On the Propagation of Light in an Expanding Universe

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The equation of the propagation of light in an expanding Universe is derived based on the definition of comoving distances. A numerical method is proposed to solve this equation jointly with the Friedmann equation. As the equation of the propagation of light in an expanding Universe defines a horizon of the visible Universe, this puts a constraint on cosmological models in order to be consistent with an upper limit for redshifts observed from galaxies. This puzzle is challenging current expansionist cosmological models.

1 Introduction

Euclidean Distances were introduced in [1] in order to derive the galactic density profile which is the evolution of galactic density over time. We define the Euclidean Distance as the equivalent distance that would be traversed by a photon between the time it is emitted and the time it reaches the observer if there were no expansion of the Universe. The comoving distance is the distance between two points measured along a path defined at the present cosmological time. The comoving distance between objects moving with the Hubble flow is deemed to remain constant in time. The Euclidean Distance is also the proper distance at the time of emission for a source of light, which is the comoving distance multiplied by the scale factor at the time of emission. From this relationship, the equation of the propagation of light in an expanding Universe is derived.

2 Equation of the propagation of light in an expanding Universe

As the Euclidean Distance is the proper distance at the time light was emitted from a source of light, it is equal to the comoving distance times the scale factor at the time of emission. By convention the scale factor is equal to one at the present time. Therefore, we have

$$y = a(t)\chi\,, \tag{1}$$

and

$$\chi = c\int_{t=T_b-T}^{T_b} \frac{dt}{a(t)}\,, \tag{2}$$

where χ is the comoving distance, y the Euclidean Distance, a the scale factor, T_b the time from the hypothetical big bang (which is the present time), and T the light travel time between observer and the source of light.

By differentiating (1) with respect to time we get:

$$\frac{dy}{dt} = \dot{a}\chi + a\dot{\chi}\,. \tag{3}$$

As $I = \int_{t_1}^{t_2} f(t)\,dt$ leads to $\frac{dI}{dt} = \frac{dt_2}{dt} f(t_2) - \frac{dt_1}{dt} f(t_1)$, from (2) we get:

$$\dot{\chi} = -\frac{c}{a(t)}\,. \tag{4}$$

As $H(t) = \dot{a}/a$, (1) leads to:

$$\dot{a}\chi = y\,H(t)\,, \tag{5}$$

Combining (3), (4) and (5) we get:

$$\frac{dy}{dt} = -c + H(t)\,y\,, \tag{6}$$

where y is the Euclidean Distance between the observer and a photon moving towards the observer.

We have just derived the equation of the propagation of light in an expanding Universe from the definition of comoving distances. This equation defines a horizon of the visible Universe at $\frac{dy}{dt} = 0$.

3 Numerical method to compute Euclidean Distances from the Friedmann equation

Equation (6) can be solved numerically using a discretization method. Let us set $t = T_b - T$ with T_b the hypothetical time since the big bang, and T the light travel time between observer and the photon. Therefore, $dt = -dT$, and (6) can be rewritten as follows:

$$\frac{dy}{dT} = c - H(T)\,y\,. \tag{7}$$

By discretization over small intervals ΔT, (7) leads to:

$$\frac{y_{n+1} - y_n}{\Delta T} = c - H(T_n)\,y_n\,. \tag{8}$$

Therefore, we obtain:

$$y_{n+1} = c\,\Delta T + y_n\,(1 - H(T_n)\,\Delta T)\,, \tag{9}$$

with initial conditions: $y_0 = 0$ and $T_0 = 0$, and $T_{n+1} = T_n + \Delta T$.

The Friedmann equation expresses H as a function of redshift z. We still need a description of H as a function of T in order to solve (9). For this purpose we compute a curve for the light travel time T versus redshift z using the Friedmann equation, with (11). Then we fit an empirical equation for $H(T)$ over the curve $H(z)$ versus T.

The light travel time versus redshift is computed as follows (derived from $dt = da/\dot{a}$):

$$T = c \int_{1/(1+z)}^{1} \frac{da}{\dot{a}}\,. \tag{10}$$

Because $H = \dot{a}/a$, (10) can be rewritten as follows:

$$T = c \int_{1/(1+z)}^{1} \frac{da}{H\,a}\,. \tag{11}$$

This integral is solved numerically using a solver such as Matlab.

The Friedmann equation that is used in this problem is as follows:

$$H = H_0\,\sqrt{\Omega_R a^{-4} + \Omega_M a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda}\,, \tag{12}$$

with Ω_R the radiation energy density today, Ω_M the matter density today, Ω_k the spatial curvature density today, and Ω_Λ a cosmological constant for the vacuum energy density today. We may alternatively express H as a function of redshift from cosmological redshift relationship by setting $a = \frac{1}{1+z}$, where the scale factor is equal to unity as the present time.

4 Results and discussion

First let us solve the above problem with the assumptions used in the lambda-cdm model [2]. The radiation energy density is generally considered neglegible, hence $\Omega_R = 0$. The common assumption in the lambda-cdm is that Ω_k is equal to zero, and $\Omega_\Lambda = 1 - \Omega_M$. To obtain a description of H as a function of T, we fit a polynomial function of order six to the $H(z)$ curve, which gives the following empirical formula for $\Omega_M = 0.3$ and $H_0 = 71\,km\,s^{-1}\,Mpc^{-1}$: $H(T) = 0.074663 - 0.049672\,T + 0.056296\,T^2 - 0.021203\,T^3 + 0.0036443\,T^4 - 0.00029054\,T^5 + 0.0000088134\,T^6$, with T in $Glyr$ and $H(T)$ in $Glyr^{-1}$. From the discretization method (9) we obtain an horizon of the visible Universe at redshift $z = 1.6$. A variant of the lambda-cdm model would be to remove the cosmological constant for the vacuum energy density ($\Omega_\Lambda = 0$), and replace this term by the spatial curvature density $\Omega_k = 1 - \Omega_M$. This variant gives almost the same result with a horizon of the visible Universe at redshift $z = 1.5$. On the other hand if H is constant over time, the horizon of the visible Universe would have a redshift that tends to infinity.

The results obtained with the equation we derived for the propagation of light solved jointly with the Friedmann equation are inconsistent with observations as it is common to observe galaxies with redshifts up to 6, and more recently beyond 8.5 [3]. This problem has been raised in the past – the recession velocity of all galaxies with $z \geq 1.5$ has been found to exceed the speed of light in all viable cosmological models [4]. A calculation based on null geodesics using gravitational radius is proposed in [5]. Their hypothesis is that the comoving distance and proper distance do not track the propagation of light through the Hubble flow. The puzzle of the propagation of light in an expanding Universe and the horizon of the visible Universe appears to be an interesting challenge for current expansionist cosmological models.

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