Can the Emdrive Be Explained by Quantised Inertia?

Michael Edward McCulloch

University of Plymouth, Plymouth, PL4 8AA, UK. E-mail: mike.mcculloch@plymouth.ac.uk

It has been shown that cone-shaped cavities with microwaves resonating within them move slightly towards their narrow ends (the emdrive). There is no accepted explanation for this. Here it is shown that this effect can be predicted by assuming that the inertial mass of the photons in the cavity is caused by Unruh radiation whose wavelengths must fit exactly within the cavity, using a theory already applied with some success to astrophysical anomalies where the cavity is the Hubble volume. For the emdrive this means that more Unruh waves are "allowed" at the wide end, leading to a greater inertial mass for the photons there. The gain of inertia of the photons when they move from the narrow to the wide end, and the conservation of momentum, predicts that the cavity must then move towards the narrow end, as observed. This model predicts the available observations quite well, although the observational uncertainties are not well known.

1 Introduction

It was first demonstrated by Shawyer (2008) that when microwaves are made to resonate within a truncated cone-shaped cavity a small, unexplained acceleration occurs towards the narrow end. In one example when 850 W of power was put into such a cavity with end diameters of 16 and 12 cm, and which had a Q value (dissipation constant) of 5900 the thrust measured was 16 mN towards the narrow end. The results from two of Shawyer's experiments are shown in Table 1 (rows 1-2). There is no explanation for this behaviour in standard physics, and it also violates the conservation of momentum, and Shawyer's own attempt to explain it using special relativity is not convincing, as this theory also should obey the conservation of momentum (Mullins, 2006).

Nethertheless, this anomaly was confirmed by a Chinese team (Juan et al., 2012) who put 80-2500 W of power into a similar cavity at a frequency of 2.45 GHz and measured a thrust of between 70 mN and 720 mN. Their result cannot however be fully utilised for testing here since they did not specify their cavity's Q factor or its geometry.

A further positive result was recently obtained by a NASA team (Brady et al., 2014) and three of their results are also shown in Table 1 (rows 3 to 5). They did provide details of their Q factor and some details of their cavity's geometry. The experiment has not yet been tried in a vacuum, but the abrupt termination of the anomaly when the power was switched off has been taken to show the phenomenon is not due to moving air.

McCulloch (2007) has proposed a new model for inertial mass that assumes that the inertia of an object is due to the Unruh radiation it sees when it accelerates, radiation which is also subject to a Hubble-scale Casimir effect. In this model only Unruh wavelengths that fit exactly into twice the Hubble diameter are allowed, so that a greater proportion of the waves are disallowed for low accelerations (which see longer Unruh waves) leading to a gradual new loss of inertia as accelerations become tiny, of order 10^{-10} m/s^2. This model, called MiHsC (Modified inertia by a Hubble-scale Casimir effect) modifies the standard inertial mass (m) as follows:

$$m_i = m\left(1 - \frac{2c^2}{|a|\Theta}\right) = m\left(1 - \frac{\lambda}{4\Theta}\right) \tag{1}$$

where c is the speed of light, Θ is twice the Hubble distance, a is the magnitude of the relative acceleration of the object relative to surrounding matter and λ is the wavelength of the Unruh radiation it sees. Eq. 1 predicts that for terrestrial accelerations (eg: 9.8 m/s^2) the second term in the bracket is tiny and standard inertia is recovered, but in low acceleration environments, for example at the edges of galaxies or in deep space (when a is small and λ is large) the second term in the bracket becomes larger and the inertial mass decreases in a new way.

In this way, MiHsC can explain galaxy rotation without the need for dark matter (McCulloch, 2012) and cosmic acceleration without the need for dark energy (McCulloch, 2007, 2010), but astrophysical tests like these can be ambiguous, since more flexible theories like dark matter can be fitted to the data, and so a controlled laboratory test like the EmDrive is useful.

Further, the difficulty of demonstrating MiHsC on Earth is the huge size of Θ in Eq. 1 which makes the effect very small unless the acceleration is tiny, as in deep space. One way to make the effect more obvious is to reduce the distance to the horizon Θ (as suggested by McCulloch, 2008) and this is what the emdrive may be doing since the radiation within it is accelerating so fast that the Unruh waves it sees will be short enough to be limited by the cavity walls in a MiHsC-like manner.

2 Method

The setup is a radio-frequency resonant cavity shaped like a truncated cone, with one round end then larger than the other. When the electromagnetic field is input in the cavity the microwaves resonate and we can consider the conservation of momentum for the light

$$\frac{\partial(mv)}{\partial t} = 0 = m\frac{\partial v}{\partial t} + v\frac{\partial m}{\partial t}. \tag{2}$$

Interpreting the first term on the right hand side as the force (mass times acceleration) that must be exerted on the light to conserve its momentum, leads to

$$F = -c\frac{\partial m}{\partial t}. \tag{3}$$

So that

$$F = -c\frac{\partial m}{\partial x}\frac{\partial x}{\partial t} = -c^2\frac{\partial m}{\partial x}. \tag{4}$$

Normally, of course, photons are not supposed to have mass in this way, but supposing we consider this? We assume the inertial mass of the microwave photons (whatever its absolute value) is affected by MiHsC, but instead of the horizon being the far-off and spherically symmetric Hubble horizon as before, the horizon is now made by the asymmetric walls of the cavity. This is possible because the photons involved are travelling at the speed of light and are bouncing very fast between the two ends of seperation s and their acceleration ($a \sim v^2/s$) is so large that the Unruh waves that are assumed to produce their inertial mass are about the same size as the cavity, so they can be affected by it, unlike the Unruh waves for a terrestrial acceleration which would be far to long to be affected by the cavity. This dependence of the inertial mass on the width of the cavity means that the inertial mass is corrected by a MiHsC-like factor (Eq. 1). Using Eq. 4, the force is modified as follows

$$F = -c^2\left(\frac{m_{bigend} - m_{smallend}}{l}\right) \tag{5}$$

where l is the axial length of the cavity. Now using eq. 1 for the inertial masses and replacing the Hubble scale with the cavity width (W) assuming for simplicity the waves only have to fit laterally, and with subscripts to refer to the big and small ends, we get

$$F = \frac{-c^2 m}{l}\left(\frac{\lambda}{4W_{big}} - \frac{\lambda}{4W_{small}}\right) \tag{6}$$

where λ is the wavelength of the Unruh radiation seen by the photons because they are being reflected back and forth by the cavity$\lambda = 8c^2/a = 8c^2/(2c/(l/c)) = 4l$ so that

$$F = -4c^2 m\left(\frac{1}{4W_{big}} - \frac{1}{4W_{small}}\right). \tag{7}$$

Using $E = mc^2$ and $E = \int Pdt$ where P is the power, gives

$$F = -\int Pdt\left(\frac{1}{W_{big}} - \frac{1}{W_{small}}\right). \tag{8}$$

Table 1: Summary of EmDrive experimental data published so far, and the predicted (Eq. 10) and observed anomalous thrust.

Expt.	P	Q	l	w_{big}/w_{small}	F_{Pred}	F_{Obs}
	W	/1000	m	metres	mN	mN
S1	850	5.9	0.156	0.16/0.1275	4.2	16
S2	1000	45	0.345	0.28/0.1289	216	80-214
B1	16.9	7.32	0.332	0.397/0.244	0.22	0.091
B2	16.7	18.1	0.332	0.397/0.244	0.53	0.05
B3	2.6	22	0.332	0.397/0.244	0.1	0.055

Integrating P over one cycle (one trip of the photons from end to end) gives Pt where t is the time taken for the trip, which is l/c, so

$$F = \frac{-Pl}{c}\left(\frac{1}{W_{big}} - \frac{1}{W_{small}}\right). \tag{9}$$

This is for one trip along the cavity, but the Q factor quantifies how many trips there are before the power dissipates so we need to multiply by Q

$$F = \frac{-PQl}{c}\left(\frac{1}{W_{big}} - \frac{1}{W_{small}}\right) \tag{10}$$

where P is the power input as microwaves (Watts), Q is the Q factor measured for the cavity, l is the length of the cavity and W_{big} and W_{small} are the diameters of the wide and narrow ends of the cavity. MiHsC then predicts that a new force will appear acting towards the narrow end of the cavity.

3 Results

We can now try this formula on the results from Shawyer (2008) (from section 6 of their paper). This EmDrive had a cavity length of 15.6 cm, end diameters of 16 cm and 12.75 cm, a power input of 850 W and a Q factor of 5900, so

$$F = \frac{850 \times 5900 \times 0.156}{3 \times 10^8}\left(\frac{1}{0.16} - \frac{1}{0.1275}\right) = 4.2 \text{ mN}. \tag{11}$$

This predicts an anomalous force of 4.2 mN towards the narrow end, which is about a third of the 16 mN towards the narrow end measured by Shawyer (2008).

We can also try values for the demonstrator engine from section 7 of Shawyer (2008) which had a cavity length of 32.5 cm, end diameters of 28 cm and 12.89 cm, a power input of 1000 W and a Q factor of 45000. So we have

$$\begin{aligned} F &= \frac{1000 \times 45000 \times 0.325}{3 \times 10^8}\left(\frac{1}{0.28} - \frac{1}{0.1289}\right) \\ &= 216 \text{ mN}. \end{aligned} \tag{12}$$

This agrees with the observed anomalous force which was between 80 and 214 mN/kW (2008) (if we also take into account the uncertainties in the model due to the simplified 1-dimensional approach used).

Table 1 is a summary of various results from Shawyer (2008) in rows 1 and 2 and Brady et al. (2014) (see the Table on their page 18) in rows 3, 4 and 5. The Juan et al. (2012) data is excluded because they did not specify their Q factor or the exact geometry in their paper. Column 1 shows the experiment (S for Shawyer (2008) and B for Brady et al. (2014)). Column 2 shows the input power (in Watts). Column 3 shows the Q factor (dimensionless, divided by 1000). Column 4 shows the axial length of the cavity. Column 5 shows the width of the big and small ends (metres). Column 6 shows the thrust predicted by MiHsC and column 7 shows the thrust observed (both in milli-Newtons).

It is unclear what the error bars on the observations are, but they are likely to be wide, looking for example at the range of values for the case S2. MiHsC predicts the correct order of magnitude for cases S1, S2, B1 and B3 which is interesting given the simplicity of the model and its lack of adjustable parameters. The anomaly is case B2 where MiHsC overpredicts by a factor of ten. This case is anomalous in other ways since the Q factor in B2 was more than doubled from that in B1 but the output thrust almost halved.

More data is needed for testing, and a more accurate modelling of the effects of MiHsC will be needed. This analysis for simplicity, assumed the microwaves only travelled along the axis and the Unruh waves only had to fit into the lateral "width" dimension, but in fact the microwaves will bounce around in 3-dimensions so a 3-d model will be needed. This approximation would become a problem for a pointed cone shape where the second term in Eq. 10 would involve a division by zero, but it is a better approximation for a truncated cone, as in these experiments.

So far, it has been assumed that as the acceleration reduces, the number of allowed Unruh waves decreases linearly, but even a small change of frequency can make the difference between the Unruh waves fitting within a cavity, and not fitting and this could explain the variation in the observations, particularly in case B2.

4 Discussion

If confirmed, Equation 10 suggests that the anomalous force can be increased by increasing the power input, or the quality factor of the cavity (the number of times the microwaves bounce between the two ends). It could also be increased by boosting the length of the cavity and narrowing it. The effect could be increased by increasing the degree of taper, for example using a pointed cone. The speed of light on the denominator of Eq. 10 implies that if the value of c was decreased by use of a dielectric the effect would be enhanced (such an effect has recently been seen).

This proposal makes a number of controversial assumptions. For example that the inertial mass of photons is finite and varies in line with MiHsC. It is difficult to provide more backing for this beyond the conclusion that it is supported by the partial success of MiHsC in predicting the EmDrive with a very simple formula.

5 Conclusions

Three independent experiments have shown that when microwaves resonate within an asymmetric cavity an anomalous force is generated pushing the cavity towards its narrow end.

This force can be predicted to some extent using a new model for inertia that has been applied quite successfully to predict galaxy rotation and cosmic acceleration, and which assumes in this case that the inertial mass of photons is caused by Unruh radiation and these have to fit exactly between the cavity walls so that the inertial mass is greater at the wide end of the cavity. To conserve momentum the cavity is predicted to move towards its narrow end, as seen.

This model predicts the published EmDrive results fairly well with a very simple formula and suggests that the thrust can be increased by increasing the input power, Q factor, or by increasing the degree of taper in the cavity or using a dielectric.

Acknowledgements

Thanks to Dr Jose Rodal and others on an NSF forum for estimating from photographs some of the emdrives' dimensions.

Submitted on December 18, 2014 / Accepted on December 19, 2014

References

1. Brady D.A., White H.G., March P., Lawrence J.T. and Davies F.J. Anomalous thrust production from an RF test device measured on a low-thrust torsion pendulum. 50th AIAA/ASME/SAE/ASEE Joint Propulsion conference, 2014.
2. Juan Y. Net thrust measurement of propellantless microwave thrusters. *Acta Physica Sinica*, 2012, v. 61, 11.
3. McCulloch M.E. The Pioneer anomaly as modified inertia. *MNRAS*, 2007, v. 376, 338–342.
4. McCulloch M.E. Can the flyby anomaly be explained by a modification of inertia? *J. Brit. Interplanet. Soc.*, 2008, v. 61, 373–378.
5. McCulloch M.E. Minimum accelerations from quantised inertia. *EPL*, 2010, v. 90, 29001.
6. McCulloch M.E. Testing quantised inertia on galactic scales. *Astro. & Space Sci.*, 2012, v. 342, 575–578.
7. Mullins J. Relativity drive: the end of wings and wheels? *New Scientist*, 2006, no. 2568, 30–34.
8. Shawyer R. Microwave propulsion — progress in the emdrive programme. 59th International Astronautical conference (IAC-2008). Glasgow, UK.