A Prediction of an Additional Planet of the Extrasolar Planetary System Kepler-62 Based on the Planetary Distances' Long-Range Order

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Recently, the discovery of the extrasolar planetary system Kepler-62 comprising five planets was reported. The present paper explores whether (i) the sequence of semi-major axis values of the planets shows a long-range order, and whether (ii) it is possible to predict any additional planets of this system. The analysis showed that the semi-major axis values of the planets are indeed characterized by a long-range order, i.e. the logarithmic positions of the planets are correlated. Based on this characteristic, an additional planet at 0.22 AU in the Kepler-62 system is predicted.

1 Introduction

In April 2013, NASA's Kepler Mission reported [1] the detection of an extrasolar planetary system comprising five planets (Kepler-62b, c, d, e and f) orbiting a star (Kepler-62) of spectral type K2, luminosity class V, 69 ± 0.02% the mass and 63 ± 0.02% the radius of the Sun. The Kepler-62 extrasolar planetary system is located in the constellation Lyra, ∼1200 light years away from Earth. The five planets have a size of 1.31, 0.54, 1.95, 1.61 and 1.41 Earth radii ($R_\oplus$). The two outermost planets (e, f) are likely to be solid planets possibly with liquid water on their surfaces since their position is within Kepler-62's Habitable Zone. The five planets were detected by analyzing the brightness variations of Kepler-62 based on images obtained by the Kepler spacecraft.

In an analysis of distances between planets of our solar system (including the dwarf planet Pluto and the asteroid Ceres) it was shown by Bohr and Olsen [2] that the sequence of distances show a long-range order on a logarithmic scale, i.e. the logarithmic positions of the planets are correlated and follow a periodic pattern; they seem to obey a "quantization". The authors tested the statistical significance of the obtained long-range order by using a permutation test, which revealed that the regularity of the distances between the planets in our solar system is very unlikely to have originated by chance.

In a subsequent study by the same authors [3], they applied their analysis to the extrasolar planetary system HD 10180 and determined that (i) the logarithmic position of the six planets show also a long-range order, and (ii) that this property is enhanced when including a seventh (hypothetically existing) planet at a position of 0.92 ± 0.05 AU, i.e. between the planets HD 10180f and HD 10180g. Based on this analysis, they postulated a possible additional planet in the HD 10180 system at a distance of 0.92 AU.

The goal of the present analysis was to apply the same data analysis approach [2, 3] to the recently discovered extrasolar planetary system Kepler-62 and thus to analyze whether (i) the semi-major axis values of the planets show a long-range order, and whether (ii) the analysis predicts additional planets of this system.

2 Materials and methods

2.1 Data

The parameter values of the Kepler-62's exoplanets were obtained from the listing in Borucki et al. [1]. In particular, two parameters were selected for the present analysis: the semi-major axis (a) and the radius (r) of each planet. For the values, see Table 1.

Planet	i	a [AU]	a [km]	r [$R_\oplus$]	r [km]	$\hat{a}$
62b	1	0.0553	8.2728×10^6	1.31	8355	2.1130
62c	2	0.0929	1.3898×10^7	0.54	3444	2.6317
62d	3	0.120	1.7952×10^7	1.95	12437	2.8877
62e	4	0.427	6.3878×10^7	1.61	10269	4.1570
62f	5	0.718	1.0741×10^8	1.41	8993	4.6767

Table 1: Kepler-62 system parameters according to [1]. i: planet number counting outwardly from the star Kepler-62, a: semi-major axis, r: radius of the planet, ($\hat{a}_i = \ln(a_i/10^6\,\text{km})$), a and r are given in two different units ([AU], [km]) and ([$R_\oplus$], [km]), respectively.

2.2 Data analysis

For the analysis, the semi-major axis value (given in units of 10^6 km) of each exoplanet was first divided by 10^6 km, then logarithmized ($\hat{a}_i = \ln(a_i/10^6\,\text{km})$) and according to these values a multimodal probability distribution function (PDF) $p(\hat{a})$, as introduced by Bohr and Olsen [2], was calculated by

$$p(\hat{a}) = \sum_{i=1}^{N} \alpha_i \, e^{-\beta}, \tag{1}$$

with $N = 5$ (i.e. the maximum number of planets of Kepler-62) and β given as

$$\beta = \frac{j - \hat{a}_i}{w_p / 2\sqrt{2\ln(2)}}, \tag{2}$$

for $j = 1, 1.01, 1.02, \ldots, 10$, with w_p the width (i.e. the full-width-at-half-maximum) of each Gaussian peak of the PDF, and α_i a scale factor. The scale factor defined the magnitude

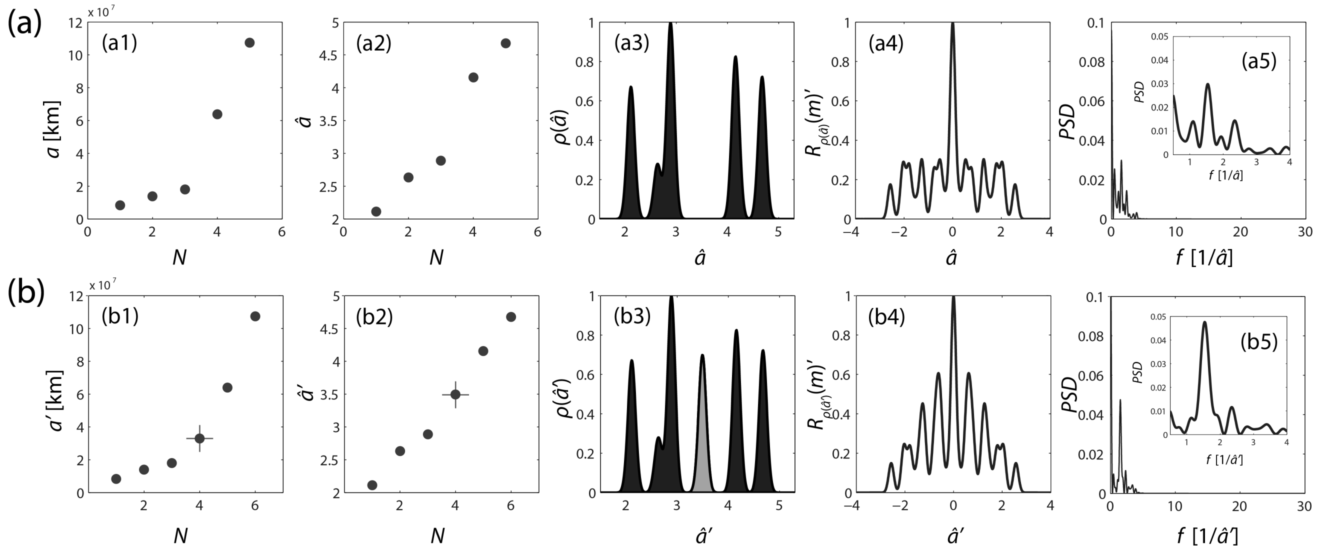

Fig. 1: Results of the analysis of the multimodal PDF $\rho(\hat{a})$ (a1-a5) and the new one $\rho(\hat{a}')$ with the additional hypothetical exoplanet (marked with a cross in Fig. (b1) and (b2), and marked with a black are of the Gaussian peak in Fig. (b3)) found using the optimization approach visualized in Fig. 2.

of each peak. For the present analysis, the scale factor was assigned to the radius of the specific planet, i.e. $\alpha_i = r_i$. The rationale for this definition is that larger planets should be contribute more to the overall multimodal PDF than smaller planets. A linear relationship was chosen rather than the non-linear one used by Bohr and Olsen [2, 3] in order to circumvent the definition of the specific type of non-linear relationship which is unknown per se. For the width of each peak, $w_p = 0.25$ was used which ensures an optimum compromise between a too strong overlap of the Gaussian peaks on the one side and to small peaks on the other. Thus, $\rho(\hat{a})$ represents a sum of Gaussian peaks located at the logarithmized planets semi-major axis values $(\hat{a})$ and weighted by (α_i), the individual radius value of the planet.

In the next step, the autocorrelation sequence of the multimodal PDF was calculated according to

$$R_{\rho(\hat{a})}(m) = \sum_{n=0}^{N-m-1} \rho(\hat{a}_{n+m})\rho(\hat{a}_n), \tag{3}$$

for $m = 1, 2, \ldots, 2N - 1$, with N the number of samples of $\rho(\hat{a})$. Then, the autocorrelation function (ACF) was determined by

$$R_{\rho(\hat{a})}(m)' = \frac{1}{R_{\rho(\hat{a})}(1)} R_{\rho(\hat{a})}(m), \tag{4}$$

i.e. $R_{\rho(\hat{a})}(m)$ was normalized by its maximum value given by $R_{\rho(\hat{a})}(1)$ so that $R_{\rho(\hat{a})}(1)' = 1$. The type and grade of the order (short- or long-range) of the input sequence can be determined using the ACF characteristics.

In order to quantify the periodicity in the ACF (i.e. the long-range order of the input sequence), in the next step the frequency-dependent power spectral density (PSD), i.e. the power spectrum (PS), of the multimodal PDF $\rho(\hat{a})$ was calculated by the periodogram method, which is the windowed discrete Fourier transform (DFT) of the biased estimate of autocorrelation sequence. For the calculation, 2^{12} points in the DFT were used by zero-padding $\rho(\hat{a})$ to a length of 2^{12} enabling a proper frequency resolution.

In order to analyze whether an additional hypothetical planet increases the long-range order, the above-mentioned signal processing steps (i.e. calculation of the multimodal PDF, the ACF and the PS) were repeated with the input signal $\rho(\hat{a})$ in which an additional Gaussian peak was inserted, corresponding to the hypothetical exoplanet's position. The high of the peak was set to the mean values of the radius of the five exoplanets. The new peak was introduced between the peaks associated with values of Kepler-62e and Kepler-62f since visual inspection reveals a gap in the multimodal PDF in this region. The semi-major axis value was varied between 0.15-0.38 AU and the corresponding ACF and PS were calculated. For each PS, the maximum PSD value of the fundamental frequency of $\rho(\hat{a})$ (i.e. the first peak after the global maximum at position 0) was calculated. From the obtained values, the maximum was determined which indicate the strongest long-range order of the corresponding sequence with the added new exoplanet. This new multimodal PDF was denoted as $\rho(\hat{a}')$, with $\hat{a}'$ the vector with the new semi-major axis values.

3 Results

The analysis of the semi-major axis values of Kepler-62's planets b-f revealed an exponential like function (Fig. 1(a1)) or a quasi linear one when logarithmized values were used (Fig. 1(a2)).

The calculated multimodal PDF is shown in Fig. 1(a3). The ACF and the PS are shown in Fig. 1(a3) and 1(a4), respectively. The search of the optimal semi-axis value of the additional (hypothetical) planet revealed that a global maximum of the PSD value in the frequency range of 1.1538 $1/\hat{a}$ ($\approx$ 0.6502 units of $\hat{a}$) can be clearly determined, as depicted in Fig. 2. Thus, the analysis predicts an additional planet at a distance of 0.22 AU from the star Kelper-62. The characteristics of the resulting new multimodal PDF $\rho(\hat{a}')$ with all six planets are shown in Fig. 1(b1-b5).

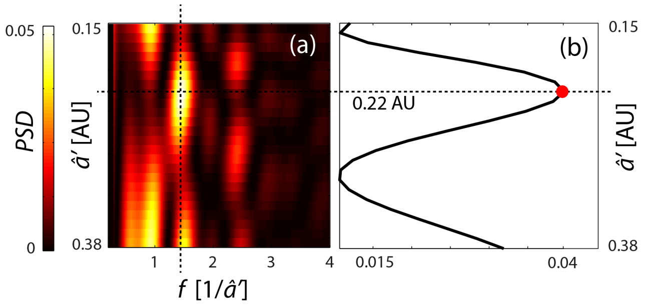

Fig. 2: (a) Color-coded visualization of the PSD values for the $\rho(\hat{a})$ function with the added hypothetical exoplanet at different positions. (b) Function of the PSD values for the frequency of 1.1538 $1/\hat{a}$. The global maximum indicates the value which corresponds to the strongest increase in the long-range order.

4 Discussion and conclusion

From the analysis conducted in the present study, the following conclusions can be drawn:

(i) The positions of the exoplanets Kepler-62a–f show a long-range order inferred from the peak-like structure (four peaks) in the ACF which is captured by the power spectrum as one single peak, corresponding to linear periodicity of the logarithmized distances between the planets.

(ii) The strength of the long-range order increases when an additional planet with a distance of 0.22 AU from the star is added to the five observed ones. This result was obtained by an optimization procedure testing all possible positions for this planet in the gap between Kepler-62e and Kepler-62f.

A prediction of possible additional planets in the Kepler-62 extrasolar system was put forward also recently by Bovaird and Lineweaver [4]. They applied a two-parameter fit to 68 different extrasolar planetary systems in total and predicted 141 additional planets. For the fitting they used a function (denoted by them as a modified Titius-Bode relation) of the form $a_n = \alpha C^n$, with a_n an the semi-major axis, two free parameters (α, C), and n a variable with the quantized values $n = 1, 2, 3, \ldots$. Based on their approach, they predicted for the Kelpler-62 system 7 additional planets with semi-major distance values of 0.07, 0.15, 0.20, 0.26, 0.33, 0.55 and 0.92 AU. Thus, the approach of Bovaird and Lineweaver predicts a finer periodicity compared to the prediction (0.22 AU) described by the present paper. Only the future will tell which approach is better in modeling the exoplanetary characteristics, i.e. the next discovery of an exoplanet of Kepler-62.

By the best of my knowledge, the two predictions (by Bovaird and Lineweave, and the present one), are the only ones published at the present concerning the extrasolar planetary system Kepler-62.

For other extrasolar planetary systems, various authors have reported a periodicity/quantization of the planetary positions and predicted additional orbits/planets based on this.

For example, Naficy et al. [5], recently compared two approaches for modelling and predicting by using either a squared model of the form $r_n = GM\,n^2/(v_0^2\,k^2)$ (with r_n the orbital radius of the n-th planet, G the gravitational constant, M the mass of a central body of the system, and the free parameters v_0^2, k, and n) or an exponential one given by $r_n = a\,e^{b\,n}$ (with a, b, n free parameters). In both cases, the parameter values of n are integers. The authors concluded that the "exponential model has a better coincidence to observational data" [5]. In addition they observed a relation between the values of the b parameter and the mass of the central star of the system, indicating a possible physical mechanism underlying the exponential model. The squared model was also used in a study analyzing extrasolar planetary systems conducted by Rubčić and Rubčić [11].

Another study based on an exponential model was conducted by Poveda and Lara [24] to examine the extrasolar planetary system 55 Cancri. However, problems with this study were pointed out later [23].

In another study, Panov [6] applied an exponential model of the type $a_n = C\,e^{2n/k}$ to extrasolar planetary systems and reported a good fit as well as predictions of additional planets.

As early as 1996, Nottale found that "the distribution of the semi-major axis of the firstly discovered exoplanets was clustered around quantized values according to the law $a/GM = (n/w_0)^2$, in the same manner and in terms of the same constant w_0 = 144 km/s as in our own inner Solar System" [7, 8]. This approach is a result of the "scale relativity" theory developed by Nottale [9, 10, 32, 33]. In 2008, an updated analysis involving 300 exoplanets was published [10] which confirmed and extended the validity of the initial analysis of 1996.*

An analysis with 443 exoplanets (i.e. all known in 2011) was conducted by Zoghbi [26]. This revealed a quantization of the planet's angular momentum which was shown to have a probability of $p < 0.024$ being due to pure chance.

*It would be worthwhile and interesting to repeat the analysis with the presently 732 confirmed exoplanets (September 2013, http://exoplanets.org).

In another study, using the equation $r_n = GM/(c\,\alpha\,e^n)$, with α the dimensionless fine structure constant of $\sim 1/137$ and c the speed of light, Pintr et al. [12] reported a strong agreement between the orbital data of the two analyzed extrasolar planetary system and the expected values. The interesting thing about this work is that the equation is derived from a physical theory describing the effects of electric and magnetic effects on the evolution of a solar system.

Finally, as mentioned earlier, employing a similar method to the one used in this paper (i.e. analysis of correlation property of the logarithmized planetary positions), Olsen and Bohr [3] analysed the extrasolar planetary HD 10180 and predicted an additional planet at 0.92 ± 0.05 AU.

Apart from analyzing extrasolar planetary systems, empirical relationships for the distances of the planets of our solar system started to be published centuries ago when J. D. Titius (1729–1796) and E. Bode (1744–1826) described an apparent regularity of the planetary radii, later known as the Titius-Bode law (expressed in 1787 in its more modern mathematical form by Wurm: $r_n = 0.4 + 0.3 \times 2^n$, $n = -\infty$ (Mercury), $0, 1, 2, \ldots$) [13]. This equation predicted the position of Uranus, but failed to fit for the planetary positions of Neptune and Pluto. Based on the many studies about regularities in planetary distances/radii conducted until now, the Titius-Bode law can be regarded as a first phenomenological description of a possible fundamental law of planetary spacing. The work of Bohr and Olsen [2, 3] in particular strongly suggests that the orbital spacing of planetary systems obey a long-range order and not a simple short-range one, supporting the notion that the quantization is not down to chance.

Concerning the physical mechanism involved in creating a long-range order in planetary systems, this issue is not resolved yet. However, important approaches have been put forward over the last decades. For example, Wells showed that the planetary distances can be "accurately predicted by the eigenvalues of the Euler-Lagrange equations resulting from the variation of the free energy of the generic plasma that formed the Sun and planets" [14, 15]. Further research of the author led him to conclude that "a unification of the morphology of the solar system" and other astrophysical phenomena "can be accomplished by a basic consideration of the minimum-action states of cosmic and/or virtual vacuum field plasmas" [16]. Finally he came to the conclusion that a unification of all physical forces can be derived based on the assumption that they are regarded "as 'fluid' or 'Magnus' forces generated by vortex structures (particles) in the virtual plasma gas" [15–17]. The work of Wells should be carefully reconsidered since it might be a key to understanding regular patterns, long-range orders and quantization of astronomical systems and structures. In addition, analysis based on the theoretical framework of stochastic electrodynamics (SED) that shed new light on the origin of the solar system [18], and also the finding of Graner and Dubrulle [19, 20] that Titius-Bode-like laws appear when assuming a scale and rotational invariance of the protoplanetary system, might also be important for an understanding of the observed patterns.

Other approaches worth exploring for further research are that based on large-scale quantization in space plasmas [22], modelling celestial mechanics using the Schrödinger equation [21, 27, 29, 39–41, 43], resonance effects [25, 28], orbital angular momentum quantization per unit mass [30, 37], fractal scaling modeling using the continued fraction method [31], conservation of mass and momentum, and stability of the angular momentum deficit [35,36], the stochastisation hypothesis [34], macroscopic quantization due to finite gravitational propagation speed [38], and the Weyl-Dirac approach to gravity [42].

One significant difficulty in explaining the observed regularities of distances is the fact that planets can migrate large distances after their formation (e.g. [44–48]). A model that gives an explanation of the regularities must include this observed fact. One possible explanation might be to regard the quantization pattern as an attractor in a phase-space of the planet's migration movements.

In conclusion, the present analysis of the extrasolar planetary system Kepler-62 reveals that (i) the semi-major axis values of the planets show a long-range order, and (ii) that there might be an additional planet at the distance of 0.22 AU between Kepler-62e and Kepler-62f.

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