}{4\pi\epsilon_0 r}\,. \tag{6}$$

The expression for the instantaneous mechanical energy has post Newton and post Einstein correction terms of all orders of c^{-2}. The relativistic dynamical equation of motion for particles of non-zero rest masses in combined electric and gravitational fields is given as [2]

$$\frac{d}{d\tau}\overline{P} = -m_p \overline{\nabla}\Phi_g - q\overline{\nabla}\Phi_e\,, \tag{7}$$

where $\overline{P}$ is the instantaneous linear momentum of the test particles . Thus, in this field, the relativistic dynamical equation of motion for test particles is given explicitly as

$$\frac{d}{d\tau}\left[\left(1-\frac{u^2}{c^2}\right)^{-1/2}\overline{u}\right] = -\left(1-\frac{u^2}{c^2}\right)^{-1/2}\overline{\nabla}\Phi_g - \frac{q}{m_0}\overline{\nabla}\Phi_e \tag{8}$$

or

$$\overline{a} + \frac{1}{2c^2}\left(1-\frac{u^2}{c^2}\right)^{-1}\frac{d}{d\tau}(u^2)\overline{u} = \frac{GM}{r^2} - \frac{q}{m_0}\left(1-\frac{u^2}{c^2}\right)^{1/2}\overline{\nabla}\Phi_e\,, \tag{9}$$

where $\overline{a}$ is the instantaneous acceleration vector of the test particles and thus the time equation of motion is obtained as

$$a_{x_0} + \frac{1}{2c^2}\left(1-\frac{u^2}{c^2}\right)^{-1}\frac{d}{d\tau}(u^2)u_{x^0} = 0\,. \tag{10}$$

The azimuthal equation of motion is

$$\dot{r}\sin\theta\,\dot{\phi} + r\cos\theta\,\dot{\theta}\,\dot{\phi} + r\sin\theta\,\ddot{\phi} + \frac{1}{2c^2}\left(1-\frac{u^2}{c^2}\right)^{-1}\frac{d}{d\tau}(u^2)\,u_\phi = 0\,. \quad (11)$$

The polar equation of motion is given as

$$r\,\ddot{\theta} + \dot{r}\,\dot{\theta} + \frac{1}{2c^2}\left(1-\frac{u^2}{c^2}\right)^{-1}\frac{d}{d\tau}(u^2)\,u_\theta = 0 \quad (12)$$

and the radial equation of motion is

$$a_r + \frac{1}{2c^2}\left(1-\frac{u^2}{c^2}\right)^{-1}\frac{d}{d\tau}(u^2)u_r = -\frac{GM}{r^2} - \frac{q}{m_0}\left(1-\frac{u^2}{c^2}\right)^{1/2}\overline{\nabla}\Phi_e\,. \quad (13)$$

As in [1], the equations have correction terms not found in the general relativistic approach.It is also worth remarking that the homogeneous charge distribution on the sphere and the charge on the test particle affects only the radial component of the motion and hence the other components are the same as those of an uncharged sphere [1].

3 Motion of photons

From [1], it can be deduced that the instantaneous gravitational potential energy of a photon is given as

$$V_g = -\frac{h\,\nu}{c^2}\frac{GM}{r}\,. \quad (14)$$

The instantaneous electric potential energy of the photon is given [2] as

$$V_e = -\frac{h\,\nu}{c^2}\,\overline{\nabla}\Phi_e \quad (15)$$

or more explicitly in this field as

$$V_e = -\frac{h\nu}{c^2}\frac{Q}{4\pi\epsilon_0 r}\,. \quad (16)$$

Also, the instantaneous kinetic energy of the photon [1] is given as

$$T = h(\nu-\nu_0)\,. \quad (17)$$

Thus, the instantaneous mechanical energy of a photon in this combined gravitational and electric field is obtained as

$$E = h(\nu-\nu_0) - \frac{h\nu}{c^2 r}\left(GM + \frac{Q}{4\pi\epsilon_0}\right). \quad (18)$$

Suppose at $r = r_0$, $E = E_0$ then

$$E_0 = -\frac{kh\nu_0}{c^2 r_0}\,, \quad (19)$$

where

$$k = GM + \frac{Q}{4\pi\epsilon_0}\,. \quad (20)$$

Thus, from the principle of conservation of mechanical energy

$$-\frac{kh\nu_0}{c^2 r_0} = h(\nu-\nu_0) - \frac{kh\nu}{c^2 r} \quad (21)$$

or

$$\nu = \nu_0\left(1-\frac{k}{c^2 r_0}\right)\left(1-\frac{k}{c^2 r}\right)^{-1}. \quad (22)$$

Equation (22) is the expression for spectral shift in this field with contributions from the gravitational and electric potentials. It has corrections of all orders of c^{-2}.

Also, for photons, the instantaneous linear momentum is given [1] as

$$\overline{P} = \frac{h\nu}{c^2}\,\overline{u}\,. \quad (23)$$

Hence, as in Newton's dynamical theory, the equation of motion of photons in this field is obtained from equation (7) as

$$\frac{d}{d\tau}\,(\nu\overline{u}) = -\nu\,\overline{\nabla}\Phi_g - \frac{qc^2}{h}\,\overline{\nabla}\Phi_e\,. \quad (24)$$

Thus the presence of an electric field introduces an additional term to the expression for the equation of motion of photons.

4 Conclusion

This article provides a crucial link between gravitational and electric fields. It also introduces, hither to unknown corrections of all orders of c^{-2} to the expressions of instantaneous mechanical energy, spectral shift and equations of motion for test particles and photons in combined spherically symmetric gravitational and electric field.

Submitted on August 17, 2012 / Accepted on August 22, 2012

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