

Electric Dipole Antenna: A Source of Gravitational Radiation

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In this article, the gravitational scalar potential due to an oscillating electric dipole antenna placed in empty space is derived. The gravitational potential obtained propagates as a wave. The gravitational waves have phase velocity equal to the speed of light in vacuum (c) at the equatorial plane of the electric dipole antenna, unlike electromagnetic waves from the dipole antenna that cancel out at the equatorial plane due to charge symmetry.

1 Introduction

Gravitational waves were predicted to exist by Albert Einstein in 1916 on the basis of the General Theory of Relativity. They are usually produced in an interaction between two or more compact masses. Such interactions include the binary orbit of two black holes, a merge of two galaxies, or two neutron stars orbiting each other. As the black holes, stars, or galaxies orbit each other, they send out waves of “gravitational radiation” that reach the Earth. A lot of efforts have been made over the years to detect these very weak waves. In this article, we show theoretically, how the gravitational potential of an electric dipole antenna placed in empty space propagates as gravitational waves.

2 Gravitational radiation from an electric dipole antenna

Recall that an electric dipole antenna is a pair of conducting bodies (usually spheres or rectangular plates) of finite capacitance connected by a thin wire of negligible capacitance through an oscillator. The charges reside on the conducting bodies (electrodes) but may travel from one to the other through the wire. The oscillator causes the charges to be built up on the electrodes such that at any time they are equal and opposite and the variation is sinusoidal with angular frequency ω [1].

Let the electric dipole antenna be represented by a pair of spheres separated by a distance s with a sinusoidal charge Q as shown in figure 1.

If the total mass of each sphere at any time is M_0 and its radius R_s and assuming an instantaneous mass distribution which varies with the motion of electrons, then at each time t , the mass density ρ_0 is given by

$$\rho_0 = \Lambda_0 + \rho_e \sin \omega t \tag{1}$$

where

$$\Lambda_0 = \frac{M_0}{4\pi R_s^3}$$

and

$$\rho_e = \frac{Nm_e}{4\pi R_s^3}$$

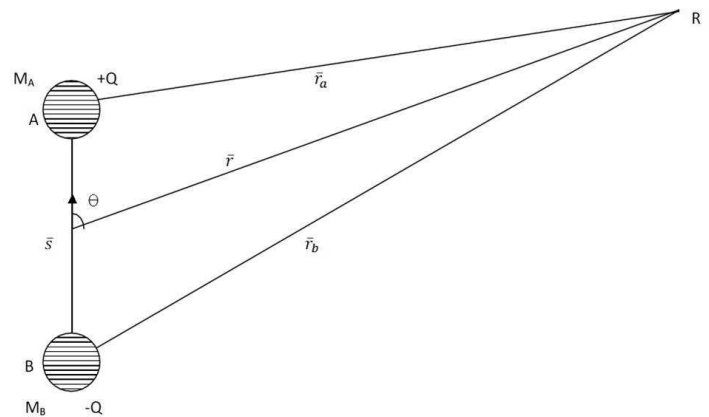


Fig. 1: Amplified diagram of an electric dipole antenna.

where N is the number of electrons moving in the dipole antenna and m_e is the electronic mass. For this mass distribution, the gravitational field equation can be written as [2]

$$\nabla^2 \Phi = \begin{cases} 0 & \text{if } r > R \\ 4\pi G\rho_0 & \text{if } r < R \end{cases} \tag{2}$$

Now, consider a unit mass placed at a point R in empty space, far off from the electric dipole as in figure 1, then by Newton’s dynamical theory, the gravitational scalar potential Φ at R at any time t can be defined as

$$\Phi(\vec{r}, t) = \frac{GM_a(\vec{r}_a, t)}{|\vec{r} - \vec{r}_a|} + \frac{GM_b(\vec{r}_b, t)}{|\vec{r} - \vec{r}_b|}. \tag{3}$$

To maintain equal and opposite charges at the electrodes, the sinusoidal movement of electrons must be in such a way that the masses of the two spheres are the same and determined at point R to be given by

$$M_a(\vec{r}_a, t) = M_b(\vec{r}_b, t) = M_0 e^{i\omega(t')}. \tag{4}$$

Thus, the gravitational potential at R becomes

$$\Phi(\vec{r}, t) = \frac{GM_0 e^{i\omega(t')}}{r_a} + \frac{GM_0 e^{i\omega(t')}}{r_b}. \tag{5}$$

Using the fact that gravitational effects propagate at the speed of light c from General Relativity [3], equation (5) can be written as

$$\Phi(\vec{r}, t) = \frac{GM_0 e^{i\omega(t-\frac{r_a}{c})}}{r_a} + \frac{GM_0 e^{i\omega(t-\frac{r_b}{c})}}{r_b}. \quad (6)$$

From figure 1 and the cosine rule it can be shown that

$$r_a \approx r - \frac{s}{2} \cos \theta = r \left(1 - \frac{s}{2r} \cos \theta\right)$$

and

$$r_b \approx r + \frac{s}{2} \cos \theta = r \left(1 + \frac{s}{2r} \cos \theta\right)$$

and assuming that $r \gg s$ then

$$t - \frac{r_a}{c} = t - \frac{r}{c} + \frac{s}{2c} \cos \theta \quad (7)$$

$$t - \frac{r_b}{c} = t - \frac{r}{c} - \frac{s}{2c} \cos \theta. \quad (8)$$

Substituting equations (7) and (8) into equation (6) yields

$$\Phi(\vec{r}, t) = \frac{GM_0}{r} e^{i\omega(t-\frac{r}{c})} \left(\frac{e^{\frac{is}{2r} \cos \theta}}{1 - \frac{s}{2r} \cos \theta} + \frac{e^{-\frac{is}{2r} \cos \theta}}{1 + \frac{s}{2r} \cos \theta} \right) \quad (9)$$

where $\lambda = \frac{c}{\omega} = \frac{c}{2\pi f}$. λ is the wavelength of the gravitational wave.

Series expansion of the exponential term and denominator of the fractions in the brackets of equation (9) to the first power of $\frac{s}{\lambda}$ and $\frac{s}{r}$ yields

$$\Phi(\vec{r}, t) = \frac{2GM_0}{r\lambda} e^{i\omega(t-\frac{r}{c})} \left(\lambda + \frac{is^2}{4r} \cos^2 \theta \right). \quad (10)$$

Equation (10) is valid provided $s \ll r$ and $s \ll \lambda$ for arbitrary s and λ .

But from complex analysis it can be shown that,

$$\lambda + \frac{is^2}{4r} \cos^2 \theta = \left(\lambda^2 + \frac{s^4 \cos^4 \theta}{16r^2} \right)^{1/2} e^{i\sigma} \quad (11)$$

where

$$\sigma = \arctan \left(\frac{s^4 \cos^4 \theta}{16r^2 \lambda^2} \right).$$

Thus equation (10) becomes,

$$\Phi(\vec{r}, t) = \frac{2GM_0}{r\lambda} e^{i\omega(t-\frac{r}{c})} \left(\lambda^2 + \frac{s^4 \cos^4 \theta}{16r^2} \right)^{1/2} e^{i\sigma} \quad (12)$$

or

$$\Phi(\vec{r}, t) = \frac{2GM_0}{r\lambda} \left(\lambda^2 + \frac{s^4 \cos^4 \theta}{16r^2} \right)^{1/2} e^{i\omega(t-\frac{r}{c} + \frac{1}{\omega}\sigma)}. \quad (13)$$

From equation (13), it is deduced that the gravitational potential propagates as a wave with phase $t - \frac{r}{c} + \frac{1}{\omega}\sigma$.

The following remarks can be deduced from the expression of gravitational potential in this field:

- For

$$s^4 \cos^4 \theta \gg 16r^2 \lambda^2$$

it is clear that

$$\arctan \left(\frac{s^4 \cos^4 \theta}{16r^2 \lambda^2} \right) \approx \frac{\pi}{2}.$$

Thus in this case, the phase velocity of the gravitational potential is c .

- If $s^4 \cos^4 \theta$ is not much greater than $16r^2 \lambda^2$ then the phase velocity of propagation is larger than c . This provides a crucial condition for the propagation of gravitational waves from an electric dipole antenna at velocities greater than the speed of light.
- At the equatorial plane of the electric dipole antenna, $\theta = \frac{\pi}{2}$ and

$$\Phi(\vec{r}, t) = \frac{2GM_0}{r} e^{i\omega(t-\frac{r}{c})}.$$

This indicates that at the equatorial plane; the gravitational wave propagates at a phase velocity of c , unlike in the case of electromagnetic waves, where fields of the two electrodes cancel out each other due to charge symmetry.

- Also, the gravitational field varies as $\frac{1}{r}$ and thus the wave dies out as one moves away from the dipole antenna. This is in agreement with the prediction by Astrophysicists that as gravitational waves travel from galaxies towards the Earth, their intensities die off and they become too weak when they get to planet Earth.

3 Conclusion

The major significance of this article is that, although the electric dipole antenna is not made up of massive compact bodies, the generation of gravitational radiation has been shown theoretically. Hence, this article highlights the fact that gravitational radiation can be produced by an interaction of two masses irrespective of their sizes. The use of gravitational potential which is a dynamical parameter also signifies that the existence of gravitational waves can also be predicted using Newton's theory of gravitation.

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