Derivation of Electromagnetism from the Elastodynamics of the Spacetime Continuum

Pierre A. Millette
University of Ottawa (alumnus), Ottawa, Canada. E-mail: PierreAMillette@alumni.uottawa.ca

We derive Electromagnetism from the Elastodynamics of the Spacetime Continuum based on the identification of the theory's antisymmetric rotation tensor with the electromagnetic field-strength tensor. The theory provides a physical explanation of the electromagnetic potential, which arises from transverse (shearing) displacements of the spacetime continuum, in contrast to mass which arises from longitudinal (dilatational) displacements. In addition, the theory provides a physical explanation of the current density four-vector, as the 4-gradient of the volume dilatation of the spacetime continuum. The Lorentz condition is obtained directly from the theory. In addition, we obtain a generalization of Electromagnetism for the situation where a volume force is present, in the general non-macroscopic case. Maxwell's equations are found to remain unchanged, but the current density has an additional term proportional to the volume force.

1 Introduction

Since Einstein first published his Theory of General Relativity in 1915, the problem of the unification of Gravitation and Electromagnetism has been and remains the subject of continuing investigation (see for example [1–9] for recent attempts). The Elastodynamics of the Spacetime Continuum [10, 11] is based on the application of a continuum mechanical approach to the spacetime continuum *(STC)*. Electromagnetism is found to come out naturally from the theory in a straightforward manner.

In this paper, we derive Electromagnetism from the Elastodynamics of the Spacetime Continuum (*STCED*). This theory thus provides a unified description of the spacetime deformation processes underlying general relativistic Gravitation [11] and Electromagnetism, in terms of spacetime continuum displacements resulting from the strains generated by the energy-momentum stress tensor.

1.1 A note on units and constants

In General Relativity and in Quantum Electrodynamics, it is customary to use "geometrized units" and "natural units" respectively, where the principal constants are set equal to 1. The use of these units facilitates calculations since cumbersome constants do not need to be carried throughout derivations. In this paper, all constants are retained in the derivations, to provide insight into the nature of the equations being developed.

In addition, we use rationalized MKSA units for Electromagnetism, as the traditionally used Gaussian units are gradually being replaced by rationalized MKSA units in more recent textbooks (see for example [12]). Note that the electromagnetic permittivity of free space ϵ_{em}, and the electromagnetic permeability of free space μ_{em} are written with "*em*" subscripts as the "0" subscripts are used in *STCED* constants. This allows us to differentiate between for example μ_{em}, the electromagnetic permeability of free space, and μ_0, the Lamé elastic constant for the shear modulus of the spacetime continuum.

2 Theory of Electromagnetism from *STCED*

2.1 Electromagnetic field strength

In the Elastodynamics of the Spacetime Continuum, the antisymmetric rotation tensor $\omega^{\mu\nu}$ is given by [11]

$$\omega^{\mu\nu} = \frac{1}{2}\left(u^{\mu;\nu} - u^{\nu;\mu}\right) \tag{1}$$

where u^μ is the displacement of an infinitesimal element of the spacetime continuum from its unstrained position x^μ. This tensor has the same structure as the electromagnetic field-strength tensor $F^{\mu\nu}$ defined as [13, see p. 550]:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \tag{2}$$

where A^μ is the electromagnetic potential four-vector $(\phi, \vec{A})$, ϕ is the scalar potential and $\vec{A}$ the vector potential.

Identifying the rotation tensor $\omega^{\mu\nu}$ with the electromagnetic field-strength tensor according to

$$F^{\mu\nu} = \varphi_0 \omega^{\mu\nu} \tag{3}$$

leads to the relation

$$A^\mu = -\frac{1}{2}\varphi_0 u^\mu_\perp \tag{4}$$

where the symbolic subscript $\perp$ of the displacement u^μ indicates that the relation holds for a transverse displacement (perpendicular to the direction of motion) [11].

Due to the difference in the definition of $\omega^{\mu\nu}$ and $F^{\mu\nu}$ with respect to their indices, a negative sign is introduced, and is attributed to (4). This relation provides a physical explanation for the electromagnetic potential: it arises from transverse (shearing) displacements of the spacetime continuum, in contrast to mass which arises from longitudinal (dilatational) displacements of the spacetime continuum [11]. Sheared spacetime is manifested as electromagnetic potentials and fields.

2.2 Maxwell's equations and the current density four-vector

Taking the divergence of the rotation tensor of (1), gives

$$\omega^{\mu\nu}{}_{;\mu} = \frac{1}{2}\,(u^{\mu;\nu}{}_{\mu} - u^{\nu;\mu}{}_{\mu}). \tag{5}$$

Recalling (28) from Millette [11], viz.

$$\mu_0 u^{\nu;\mu}{}_{\mu} + (\mu_0 + \lambda_0) u^{\mu}{}_{;\mu}{}^{\nu} = -X^{\nu} \tag{6}$$

where X^ν is the volume force and λ_0 and μ_0 are the Lamé elastic constants of the spacetime continuum, substituting for $u^{\nu;\mu}{}_{\mu}$ from (6) into (5), interchanging the order of partial differentiation in $u^{\mu;\nu}{}_{\mu}$ in (5), and using the relation $u^{\mu}{}_{;\mu} = \varepsilon^{\mu}{}_{\mu} = \varepsilon$ from (19) of [11], we obtain

$$\omega^{\mu\nu}{}_{;\mu} = \frac{2\mu_0 + \lambda_0}{2\mu_0}\,\varepsilon^{;\nu} + \frac{1}{2\mu_0}\,X^{\nu}. \tag{7}$$

As seen in [11], in the macroscopic local case, the volume force X^ν is set equal to zero to obtain the macroscopic relation

$$\omega^{\mu\nu}{}_{;\mu} = \frac{2\mu_0 + \lambda_0}{2\mu_0}\,\varepsilon^{;\nu} \tag{8}$$

Using (3) and comparing with the covariant form of Maxwell's equations [14, see pp. 42–43]

$$F^{\mu\nu}{}_{;\mu} = \mu_{em}\, j^{\nu} \tag{9}$$

where j^ν is the current density four-vector $(c\varrho, \vec{j})$, ϱ is the charge density scalar, and $\vec{j}$ is the current density vector, we obtain the relation

$$j^{\nu} = \frac{\varphi_0}{\mu_{em}}\,\frac{2\mu_0 + \lambda_0}{2\mu_0}\,\varepsilon^{;\nu}. \tag{10}$$

This relation provides a physical explanation of the current density four-vector: it arises from the 4-gradient of the volume dilatation of the spacetime continuum. A corollary of this relation is that massless (transverse) waves cannot carry an electric charge or produce a current.

Substituting for j^ν from (10) in the relation [15, see p. 94]

$$j^{\nu} j_{\nu} = \varrho^2 c^2, \tag{11}$$

we obtain the expression for the charge density

$$\varrho = \frac{1}{2}\,\frac{\varphi_0}{\mu_{em} c}\,\frac{2\mu_0 + \lambda_0}{2\mu_0}\,\sqrt{\varepsilon^{;\nu}\varepsilon_{;\nu}} \tag{12}$$

or, using the relation $c = 1/\sqrt{\epsilon_{em}\mu_{em}}$,

$$\varrho = \frac{1}{2}\,\varphi_0 \epsilon_{em} c\,\frac{2\mu_0 + \lambda_0}{2\mu_0}\,\sqrt{\varepsilon^{;\nu}\varepsilon_{;\nu}}. \tag{13}$$

Up to now, our identification of the rotation tensor $\omega^{\mu\nu}$ of the Elastodynamics of the Spacetime Continuum with the electromagnetic field-strength tensor $F^{\mu\nu}$ has generated consistent results, with no contradictions.

2.3 The Lorentz condition

The Lorentz condition can be derived directly from the theory. Taking the divergence of (4), we obtain

$$A^{\mu}{}_{;\mu} = -\frac{1}{2}\,\varphi_0 u_{\perp}{}^{\mu}{}_{;\mu}. \tag{14}$$

From (23) of [11], viz.

$$\omega^{\mu}{}_{\mu} = u_{\perp}{}^{\mu}{}_{;\mu} = 0, \tag{15}$$

(14) simplifies to

$$A^{\mu}{}_{;\mu} = 0. \tag{16}$$

The Lorentz condition is thus obtained directly from the theory. The reason for the value of zero is that transverse displacements are massless because such displacements arise from a change of shape (distortion) of the spacetime continuum, not a change of volume (dilatation).

2.4 Four-vector potential

Substituting (4) into (5) and rearranging terms, we obtain the equation

$$\nabla^2 A^{\nu} - A^{\mu;\nu}{}_{\mu} = \varphi_0 \omega^{\mu\nu}{}_{;\mu} \tag{17}$$

and, using (3) and (9), this equation becomes

$$\nabla^2 A^{\nu} - A^{\mu;\nu}{}_{\mu} = \mu_{em}\, j^{\nu}. \tag{18}$$

Interchanging the order of partial differentiation in the term $A^{\mu;\nu}{}_{\mu}$ and using the Lorentz condition of (16), we obtain the well-known wave equation for the four-vector potential [14, see pp. 42–43]

$$\nabla^2 A^{\nu} = \mu_{em}\, j^{\nu}. \tag{19}$$

The results we obtain are thus consistent with the macroscopic theory of Electromagnetism, with no contradictions.

3 Electromagnetism and the volume force X^ν

We now investigate the impact of the volume force X^ν on the equations of Electromagnetism. Recalling (7), Maxwell's equation in terms of the rotation tensor is given by

$$\omega^{\mu\nu}{}_{;\mu} = \frac{2\mu_0 + \lambda_0}{2\mu_0}\,\varepsilon^{;\nu} + \frac{1}{2\mu_0}\,X^{\nu}. \tag{20}$$

Substituting for $\omega^{\mu\nu}$ from (3), this equation becomes

$$F^{\mu\nu}{}_{;\mu} = \varphi_0\,\frac{2\mu_0 + \lambda_0}{2\mu_0}\,\varepsilon^{;\nu} + \frac{\varphi_0}{2\mu_0}\,X^{\nu}. \tag{21}$$

The additional X^ν term can be allocated in one of two ways:

1. either j^ν remains unchanged as given by (10) and the expression for $F^{\mu\nu}{}_{;\mu}$ has an additional term as developed in Section 3.1 below,
2. or $F^{\mu\nu}{}_{;\mu}$ remains unchanged as given by (9) and the expression for j^ν has an additional term as developed in Section 3.2 below.

Option 2 is shown in the following derivation to be the logically consistent approach.

3.1 j^ν unchanged (contradiction)

Using (10) (j^ν unchanged) into (21), Maxwell's equation becomes

$$F^{\mu\nu}{}_{;\mu} = \mu_{em} j^\nu + \frac{\varphi_0}{2\mu_0} X^\nu. \qquad (22)$$

Using (20) into (17) and making use of the Lorentz condition, the wave equation for the four-vector potential becomes

$$\nabla^2 A^\nu - \frac{\varphi_0}{2\mu_0} X^\nu = \mu_{em} j^\nu. \qquad (23)$$

In this case, the equations for $F^{\mu\nu}{}_{;\mu}$ and A^ν both contain an additional term proportional to X^ν.

We show that this option is not logically consistent as follows. Using (10) into the continuity condition for the current density [14]

$$\partial_\nu j^\nu = 0 \qquad (24)$$

yields the expression

$$\nabla^2 \varepsilon = 0. \qquad (25)$$

This equation is valid in the macroscopic case where $X^\nu = 0$, but disagrees with the general case (non-zero X^ν) given by (35) of [11], viz.

$$(2\mu_0 + \lambda_0)\nabla^2 \varepsilon = -X^\nu{}_{;\nu}. \qquad (26)$$

This analysis leads to a contradiction and consequently is not valid.

3.2 $F^{\mu\nu}{}_{;\mu}$ unchanged (logically consistent)

Proper treatment of the general case requires that the current density four-vector be proportional to the RHS of (21) as follows ($F^{\mu\nu}{}_{;\mu}$ unchanged):

$$\mu_{em} j^\nu = \varphi_0 \frac{2\mu_0 + \lambda_0}{2\mu_0} \varepsilon^{;\nu} + \frac{\varphi_0}{2\mu_0} X^\nu. \qquad (27)$$

This yields the following general form of the current density four-vector:

$$j^\nu = \frac{1}{2} \frac{\varphi_0}{\mu_{em}\,\mu_0} [(2\mu_0 + \lambda_0)\varepsilon^{;\nu} + X^\nu]. \qquad (28)$$

Using this expression in the continuity condition for the current density given by (24) yields (26) as required.

Using (28) into (21) yields the same covariant form of the Maxwell equations as in the macroscopic case:

$$F^{\mu\nu}{}_{;\mu} = \mu_{em} j^\nu \qquad (29)$$

and the same four-vector potential equation

$$\nabla^2 A^\nu = \mu_{em} j^\nu \qquad (30)$$

in the Lorentz gauge.

3.3 Homogeneous Maxwell equation

The validity of this analysis can be further demonstrated from the homogeneous Maxwell equation [14]

$$\partial^\alpha F^{\beta\gamma} + \partial^\beta F^{\gamma\alpha} + \partial^\gamma F^{\alpha\beta} = 0. \qquad (31)$$

Taking the divergence of this equation over α,

$$\partial_\alpha \partial^\alpha F^{\beta\gamma} + \partial_\alpha \partial^\beta F^{\gamma\alpha} + \partial_\alpha \partial^\gamma F^{\alpha\beta} = 0. \qquad (32)$$

Interchanging the order of differentiation in the last two terms and making use of (29) and the antisymmetry of $F^{\mu\nu}$, we obtain

$$\nabla^2 F^{\beta\gamma} + \mu_{em}(j^{\beta;\gamma} - j^{\gamma;\beta}) = 0. \qquad (33)$$

Substituting for j^ν from (28),

$$\nabla^2 F^{\beta\gamma} = -\frac{\varphi_0}{2\mu_0} [(2\mu_0 + \lambda_0)(\varepsilon^{;\beta\gamma} - \varepsilon^{;\gamma\beta}) + (X^{\beta;\gamma} - X^{\gamma;\beta})]. \qquad (34)$$

(42) of [11], viz.

$$\mu_0 \nabla^2 \varepsilon^{\mu\nu} + (\mu_0 + \lambda_0)\varepsilon^{;\mu\nu} = -X^{(\mu;\nu)} \qquad (35)$$

shows that $\varepsilon^{;\mu\nu}$ is a symmetrical tensor. Consequently the difference term $(\varepsilon^{;\beta\gamma} - \varepsilon^{;\gamma\beta})$ disappears and (34) becomes

$$\nabla^2 F^{\beta\gamma} = -\frac{\varphi_0}{2\mu_0} (X^{\beta;\gamma} - X^{\gamma;\beta}). \qquad (36)$$

Expressing $F^{\mu\nu}$ in terms of $\omega^{\mu\nu}$ using (3), the resulting equation is identical to (39) of [11], viz.

$$\mu_0 \nabla^2 \omega^{\mu\nu} = -X^{[\mu;\nu]} \qquad (37)$$

confirming the validity of this analysis of Electromagnetism including the volume force.

(28) to (30) are the self-consistent electromagnetic equations derived from the Elastodynamics of the Spacetime Continuum with the volume force. In conclusion, Maxwell's equations remain unchanged. The current density four-vector is the only quantity affected by the volume force, with the addition of a second term proportional to the volume force. It is interesting to note that the current density obtained from the quantum mechanical Klein-Gordon equation with an electromagnetic field also consists of the sum of two terms [16, see p. 35].

4 Discussion and conclusion

In this paper, we have derived Electromagnetism from the Elastodynamics of the Spacetime Continuum based on the identification of the theory's antisymmetric rotation tensor $\omega^{\mu\nu}$ with the electromagnetic field-strength tensor $F^{\mu\nu}$.

The theory provides a physical explanation of the electromagnetic potential: it arises from transverse (shearing) displacements of the spacetime continuum, in contrast to mass which arises from longitudinal (dilatational) displacements of the spacetime continuum. Hence sheared spacetime is manifested as electromagnetic potentials and fields.

In addition, the theory provides a physical explanation of the current density four-vector: it arises from the 4-gradient of the volume dilatation of the spacetime continuum. A corollary of this relation is that massless (transverse) waves cannot carry an electric charge or produce a current.

The transverse mode of propagation involves no volume dilatation and is thus massless. Transverse wave propagation is associated with the distortion of the spacetime continuum. Electromagnetic waves are transverse waves propagating in the *STC* itself, at the speed of light.

The Lorentz condition is obtained directly from the theory. The reason for the value of zero is that transverse displacements are massless because such displacements arise from a change of shape (distortion) of the spacetime continuum, not a change of volume (dilatation).

In addition, we have obtained a generalization of Electromagnetism for the situation where a volume force is present, in the general non-macroscopic case. Maxwell's equations are found to remain unchanged, but the current density has an additional term proportional to the volume force X^ν.

The Elastodynamics of the Spacetime Continuum thus provides a unified description of the spacetime deformation processes underlying general relativistic Gravitation and Electromagnetism, in terms of spacetime continuum displacements resulting from the strains generated by the energy-momentum stress tensor.

Submitted on December 9, 2012 / Accepted on December 10, 2012

References

1. Horie K. Geometric Interpretation of Electromagnetism in a Gravitational Theory with Space–Time Torsion. arXiv: hep-th/9409018v1.
2. Sidharth B.G. The Unification of Electromagnetism and Gravitation in the Context of Quantized Fractal Space Time. arXiv: gen-ph/0007021v1.
3. Wu N. Unification of Electromagnetic Interactions and Gravitational Interactions. arXiv: hep-th/0211155v1.
4. Rabounski D. A Theory of Gravity Like Electrodynamics. *Progress in Physics*, 2005, v. 2, 15–29.
5. Wanas M.I. and Ammar S.A. Space-Time Structure and Electromagnetism. arXiv: gr-qc/0505092v2.
6. Shahverdiyev S.S. Unification of Electromagnetism and Gravitation in the Framework of General Geometry. arXiv: gen-ph/0507034v1.
7. Chang Y.-F. Unification of Gravitational and Electromagnetic Fields in Riemannian Geometry. arXiv: gen-ph/0901.0201v1.
8. Borzou A. and Sepangi H.R. Unification of Gravity and Electromagnetism Revisited. arXiv: gr-qc/0904.1363v3.
9. Chernitskii A.A. On Unification of Gravitation and Electromagnetism in the Framework of a General-Relativistic Approach. arXiv: gr-qc/0907.2114v1.
10. Millette P.A. On the Decomposition of the Spacetime Metric Tensor and of Tensor Fields in Strained Spacetime. *Progress in Physics*, 2012, v. 4, 5–8.
11. Millette P.A. The Elastodynamics of the Spacetime Continuum as a Framework for Strained Spacetime. *Progress in Physics*, 2013, v. 1, 55–59.
12. Baylis W.E. Electrodynamics, A Modern Approach. Birkhäuser, Boston, 2002.
13. Jackson J.D. Classical Electrodynamics, 2nd ed. John Wiley & Sons, New York, 1975.
14. Charap J.M. Covariant Electrodynamics, A Concise Guide. The John Hopkins University Press, Baltimore, 2011.
15. Barut A.O. Electrodynamics and Classical Theory of Fields and Particles. Dover Publications, New York, 1980.
16. Greiner W. Relativistic Quantum Mechanics, Wave Equations. Springer-Verlag, New York, 1994.