

LETTERS TO PROGRESS IN PHYSICS

## The Infinite as a Hegelian Philosophical Category and Its Implication for Modern Theoretical Natural Science

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The concept of the infinite as a mathematical, a scientific and as a philosophical category is differentiated. A distinction between Hegel's dialectical concept of the infinite as opposed to his idealist-philosophical "system" of the "Absolute Idea" as the "True Infinite" is emphasized.

### 1 The infinite as a mathematical category

The concept of the infinite as a mathematical category arose naturally enough with the invention of the numerical system by the Sumerians around 3000 B.C. and the subsequent developments of the concepts of geometry, the measure of time, mathematical operations (arithmetic, algebraic, exponentials etc.), One could always add or subtract a unit of number, length or time to get a new one ad infinitum without an end. This infinite is undetermined, has no characterization and was termed the "spurious" or the "false" infinite (*bad infinity*) by G. W. F. Hegel (1770–1831 A.D.), as opposed to the "True Infinite" (to be discussed later).

"The spurious infinite" according to Hegel [1],

"...seems to superficial reflection something very grand, the greatest possible. ... When time and space for example are spoken of as infinite, it is in the first place the infinite progression on which our thoughts fasten ... the infinity of which has formed the theme of barren declamation to astronomers with a talent for edification. In an attempt to contemplate such an infinite our thought, we are commonly informed, must sink exhausted. It is true indeed that we must abandon the unending contemplation, not however because the occupation is too sublime, but because it is too tedious ... the same thing is constantly recurring. We lay down a limit: then pass it: next we have a limit once more, and so for ever."

The infinite as a mathematical category took a mystical form once Pythagoras of Samoa (580?–520 B.C.), and later Plato (429–347 B.C.) idealized the numbers, their relations and geometry into their philosophical system, where the infinite along with the numbers and the forms were universals that exists in a realm beyond space and time for all eternity, a realm that sense perception cannot reach; it is only given to thought and intuition.

As Frederick Engels [2, p. 46] wrote,

"Like all other sciences, mathematics arose out of the *need* of man; from measurement of land and of the content of vessels, from computation of time and mechanics. But, as in every department of thought, at a certain stage of development, the laws abstracted from the real world become divorced from the real world and are set over against it as something independent, as laws coming from outside to which the world has to conform. This took place in society and in the state, and in this way, and not otherwise, *pure* mathematics is subsequently *applied* to the world, although it is borrowed from this same world and only represents one section of its forms of interconnection — and it is only just precisely because of this that it can be applied at all".

The mathematical pursuit of the infinite therefore, of necessity became a spiritual endeavor. In his attempt to know the infinite and to prove his continuum hypothesis, Georg Cantor (1845–1914 A.D.) for example, was eventually compelled to make a distinction between *consistent* and *inconsistent* collections; for him only the former were *sets*. Cantor called the *inconsistent* collections the *absolute infinite* that God alone could know. His idea of an "actual infinite" attracted theological interest because of its implication for an all-encompassing God; but at the same time it inspired scorn of the contemporary mathematicians. What Cantor, other mathematicians and natural science pursued in reality is the "spurious infinite" of Hegel. An infinite series starting with a first term is also undefined, because there is no end to the other side, and one cannot come back to the first term starting from the other end. Cantor's pursuit of the infinite led him to the ridiculous idea of the *infinity* of infinities, and no other mathematicians followed his steps. If there is more than one infinite then by definition they become mere finites. Mathematicians of all ages had no clue as to the nature of the infinite; some denied its existence all together; while others maintained (following Plato) that mathematical entities can-

not be reduced to logical propositions, originating instead in the intuitions of the mind.

## 2 The infinite as a scientific category

Historically, natural science took a rather pragmatic and an opportunistic approach towards infinity, i.e., *reductio ad absurdum* argument which avoids the use of the infinite. It truncates infinity by putting an arbitrary limit as Georg Cantor did, and calls the rest the “absolute infinite” that is known only to infinite God. It deals with infinity with some arbitrary mathematical tricks, for example, a circle is the limit of regular polygons as the number of sides goes to infinity; an infinite series starts with a first term; in renormalization, one set of infinite is cancelled by invoking another set of infinite to get a finite result that was desired in the first place and so on.

Isaac Newton (1642–1727 A.D.) and Albert Einstein (1879–1955 A.D.) faced the same conceptual problems of the infinite universe in formulating their theories of gravity. Einstein declared, “Only the closed ness of the universe can get rid of this dilemma” [3]. He then set himself to develop a theory of gravity based on geometry, because geometry deals with closed space!

But an attempt to truncate infinity this way can only lead us back to medieval geocentric cosmology. The unpleasant fact is that, by definition a truncated infinite is also infinity and any mathematical operation on infinity leaves it unchanged as Galileo asserted in his famous 1638 pronouncement on infinity that, “Equal”, “greater”, and “less” cannot apply to infinite quantities [4]. The arbitrary renormalization process and *reductio ad absurdum* practiced by natural science cannot resolve the contradiction of the infinite; it only leads to more and more contradictions and a dependence on ever more mysteries and theology, as we observe in modern theoretical natural science. The reason why Albert Einstein chose a finite and closed universe as opposed to the open ones was not only to make his equations meaningful and/or because of his love for *simplicity* and *aesthetics*, as reductionist ideologues and worshipers of symmetry would have us believe, but also because of his sober realization that his Machean-philosophy based cosmology collapses in an infinite universe. If Mach’s principle is followed, then an infinite universe means that the inertia and the mass of atoms etc. also become infinite. To keep the world as we see it now (inertia, mass, etc.); all Mach based cosmologies must have the universe started at a finite past and also must have a finite extension. So this way the contradiction of infinity is not solved.

The notion of the infinite in natural science became ever more clouded after Albert Einstein established the primary role of mathematics in natural science. Natural science became seduced to the idea that where experimental evidence and empirical data is difficult and/or impossible to obtain

“logical consistency of mathematics” will lead the way. The stunning success of the theories of relativity in early 20th century, led Einstein to revive Pythagoras’s notion of mathematics. “How can it be” he wondered, “that mathematics being a *product of human thought* which is *independent of experience*, is so admirably appropriate to the objects of reality?” [5].

The theory of general relativity is a classic example where the power of mathematics, pure thought and aesthetics devoid of any empirical content is purported to have conceived the ultimate reality of the universe. “Our experience hitherto justifies us in believing that nature is the realization of the simplest conceivable mathematical ideas. I am convinced that we can discover by means of purely mathematical constructions the concepts and the laws connecting them with each other, which furnish the key to the understanding of natural phenomena. ... In a certain sense, therefore, I hold it true that pure thought can grasp reality, as the ancients dreamed”, declares Albert Einstein [6].

With his mathematical idealism Einstein erased the difference between the *pure* mathematics, whose program is the *exact* deduction of consequences from logically independent postulates, and the *applied* mathematics of *approximation* needed for science. Natural science uses approximate empirical data, which are fitted on in various ways to *analytic functions* of *pure* mathematics that helps in the systematization, generalization, and the formulation of tentative theories. But the results and the inferences are only valid in a narrow range of the data values for the argument for which approximate empirical information is available.

A convenient property of the analytic functions (such as the field equations) is that, such functions are known for all values of their argument when their values in any small range of the argument values are known and thereby allowing an unlimited extension of this procedure from the macrocosm to the microcosm. Thus, the *a priori* assumption that the laws of Nature involve *analytic functions* leads to a complete mechanistic determination of the world based on their experimentally determined value in a narrow range only. But the validity of such a procedure of unlimited extension of mathematical functions for the real world, were questioned both by mathematician/philosophers such as Bridgman [7] and scientists like Klein [8] at the advent of quantum mechanics; based as they argued (on different grounds) on the unavoidable inaccuracies of empirical knowledge. And as quantum mechanics clearly shows, there is uncertainty in the ontological nature of reality itself at micro level. So, our epistemological knowledge must always be defective, tentative and approximate, increasing in scope from one generation of humanity to the next; like an infinite mathematical series, without ever coming to a termination or without ever reaching one final and ultimate truth.

The quantum phenomena and the failure so far [9]; (in spite of over a century-long intense efforts by some of

the most brilliant mathematicians including Einstein) to unify “ALL” the particles and “ALL” the forces of Nature into a simple and reductionistic “theory of everything” demonstrate the folly of this kind of naïve and over-simplified extrapolation of idealized mathematics to the real world at the two opposite directions of infinity, i.e., macrocosm and microcosm.

### 3 The infinite as a philosophical category

The concept of the infinite was implicit in the early philosophical developments especially among the early Greek thinkers that centered around the basic questions of the primacy of spirit or nature, unity or multiplicity, stasis or motion. This debate divided the philosophers into two great camps. Those who asserted the primacy of spirit, unity and stasis formed the camp of idealism; the contrary camp formed the various schools of materialism.

The earliest idealist Greek philosophers (the Eleatics) denied the reality of becoming, multiplicity or motion; these characteristics they maintained, are of the sense-world or physical Nature. These they argued are not *real* but only *appearances* and hence these are illusions. For Parmenides (515–450 B.C.) for example the sole reality is Being, Being is One, only the One is; the Many not. This Being cannot be perceived by senses, it is given only to thought or mind. This line of thinking permeates the range of idealist philosophers like Plato, Aristotle, Berkeley, Hume, Hegel and all monotheistic religions. The Unity of Being in this view means that the infinite must be contained in this one Being. The Being meaning God in theological terms, the infinite, then became associated with abstract God. The idealist view of infinity was later incorporated into mathematics and theoretical natural science.

But the dialectically opposite and the materialist view of reality — i.e. the validity of the sense perception of change, multiplicity and motion in material Nature also arose simultaneously in early Greek philosophy. The founder of the dialectical view, Heraclitus (544–483 B.C.) on the contrary saw the world as a process — as changing eternally. For him Unity is not a homogenous unity, but is a “unity of the opposites or of opposite tendencies”. The Unity is a complex entity that contains at least two dominant opposite fragments that are in constant conflict with each other and renders this unity susceptible to diversity, change and movement. The concept of the infinite in this view is therefore, open ended. Epicurus (341~270 B.C.) following the tradition of Heraclitus was the first to assert that the universe is infinite in its extension in all directions and that multiplicity, time and motion are endless.

Benedict Spinoza (1632–1677 A.D.) made an important advance on the concept of infinity along the dialectical tradition which helped Hegel (himself an idealist) to formulate in a comprehensive way the dialectical view of the infinite in particular and his dialectical method in general. Spinoza formulated the profound idea that to define something is to set

boundaries for it; i.e., to determine is to limit. The infinite then is something that is undetermined or that has no limit or boundary. In other words the Infinite is limited only by itself and like God is “self-determined”.

In popular concept, God is supposed to be infinite. Spinoza’s idea of the infinite led to an insurmountable difficulty for conventional philosophy and theology which regarded the infinite and the finite as mutually exclusive opposites; absolutely cut off from each other. How then the infinite can be conceived; how infinite God can have contact with finite man, since it will limit His infiniteness. Finiteness of the world became a primary requirement for medieval theology. The inquisition did not hesitate to spill blood and torture victims to defend its doctrine. Hegel, following Spinoza called the “Absolute Idea” of his philosophy the “True Infinite” which is self-determined. For him the material world or Nature is a crude replica — an alienated form of the “Absolute Idea”.

The fundamental difference between these two world-views and hence their implication for the concept of infinity gets its concrete expression in the question of *matter* and *motion*. While Newton recognized matter as a *real entity*, for Einstein matter is a particular representation of an all pervading (space-time) reality (“Being” of Parmenides?). Einstein expressed this point of view in an unambiguous way, “Since the theory of general relativity (GR) implies the representation of physical reality by a continuous field, the concept of particles and material points cannot play a fundamental part and neither can the concept of motion. The particle can only appear as a limited region in space in which the field strength or energy density is particularly high” [10]. Motion in the view of both Newton and Einstein could only arise from an *impulse* from without — from God — the “unmoved mover”. And why energy density at particular points must arbitrarily be high to form material points must also depend on intervention by Providence. For dialectics (and quantum mechanics) on the contrary, matter and motion are the fundamental elements and the primary conditions of all physical reality; *motion is the mode of existence of matter. Matter without motion is as inconceivable as motion without matter.*

The only way the conceptual problem of infinity can be resolved is through the dialectics of Hegel — the law of the unity of the opposites. The notion that the finite and the infinite reside together in a contradiction; that they are united as well as are in opposition to each other. That, the finite **is** the infinite and vice versa. That this contradiction resolves itself continuously in the never-ending development in time and extension in space of the universe, in the same way as for example intellectual advance find its resolution in the progressive evolution of humanity from one particular generation to the next. Just as Nature or the universe (ontologically) is incapable of reaching a final, ever lasting, unchanging or an ideal state so is thought (which is only a reflection of Nature in the mind of man) epistemologically is incapable of comprehending a completed, exhaustive or immutable knowledge

— the so-called absolute truth of the world. For dialectics, “eternal change” (with temporary stages of infinite number of leaps) is the only thing that is permanent and the only absolute. Hegel’s dialectics therefore, is a condemnation of all claims to absolute truth by all idealism including the mathematical idealism of modern official natural science, which is but a reincarnation or rather restoration of the old idealism. In human history, as well as in the history of natural science, hitherto all claims to the “final truth” are but the partial masquerading as the complete.

The continuous resolution of the contradiction of the finite and the infinite like the other evolutionary processes are not only dialectical but they also develop historically following the three general laws i.e. i) transformation of quantity into quality and vice versa, ii) interpenetration of the opposites and iii) the negation of the negation. Engels [11] summarized these three laws from Hegel’s *Logic*, where the first law comprises the *Doctrine of Being*, the second, the *Doctrine of Essence*, while the third constitutes the fundamental law for the construction of the whole system. Hegel deduced his philosophy from the history of Nature, of society and of thought. The infinite universe is not a mere abstract, quality less, boring, endless extension of uniformity (spurious or *bad infinity*), it includes a variety of qualitative contents with different forms of movements passing one into the other and developing historically. The infinite space is adorned with the drama of things “coming into being” and “passing out of existence” in each of the innumerable island universes; each island universe with innumerable galaxies and each galaxy in turn with innumerable stars and planets. Under favorable conditions, galaxies propagate [12, 13]; the stars produce the higher elements; the planets give rise to the evolution of molecules, to organic life and finally to the thinking brain through which infinite Nature (for a brief period of time) *becomes conscious of itself* ! Self-consciousness is therefore, the property of the highest developed form of matter, which like everything else comes into being and passes out of existence as temporary bubbles in the eternal and infinite universe.

The knowledge of the infinite is therefore proportional to the knowledge of the finite. This knowledge is necessarily a historical and an iterative process progressing through successive generations of mankind without ever terminating in one final or absolute truth a quest of which was the aim of all idealism — mathematical, scientific or philosophical. A progressively better understanding of the infinite universe can only come about by studying the finite around us guided by the general laws of dialectics.

There are innumerable number of water and other molecules and atoms on earth and yet we understand (in a limited sense) and live at ease with these! The properties of matter and its structure under the various conditions in terrestrial nature must be the same that exists under similar conditions billions of light years away. In fact, one sun with its planets and its life supporting earth and one Milky Way galaxy with

its surrounding family group form the essential basis for an understanding of the universe. Beyond 15 billion light years there is no wonderland or lurking monsters to be seen. What we will see there is more or less the same we now see within a few million light years around us! The same applies to the micro-world. There is no limit of space, time or length in any direction; up-down, left-right; back-front, at least up to the level beyond which the terms mass, time or length lose their meaning (in the usual sense of the term) because of quantum uncertainty and due to other yet unknown effects. The limits from quasars (at the ultimate boundary of the universe?) to the quarks at the lowest end, set by Official Science must therefore be false; because this represents an arbitrary limitation of infinity, conditioned by the limitation of the empirical knowledge of our time.

#### 4 The “Absolute Idea” of Hegel as the “True Infinite”

As Engels pointed out [14], the dialectical view of the infinite as discussed above, are necessary logical conclusions from the dialectical method of Hegel; but conclusions he himself never expressed so explicitly. Hegel was an idealist and above all he was the official philosopher of the Royal Prussian court of Frederick William III. His task was to make a system of philosophy that must specify one absolute truth or a “first cause” of the world, as tradition demanded it. Therefore, even though Hegel, especially in his *Logic* emphasized that this absolute truth is nothing but the logical. i.e., historical *process* itself, he nevertheless found it necessary to bring his dialectical process to a termination in the “Absolute Idea”. For his philosophical “system” his dialectical “method” had to be untrue. Hegel also turned his philosophy upside down, where the “Absolute Idea” (like all idealism) became primary and nature only a crude reflection of the “Idea”, even though (through unprecedented detail and encyclopedic work) he extracted the laws of dialectics from the history of the material and the human world.

But nevertheless, the dialectical method of Hegel helped him to overcome the impossible contradiction of the infinite and the finite faced by Spinoza, theology and all previous idealist philosophies. For Hegel, the finite and the infinite are no independent entities separated from each other by an unbridgeable gap in between, as old philosophy asserted; but these are the integral components of a single unity within which the two opposites reside together in active unity and opposition, and hence in a logical contradiction. A resolution of this contradiction to an ever new “unity of the opposites” and so on — *the negation of the negation* is what gives rise to motion, change, development, and historical evolution of the universe as a never ending process.

Idealist Hegel can terminate the infinite process of change by making his “Absolute Idea” (the self-determined, the True Infinite”) as the ultimate end result of all change, motion, development or history, and making it the beginning again, i.e.

the end as the true beginning. For Hegel, the finite Nature or man IS the infinite “Absolute Idea” itself! The “Absolute Idea” alienates and disguises itself into Nature, evolves historically through all the usual twists and turns following the laws of dialectics and comes back to itself again through the consciousness of man and particularly through the philosophy of Hegel himself, who for the first time in the history of mankind perceived in thought the ultimate truth of this dialectical movement, in absolute profoundness. For Hegel the “Absolute Idea” which is the end result of all change, development, motion, history etc. — the static reality of Parmenides, the abstract God of theology, the self-determined entity of Spinoza, is the “True Infinite” and the absolute truth of the world.

But this “Absolute Idea” or the “True Infinite” of Hegel like the mathematical “Absolute Infinite” of Cantor; are only absolutes in the sense that they have absolutely nothing to say about it! Thus in spite of his prodigious intellect and in spite of the logical implication of his profound dialectical “method” to the contrary, Hegel unfortunately pursued the illusion of an absolute truth, like all the other idealist philosophers and all theological prophets of all times. The mathematical idealism and reductionism of modern official theoretical natural science inherited this illusion — i.e., the empty shell of all idealism but not the kernel — the dialectical “method” of this great idealist thinker.

## 5 Conclusion

During the last few centuries especially since Copernicus (1473–1543), natural science accumulated impressive empirical evidence and gained variable degrees of understanding of the terrestrial nature; that collectively vindicate Hegel’s assertion that *change* is the only absolute truth and that the dialectical laws are the only eternal laws that govern the development and the transformation of matter and life. But ironically, natural science claims its own invariable truth exactly in the areas where it possesses the least empirical evidence! As intoxicated modern official natural science celebrates its achievement of a definitive knowledge of one single event i.e., the “Big Bang” origin of the universe and the triumph of its mathematical idealism; with the award of Nobel Prizes, and as the world awaits in breathless anticipation the imminent discovery of a “theory of everything” that will bring an “End of Physics” and possibly the end of all knowledge (by “knowing the mind of God”, according to one of the leading physicists Stephen Hawking [15]); it would be instructive for us to remember the sober dialectical assessment of Frederick Engels [2, pp. 43–44] — one of the greatest inheritors of Hegel’s philosophy:

“The perception that all the phenomena of Nature are systematically interconnected drives science to prove this interconnection throughout, both in general and in detail. But an adequate,

exhaustive scientific statement of this interconnection, the formulation in thought of an exact picture of the world system in which we live, is impossible for us, and will always remain impossible. If at any time in the evolution of mankind such a final, conclusive system of the interconnections within the world — physical as well as mental and historical — were brought to completion, this would mean that human knowledge had reached its limit, and, from the moment when society had been brought into accord with that system, further historical evolution would be cut short — which would be an absurd idea, pure nonsense. Mankind therefore finds itself faced with a contradiction; on the one hand, it has to gain an exhaustive knowledge of the world system in all its interrelations; and on the other hand, because of the nature both of man and of the world system, this task can never be completely fulfilled. But this contradiction lies not only in the nature of the two factors — the world, and man — it is also the main lever of all intellectual advance, and finds its solution continuously, day by day, in the endless progressive evolution of humanity. . .”.

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