# Proton-Electron Mass Ratio: A Geometric Inference

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In this paper we propose that the inertial masses of the proton and of the electron can be associated to volumes of the unit cells of hyper-cubic lattices constructed in the momentum space. The sizes of the edges of these cells are given by the Planck's momentum in the case of the electron, and by a modified Planck's momentum in the case of the proton. We introduce a "conservation of information principle" in order to obtain the wave function which leads to this modified momentum. This modification is attributed to the curvature of the space-time, and in doing this, the concept of the entropy of a black hole has been considered. The obtained proton-electron mass ratio reproduces various results of the literature, and compares well with the experimental findings.

## 1 Introduction

The volumes of certain associated symmetric spaces have been used as a means to estimate the proton-electron mass ratio, besides the ratios among leptons and mesons masses [1–7]. Some of these papers [1–4] claim to present more consistent physical interpretations of the particles mass ratio, obtained through these geometric approaches. As was pointed out by González-Martin, Smilga [1,4] obtained a volume factor from the decomposition of SO(3, 3) with respect to the product group SO(3, 1) × SO(2). He calculated this volume factor that when compared with the volume factor of the electron furnishes a proton-electron mass ratio very close to known experimental result. The same evaluation was done earlier by Wyler [7].

In this work we intend to pursue further on this subject, by associating the masses of the proton and of the electron to the volumes of unit cells in the momentum space, with each unit cell having its appropriate size. For appropriate size we mean that, the unit cell edge associated to the electron mass is given by a characteristic momentum of the Planck's scale. On the other hand the unit cell related to the proton mass is also evaluated with the aid of a Planck's scale momentum, but modified by the curvature of the space-time. The reason to establish such differences is that the electron is usually described through Quantum Electrodynamics (*QED*) [8], an abelian field theory. Meanwhile the proton is described by Quantum Chromodynamics (*QCD*) [9], a non-abelian field theory, and we propose that this feature introduces a curvature in space-time modifying the size of the cell of the momentum space.

## 2 A conjecture about the conservation of the information

If we consider a black hole of radius $r$, its entropy is given by the well known Bekenstein-Hawking [10–12] formula

$$S = \frac{A}{4} = \frac{\pi r^2}{L_{Pl}^2}, \tag{1}$$

where $L_{Pl}$ is Planck's length.

Let us write a "law of the conservation of the information" in the form

$$S + I = C. \tag{2}$$

In (2), $C$ is a constant. Now we propose to associate the quantity of information, $I$, to the logarithm of a density of probability $\Psi^2$, where $\Psi$ is a wave function associated to this curved space-time. We have

$$\frac{\pi r^2}{L_{Pl}^2} + \ln(\Psi^2) = C. \tag{3}$$

Equation (3) leads to

$$\Psi = \Psi_0 \exp\left(-\frac{\pi\, r^2}{L_{Pl}^2}\right). \tag{4}$$

In order to better examine the content of $\Psi$ it is convenient to interpret it as a ground-state wave function of a kind of one-dimensional harmonic oscillator. Inserting this function and its second derivative in a Schrödinger equation for a particle of mass $M$, we have

$$-\frac{\hbar^2}{2M}\left(\frac{\pi^2 r^2}{L_{Pl}^4}\right)\Psi + \frac{\hbar^2}{2M}\left(\frac{\pi}{L_{Pl}^2}\right)\Psi + V\Psi = \epsilon_0 \Psi. \tag{5}$$

Making the identification of the "$r$-squared" and the "independent of $r$" terms, we have

$$\frac{1}{2}\,\frac{\hbar^2\pi^2}{ML_{Pl}^4}\, r^2 = \frac{1}{2} k r^2 = V(r) \tag{6}$$

and

$$\frac{\hbar^2 \pi}{2ML_{Pl}^2} = \epsilon_0 = \frac{1}{2}\hbar\omega. \tag{7}$$

By taking

$$L_{Pl} = \frac{\hbar}{M_{Pl}} \quad \text{and} \quad M \equiv M_{Pl}, \tag{8}$$

we get

$$\hbar\omega = \pi M_{Pl}\, c^2 = \langle p \rangle\, c \tag{9}$$

with

$$<p> = \pi M_{Pl}\, c. \quad (10)$$

We interpret (10) as the size of the unit cell in the curved momentum space. Equation (9) can be seen as the difference in energy levels in the curved space, namely $\hbar\omega$ being related to the emission (absorption) of a boson of momentum $<p>$.

## 3 Estimate of the proton-electron mass ratio

As was pointed out by Wesson [13], Einstein's Equivalence Principle (EEP) may be a direct consequence of an extra dimension. Yet according to Wesson, a null path in five space-time dimensions (5-D) can describe a massive particle which usually lives in four dimensions. This null path conditions in 5-D can encompass both the gravitational mass of this particle (related to its Schwarzschild radius) as well its inertial mass (related to its Compton length).

Partially inspired in Wesson work [13], we will assume that particle masses are tied to some type of unit cell in a five-dimensional momentum space lattice. First let us consider the electron. The field theory which deals with the electron is the (abelian) QED [8]. We imagine that the amount of inertial mass of the electron ($m_e$) is proportional to the five-dimensional volume of the unit cell in the momentum space lattice, which size is given by the Planck's characteristic momentum, namely

$$p = M_{Pl}\, c, \quad (11)$$

and

$$V_5 = p^5 = (M_{Pl}\, c)^5 . \quad (12)$$

Therefore we write

$$m_e = K V_5 = K\, (M_{Pl}\, c)^5 . \quad (13)$$

On the other hand the proton is a hadron which structure is described by QCD [9, 14], a non-abelian field theory. QCD has in common with General Relativity (GR), the fact that both are known to be non-linear theories. It seems that in evaluating the proton mass, a curved space-time must be considered. This leads to a modified size of the unit cell in the momentum space lattice. Looking at the wave function given by (4) and the structure of energy levels implied by it, we have obtained $<p>$ given by (10). But the curvature of a space seems not to be displayed by a mathematical object such as a volume. Then we propose that the inertial mass of the proton $m_p$ is proportional to a five-surface area in the curved momentum space lattice, this surface area being a derivative from a six-volume. Therefore we write

$$<V_6> = <p>^6 \quad (14)$$

$$<S_5> = \frac{\mathrm{d}<V_6>}{\mathrm{d}<p>} = 6<p>^5 \quad (15)$$

and

$$m_p = K<S_5> = K\, 6\pi^5\, (M_{Pl}\, c)^5 . \quad (16)$$

In writing (16) we have used (10), and considered that the proportionality constant $K$ is the same as that used in determining the electron mass. By comparing (13) and (16), we finally obtain

$$\frac{m_p}{m_e} = 6\pi^5 \approx 1836.12. \quad (17)$$

The ratio given by (17) has been previously obtained by various authors, and compares relatively well with the experimental values (please see [1, 3, 4] and references cited in those papers).

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