# Nuclear Phase Transition from Spherical to Axially Symmetric Deformed Shapes Using Interacting Boson Model

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The interacting boson model (sd-IBM1) with intrinsic coherent state is used to study the shape phase transitions from spherical U(5) to prolate deformed SU(3) shapes in Nd-Sm isotopic chains. The Hamiltonian is written in the creation and annihilation form with one and two body terms.For each nucleus a fitting procedure is adopted to get the best model parameters by fitting selected experimental energy levels, B(E2) transition rates and two-neutron separation energies with the calculated ones.The U(5)-SU(3) IBM potential energy surfaces (PES's) are analyzed and the critical phase transition points are identified in the space of model parameters.In Nd-Sm isotopic chains nuclei evolve from spherical to deformed shapes by increasing the boson number. The nuclei $^{150}$Nd and $^{152}$Sm have been found to be close to critical points.We have also studied the energy ratios and the B(E2) values for yrast band at the critical points.

## 1 Introduction

The interacting boson model (IBM) [1] describes the low energy quadruple collective states of even-even nuclei in terms of bosons with angular momentum 0 and 2 so called s and d bosons. The bosonic Hamiltonian is assumed to have a general form with one- and two-body terms and must be invariant under some fundamental symmetries. The algebraic formulation of the IBM allows one to find analytical solutions associated with breaking the U(6) into three dynamical symmetries called U(5), SU(3) and O(6) limits of the model, corresponding to spherical (vibrational), axially symmetric prolate deformed (rotational)and soft with respect to axial symmetric ($\gamma$-unstable) shapes respectively.

Phase transitions between the three shapes of nuclei are one of the most significant topics in nuclear structure research [2-11]. These shape phase transitions were considered in the framework of the geometric collective model [12], resulting in the introduction of the critical point symmetries E(5) [13] X(5) [14]. Y(5) [15], Z(5) [16] and E(5/4) [17]. The E(5) corresponds to the second order transition between U(5) and O(6), while X(5) corresponds to the first order transition between U(5) and SU(3). The symmetry at the critical point is a new concept in the phase transition theory, especially for a first order transition. From the classical point of view, in a first order transition, the state of the system changed discontinuously and a sudden rearrangement happens, which means that there involves an irregularity at critical point [18].

Empirical evidence of these transitional symmetries at the critical points has been observed in several isotopes.
The study of the shape phase transitions in nuclei can be best done in the IBM, which reproduces well the data in several transitional regions [8, 11].

In this paper we use the IBM with intrinsic coherent states to study the spherical to prolate deformed shape transition in the Nd-Sm isotopic chains. Section 2 outlines the theoretical approach and the main features of the U(5)-SU(3) model, the model Hamiltonian under study is introduced in subsection 2.1. In subsection 2.2 the intrinsic coherent states are given as energy states of the model Hamiltonian.In section 3 we present the numerical results of PES's for Nd-Sm isotopic chains and gives some discussions. Finally a conclusion is given in section 4.

## 2 Outline of the theoretical approach

### 2.1 The general Hamiltonian of the sd-IBM

In order to study the geometric shapes associated with the sd-IBM, we consider the most standard one and two body IBM Hamiltonian [1]

$$\begin{aligned} H &= \epsilon_s \hat{n}_s + \epsilon_d \hat{n}_d \\ &+ \sum_L \frac{1}{2}\sqrt{2L+1}\, C_L \Big[[d^\dagger \times d^\dagger]^{(L)} \times [\tilde{d} \times \tilde{d}]^{(L)}\Big]^{(0)} \\ &+ \frac{1}{\sqrt{2}} v_2 \left(\Big[[d^\dagger \times d^\dagger]^{(2)} \times \tilde{d}s\Big]^{(0)} + \Big[s^\dagger d^\dagger \times [\tilde{d}\times\tilde{d}]^{(2)}\Big]^{(0)}\right) \\ &+ \frac{1}{2} v_0 \left(\Big[[d^\dagger \times d^\dagger]^{(0)} \times ss\Big]^{(0)} + \Big[s^\dagger s^\dagger \times [\tilde{d}\times\tilde{d}]^{(0)}\Big]^{(0)}\right) \\ &+ u_2 [d^\dagger s^\dagger \times \tilde{d}s]^{(0)} + \frac{1}{2} u_0 [d^\dagger s^\dagger \times ss]^{(0)} \end{aligned} \quad (1)$$

with

$$C_L = \langle ddL|v|ddL\rangle, \quad (2)$$

$$v_2 = \sqrt{\frac{5}{2}}\,\langle dd2|v|ds2\rangle, \quad (3)$$

$$v_0 = \langle dd0|v|ss0\rangle, \quad (4)$$

$$u_2 = 2\sqrt{5}\,\langle ds2|v|ds2\rangle, \quad (5)$$

$$u_0 = \langle ss0|v|ss0\rangle, \quad (6)$$

where $s^\dagger(s)$ and $d^\dagger(\tilde{d})$ are the creation and annihilation operators of the s and d bosons. $\tilde{d}$ is the annihilation operator of the d boson with the time reversal phase relation $\tilde{d}_{2k} = (-1)^{2+k} d_{2,-k}$ .

### 2.2 The intrinsic coherent state

The geometric picture of the IBM can be investigated by introducing the intrinsic coherent state which is expressed as a boson condensate [19]:

$$|N\beta\gamma\rangle = \frac{1}{\sqrt{N!}}({b_c}^\dagger)^N|0\rangle, \quad (7)$$

$${b_c}^\dagger = \frac{1}{\sqrt{1+\beta^2}}\left[s^\dagger + {d_0}^\dagger\,\beta\cos\gamma + \frac{1}{\sqrt{2}}({d_2}^\dagger + {d_{-2}}^\dagger)\beta\sin\gamma\right], \quad (8)$$

where N is the boson number, $\beta$ and $\gamma$ are the intrinsic deformation parameters which determine the geometrical shape of the nucleus. $|0\rangle$ is the boson vacuum. Here $\beta \geq 0$, $0 \leq \gamma \leq \frac{\pi}{3}$.

### 2.3 The Potential Eneryg Surface (PES)

The PES associated with the classical limit of IBM Hamiltonian (1) is given by its expectation value in the intrinsic coherent state (7)

$$\begin{aligned} E(N,\beta,\gamma) &= \langle N\beta\gamma|H|N\beta\gamma\rangle = \epsilon_s\frac{N}{1+\beta^2} + \epsilon_d\frac{N\beta^2}{1+\beta^2} + \\ &\left(\frac{1}{10}C_0 + \frac{1}{7}C_2 + \frac{9}{35}C_4\right) N(N-1)\frac{\beta^4}{(1+\beta^2)^2} - \\ &\frac{2}{\sqrt{35}}v_2 N(N-1)\frac{\beta^3\cos 3\gamma}{(1+\beta^2)^2} + \frac{1}{\sqrt{5}}(v_0+u_2)N(N-1) \\ &\frac{\beta^2}{(1+\beta^2)^2} + \frac{1}{2}u_0 N(N-1)\frac{1}{(1+\beta^2)^2}. \end{aligned} \quad (9)$$

If the parameter $v_2 = 0$, then the PES is independent of $\gamma$. If $v_2 \neq 0$ then for every $\beta > 0$ the PES has a minimum at $\gamma = 0$, if $v_2 > 0$ (axially symmetric case with prolate shape) or $\gamma = \frac{\pi}{3}$ if $v_2 < 0$ (oblate shape).

The PES equation (9) can be written in another form as:

$$\frac{E(N,\beta,\gamma)}{N} = \frac{A_2\beta^2 + A_3\beta^3\cos 3\gamma + A_4\beta^4}{(1+\beta^2)^2} + A_0 \quad (10)$$

Table 1: Equilibrium values of the parameters $A_2, A_3, A_4$ in the large N limit for transition from dynamical symmetry limit U(5) to dynamical symmetry limit SU(3) as an illustrative example.

| Set | $A_2$ | $A_3$ | $A_4$ |
|---|---|---|---|
| a | 500 | -283 | 850 |
| b | 102 | -508 | 703 |
| c | 91 | -514 | 727 |
| d | 0 | -566 | 700 |
| e | -250 | -707 | 625 |
| f | 95 | -512 | 728 |
| g | 85 | -517 | 725 |

with

$$A_2 = \epsilon_d - \epsilon_s - u_0 + (N-1)\frac{1}{\sqrt{5}}(u_2 + v_0), \quad (11)$$

$$A_3 = -\frac{2}{\sqrt{35}}(N-1)v_2, \quad (12)$$

$$A_4 = \epsilon_d - \epsilon_s - \frac{1}{2}u_0 + (N-1)\left(\frac{1}{10}C_0 + \frac{1}{7}C_2 + \frac{9}{35}C_4\right), \quad (13)$$

$$A_0 = \frac{1}{2}u_0. \quad (14)$$

To determine the critical values of the order parameters of the system, one needs to determine the locus of points for which the conditions $\frac{\partial E}{\partial \beta} = 0$ and $\frac{\partial^2 E}{\partial \beta^2} = 0$ are fulfilled.

The equilibrium value of $\beta$ is determined by:

$$\frac{\partial E(N,\beta)}{\partial \beta} = 0, \quad (15)$$

leading to

$$\beta\left[2A_2 + 3A_3\beta + (4A_4 - 2A_2)\beta^2 - A_3\beta^3\right] = 0. \quad (16)$$

Figure (1) (with the parameters listed in table (1)) illustrates the critical points: For $A_2 = 1, A_3 = A_4 = 0$, the nucleus is in the symmetric phase since the PES has a unique minimum at $\beta = 0$ when $A_3$ and $A_4$ not vanish and $A_2$ decreases, a second nonsymmetric minimum arises (set b) at $\beta \neq 0$. This non symmetric minimum take the same depth of the symmetric one at the critical point (set c). Beyond this value, the symmetric minimum at $\beta = 0$ becomes unstable point (set d). (Sets g, h) show two cases in the coexistence region.

## 3 Application to Nd–Sm isotopic chains

Nuclei in rare-earth region are well-known examples of the U(5)-SU(3). The validity of the present technique is examined for the rare earth isotopic chains $^{144-154}$Nd and $^{146-162}$Sm. The optimized values of the nine parameters of the Hamiltonian $\epsilon_s, \epsilon_d, c_0, c_2, c_4, u_0, u_2, v_0, v_2$ which are truncated to four parameters $A_2, A_3, A_4, A_0$ are adjusted by fitting

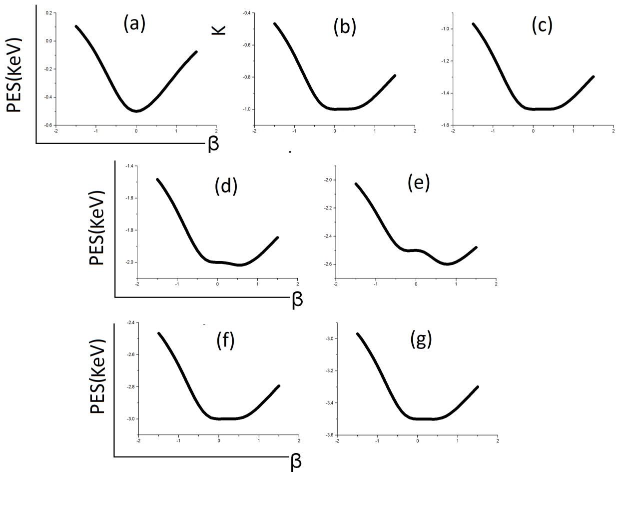


Fig. 1: The scaled PES's as a function of the deformation parameter $\beta$ for the model parameters listed in table (1). The curves (b, c, d) represents the spinodal, critical and antispinodal points respectively. The curves (f, g) show two cases on the coexistence region.

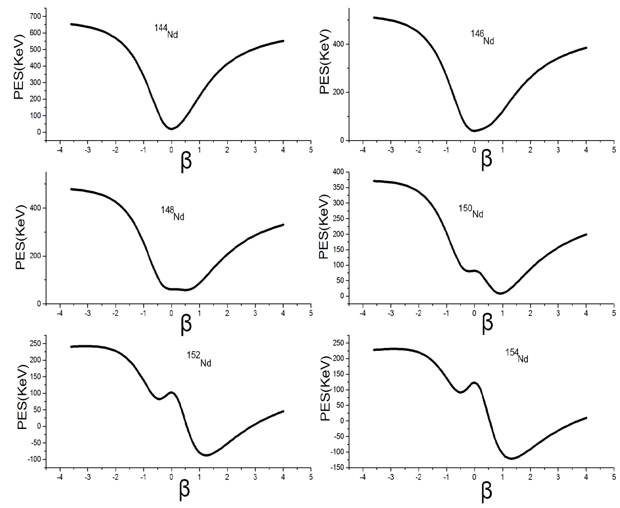


Fig. 2: The PES's (in the $\gamma = 0$ plane given by the IBM as a function of deformation parameter $\beta$ , to describe the U(5)-SU(3) transition in $^{144-154}$Nd isotopic chain. The calculations are for $\chi = -\sqrt{7}/2$.

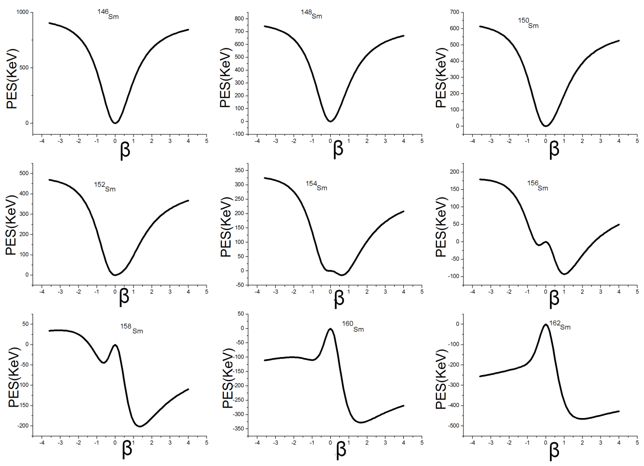


Fig. 3: The same as Fig.2 but for $^{146-162}$Sm isotopic chain.

procedure using a computer simulated search program in order to describe the gradual change in the structure as neutron number varied (number of bosons) and to reproduce ten positive parity experimental levels namely ($2_1^\dagger$, $4_1^\dagger$, $6_1^\dagger$, $8_1^\dagger$, $0_2^\dagger$, $2_3^\dagger$, $4_3^\dagger$, $2_2^\dagger$, $3_1^\dagger$ and $4_2^\dagger$), the B(E2) values and the two neutron separation energies for each nucleus in each isotopic chain. The effect of $\epsilon_s$ be ignored also the parameter $u_0$ is kept zero because it can be absorbed in the three parameters. The resulting model parameters are listed explicitly in Table (2). The PES's $E(N,\beta)$ as a function of the deformation parameter $\beta$for our Nd-Sm isotopic chains evolving from spherical to axially symmetric well deformed nuclei are illustrated in the Figures 2, 3. At the critical points ($^{150}$Nd, $^{152}$Sm) the spherical and deformed minima must coexist and be degenerated in order to obtain a first order phase shape transition. To identify the shape phases and their transition it is helpful to examine the correspondence between the interaction strengths in the microscopic model and the dynamical symmetry in the IBM.

Phase transitions in nuclei can be tested by calculating the energy ratios

$$R_{I/2} = E(I_1^+)/E(2_1^+). \tag{17}$$

For $I = 4$, the ratio $R_{4/2}$ varied from the values which correspond to vibrations around a spherical shape$R_{4/2} = 2$ to the characteristic value for excitations of a well deformed rotor $R_{4/2} = 3.33$. Figure (4) shows the $R_{I/2}$ for $^{150}$Nd and $^{152}$Sm compared to U(5) and SU(3) prediction.

Now, we discuss the electric quadruple transition probabilities. The general form of the E2 operator was used

$$T^{(E2)} = \alpha\left(\left[d^\dagger \times \tilde{s} + s^\dagger \times d^\dagger\right]^{(2)} + \beta\left[\tilde{d} \times \tilde{d}\right]^{(2)}\right) \tag{18}$$

where $\alpha$ is the boson effective charge and $\beta$ is the structure parameter. The parameters $\alpha$ and $\beta$ have been determined directly from the least square fitting to the observed $\beta(E2)$. $\alpha = 0.135$ and $\beta = -0.115$. The ratios of the E2 transition rates for the U(5) and SU(3) are given by

$$\begin{aligned} B_{(I+2)/2} &= B(E2, I+2 \rightarrow I)/B(E2, 2_1^+ \rightarrow 0_1^+), \\ &= \frac{1}{2}(I+2)\left(1 - \frac{I}{2N}\right) \qquad \text{for U(5)}, \\ &= \frac{15}{2}\frac{(I+2)(I+1)}{(2I+3)(2I+5)}\left(1 - \frac{I}{2N}\right)\left(1 + \frac{I}{2N+3}\right) \\ &\quad \text{for SU(3)}. \end{aligned} \tag{19}$$

In Figure (5), the $B_{(I+2)/2}$ ratios are shown for the best candi-

Table 2: The adopted best model parameters in (keV) for our selected Nd-Sm isotopic chains.

| | $N_B$ | $A_2$ | $A_3$ | $A_4$ | $A_0$ |
|---|---|---|---|---|---|
| $^{144}$Nd | 6 | 400.132 | -242.551 | 636.717 | 18.936 |
| $^{146}$Nd | 7 | 168.175 | -291.061 | 452.077 | 39.874 |
| $^{148}$Nd | 8 | 54.518 | -339.571 | 385.737 | 60.812 |
| $^{150}$Nd | 9 | -140.338 | -388.081 | 238.197 | 81.751 |
| $^{152}$Nd | 10 | -359.495 | -436.591 | 66.357 | 102.689 |
| $^{154}$Nd | 11 | -452.052 | -485.102 | 21.117 | 123.627 |
| $^{146}$Sm | 7 | 748.245 | -160.541 | 946.905 | 0.0 |
| $^{148}$Sm | 8 | 554.405 | -187.298 | 786.175 | 0.0 |
| $^{150}$Sm | 9 | 360.565 | -214.055 | 625.445 | 0.0 |
| $^{152}$Sm | 10 | 166.725 | -240.812 | 464.715 | 0.0 |
| $^{154}$Sm | 11 | -27.115 | -267.569 | 303.985 | 0.0 |
| $^{156}$Sm | 12 | -220.955 | -294.326 | 143.255 | 0.0 |
| $^{158}$Sm | 13 | -414.795 | -321.083 | -17.475 | 0.0 |
| $^{160}$Sm | 14 | -608.635 | -347.839 | -178.205 | 0.0 |
| $^{162}$Sm | 15 | -802.475 | -374.596 | -338.935 | 0.0 |

date $^{152}$Sm compared to the U(5) and SU(3) predictions and the experimental data.

## 4 Conclusion

The shape transition U(5)-SU(3) in $^{144-154}$Nd and $^{146-162}$Sm isotopic chains in the rare earth region is studied in the framework of sd IBM1 using the most general Hamiltonian in terms of creation and annihilation operators using the method of the intrinsic states.

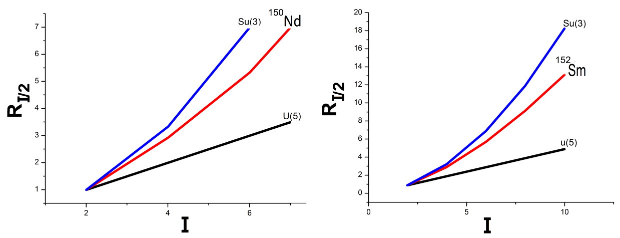


Fig. 4: Normalized excitation energies $R_{I/2} = E((I_1^{\dagger})/E((I_2^{\dagger})$ for $^{150}$Nd and $^{152}$Sm nuclei compared to U(5) and SU(3) predictions.

The optimized model parameters have been deduced by using a computer simulated search program in order to obtain a minimum root mean square deviation of the calculated some excitation energies, the two neutron separation energies and some B(E2) values from the measured ones. The PES's are analyzed and the location of the critical points are obtained. In our Nd and Sm chains, nuclei evolve from spherical to prolate deformed shape transition. The lighter nuclei are spherical and the heavier are well deformed. The $^{150}$Nd and the $^{152}$Sm have been found to be critical point nuclei, that is the

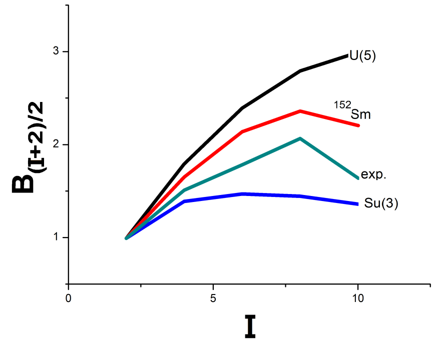


Fig. 5: Comparison of the $B_{I+2/2} = B(E2, I+2 : I)/B(E2, (2_1^{\dagger}, 0_1^{\dagger})$ ratios of the ground state band in $^{152}$Sm (N=11) compared to the U(5) and SU(3) predictions and the experimental ratio.

transition from the spherical to deformed occurs between boson number N=9 and N=10. The energy ratios and the B(E2) values are also studied.

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