A New Model of Black Hole Formation

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The formation of a black hole and its event horizon are described. Conclusions, which are the result of a thought experiment, show that Schwarzschild [1] was correct: A singularity develops at the event horizon of a newly-formed black hole. The intense gravitational field that forms near the event horizon results in the mass-energy of the black hole accumulating in a layer just inside the event horizon, rather than collapsing into a central singularity.

1 Introduction

This article describes the formation of a black hole and the physics of event horizon formation. In early 1916, a German physicist, Karl Schwarzschild, published a short paper in which he gave a solution to Einstein's general relativity field equations for spherically symmetric objects. Schwarzschild's solution "contains a coordinate singularity on a surface that is now named after him. In Schwarzschild coordinates, this singularity lies on the sphere of points at a particular radius, called the Schwarzschild radius" [1] (emphasis added). The significance of this paper has not been generally appreciated, although it led physicists eventually to accept black holes as real physical objects. Many black holes have been detected in recent years using astronomical techniques. But physicists in general have concluded that the singularity lies at the center of the black hole rather than on its event horizon. They have mostly ignored the results of Schwarzschild, who found that the singularity occurred at the event horizon itself rather than at the center of the spherical space enclosed by the event horizon. In this article I show by means of a suitably chosen thought experiment that Schwarzschild was correct.

2 A collapsing star

Following the occurrence of a Type 1a supernova, a neutron star is usually formed. For neutron stars with a mass greater than the Tolman-Oppenheimer-Volkoff limit (about 3 to 4 solar masses), the star will collapse to form a black hole. We need to follow the history of some points on and within the collapsing star in order to find out what really happens when a black hole is formed. To establish some boundary conditions, note that a point at the center of the collapsing star will not move with respect to a coordinate system centered on the star; the center of the system does not participate in the collapse. Of more interest is a point on the surface of the collapsing star. This point will have a velocity vector directed toward the center of the star with a speed that depends on the time from the initiation of collapse until the formation of the event horizon, at which time its speed is assumed to be the speed of light, c. Assume that a point halfway between the surface and the center will also have an inwardly directed velocity with half the speed of the surface point. In other words, the con-

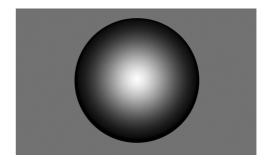


Fig. 1: Radial velocities in a collapsing star.

traction is radially linear. Some departure from this linearity will not severely affect my conclusions.

Figure 1 shows qualitatively what these radial velocities look like. The size of the star in the illustration is assumed to be approaching the Schwarzschild radius. The black colors indicate high radial velocity and white indicates small or zero velocities. The figure was constructed using the gradient tool in Photoshop and is linear in value from the center to the outer boundary. In reality, the darkest black should be confined to the very outer edges of the star and most of the interior should be either white or light gray. Nevertheless, the picture does give a good idea of the kind of radial velocities one would find in the cross-section of a collapsing star.

Figure 2 shows the situation at the moment when the event horizon forms. Note that the points at $0.995 R_s$, where R_s is the Schwarzschild radius, have 10 times their normal, or rest, mass. The asymptote on the right goes to infinity at the Schwarzschild radius, R = 1.0 in the illustration. This is the singularity that Karl Schwarzschild discovered when he solved Einstein's field equations for a symmetrical, nonrotating body. The equation used to plot the points for the mass as a function of the radius is:

$$\frac{m}{m_0} = \frac{1}{\sqrt{1 - v^2/c^2}} \equiv \frac{1}{\sqrt{1 - R^2}}, \quad 0 \le R < 1.$$
(1)

The validity of this special relativity equation under the conditions in the formation of an event horizon is unsure, but since a singularity is a singularity, and this equation defines one for v = c, it is likely as good as some other measure.

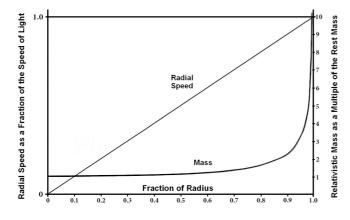


Fig. 2: Mass distribution in a newly-formed black hole. Drawing by the author.

The essential point is that most of the mass will be concentrated near the event horizon as soon as it forms. Thus the gravitational field will be quickly reversed, and with it, the velocity field inside the event horizon. Particles in the interior of the new black hole will be strongly attracted to the event horizon, since that is where most of the mass is located. This implies that the entire mass of the collapsed star could end up in a shallow region just inside the event horizon. There is no way to determine from the outside whether or not this happens.

In this scenario, the mass M is contained in a very thin layer at the radius R and the interior is empty. But how does it get there? According to Susskind [2, see p. 238] anything that impacts the event horizon of a black hole is absorbed by it, spreading over the entire extent of the event horizon the way a drop of ink dissolves rapidly in a basin of warm water. What if the event horizon itself comprises all of the mass contained in the black hole, held in a layer perhaps one Planck length in thickness? (Admittedly, that's a guess on my part.) From the outside, it would still behave like a black hole. All differences would be on the inside.

In my model the material of a collapsing star would, as soon as it has compacted enough to form a black hole, begin to migrate to the event horizon, like iron filings attracted to a magnet. The only place where the gravity of the material comprising the event horizon layer is neutral would be the exact, precise center of the black hole. But even so small a particle as a hadron would, sooner or later, wander off center — if for no other reason, because of the Heisenberg uncertainty principle. It would then be instantly attracted to the event horizon and would stick there like a bug on fly paper. Eventually the entire inside of the black hole would be empty. The layer comprising the event horizon layer may be extremely thin, but it is most definitely not a singularity, a mere mathematical point.

I recently discarded this possibility, but it appears that I may have been too hasty in doing so.

3 What happens to the matter in a black hole?

In this reconsidered theory, the singularity at the event horizon is only mathematical, not real. The mass of the collapsed star is contained in a thin layer just inside the event horizon, perhaps only a single Planck length thick. There is an external complement to this idea. Leonard Susskind [2, see pp. 233–234] writes:

The only [solution] consistent with the laws of physics would be to assume that some kind of super-heated layer exists just above the horizon, perhaps no more than a Planck length thick... the layer must be composed of tiny objects, very likely no bigger than the Planck length. The hot layer would absorb anything that fell onto the horizon, just like drops of ink dissolving in water... This hot layer of stuff needed a name. Astrophysicists had already coined the name that I eventually settled on... They had used the idea of an imaginary membrane covering the black hole just above its horizon to analyze certain electrical properties of black holes. [They] had called this imaginary surface the stretched horizon, but I was proposing a real layer of stuff, located a Planck length above the horizon, not an imaginary surface.

I liked the sound of "stretched horizon" and adopted it for my own purposes. Today the stretched horizon is a standard concept in black hole physics. It means the thin layer of hot microscopic "degrees of freedom" located about one Planck distance above the horizon.

I propose the name "Shell Theory" for my explanation of black hole formation.* This theory posits a one-to-one correspondence between the bits of entropy on the surface of the event horizon of a black hole and the particles of the collapsed star in the shell layer just inside the event horizon. The gravitational field and other external properties of the black hole will be exactly the same as if an infinite singularity existed at the center, because the amount of mass-energy in each case will be identical. All that is necessary for this condition to be true is that the distribution of mass inside the event horizon is spherically symmetrical. The shell theory has the same spherical symmetry as conventional theory with a singularity at the center of the black hole.

In the shell theory evolution of a black hole, the collapsing of the remnant star must stop as soon as the event horizon is formed. The reversal would start at a time somewhat prior to the formation of the event horizon. In figure 1 it is apparent that even before the outer layer of particles achieves a velocity magnitude equal to the speed of light, the distribution of

^{*}For the purposes of this article, a "shell" is defined as the volume enclosed between concentric spheres of different radii.

mass within the collapsing object would favor the outer layers over the inner layers. This differential in the gravitational field would build up rapidly as the size of the collapsing star approached the Schwarzschild limit, so it would not be an instantaneous reversal.

The mass of a differential shell from the collapsing star as a function of the radius, assuming that the radial velocity of a point inside the object is a linear function of the radius up to the limit of v = c, at $R = R_s$, is:

$$d\frac{m}{m_0} = 4\pi R^2 \frac{m}{m_0} \, dR \,, \tag{2}$$

where

$$\frac{m}{m_0} = \frac{1}{\sqrt{1 - R^2}} \,. \tag{3}$$

Therefore the total relative mass of a spherical shell is given by the integral:

$$\frac{m}{m_0} = \int \frac{4\pi R^2 dR}{\sqrt{1-R^2}} = 4\pi \left[\frac{1}{2}\sin^{-1}(R) - \frac{1}{2}R\sqrt{1-R^2}\right].$$
 (4)

This result must be evaluated at three points: R = 0; R = R; and $R = R_s$. The result for R = 0 is simple: 0. For $R = R_s$ the term $(1 - R^2)$ becomes zero, and $\sin^{-1}(1)$ is $\frac{\pi}{2}$; so the result for $R = R_s$ is $\frac{\pi}{4} (\times 4\pi)$. Subtracting the two solutions from each other (ignoring the common factor of 4π) and setting the results equal to each other — so that we obtain the radius within which and without which there is equal mass — we have, after rearranging terms, the equation:

$$\sin^{-1}(R) = \frac{\pi}{4} + R\sqrt{1 - R^2}.$$
 (5)

This equation, (5), is difficult to evaluate in closed form, but the result can be obtained easily through the process of successive iterations. The solution is approximately R = 0.915(the difference between the two sides of the equation is 9×10^{-4} out of 1.155), meaning that the outer 8.5% of the sphere contains as much relativistic mass as the entire inner 91.5%.* This amply demonstrates that what was initially the inward implosion of a neutron star will now be a radially outward "explosion" within the confines of the event horizon the surface implied by Schwarzschild's results.

4 Results and discussion

The likely end result will be that all of the mass-energy of a collapsed star ends up confined to a very thin layer — probably only one Planck length thick — just inside the event horizon. There may be a "black hole" there, but its matter will not be located in an infinitely dense singularity at the center point.

Also notice that for a solid body of uniform density, the gravitational field outside the surface is inversely proportional

to the square of the distance from the center of the body, but for points inside the body the gravitational field is linear, diminishing to zero at the center. This reinforces the assumption that the collapse of the neutron star should be linear in nature. The effect as the radius of the shrinking star approaches and attains the Schwarzschild radius is to change this linear gravitational potential into a hyperbolic gravitational field, asymptotic to infinity at R_s .

The singularity at the Schwarzschild radius, or event horizon, is mathematical only and does not affect any real particles. The event horizon is described by a metric of points distributed over a spherical manifold, and the term "point mass" is an oxymoron since a point cannot have mass or any other physical property. It is nothing more than a mathematical position in space-time. In this context, note that the integration in equation (4) does not diverge at $R = R_s$, as it would if there were a true infinity at that point.

Where I have written the word "point" or "points", this term should not be taken literally. The reader should imagine a tiny amount of matter, perhaps a cubic Planck length (Planck volume) in size, located at a particular point in spacetime. An actual point has no dimensions and therefore cannot have mass or any other physical property. The Planck volume is believed by many to be a quantum unit of space.

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^{*}A more precise result is $0.914554 \pm 2 \times 10^{-6}$.