The de Broglie Relations Derived from the Electron and Proton Coupling to the Planck Vacuum State

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This paper argues that the de Broglie relations for the electron and proton are the result of their coupling to the Planck vacuum state, the continuum nature of that state impressing a wave-like behavior onto the free-space-particle aspect of the two particles. Lorentz transforming the vanishing of their corresponding particle/vacuum coupling forces at their respective Compton radii, treated as Lorentz invariant constants, leads to their space-direction and time-direction de Broglie relations. Results: explain the peculiar form of the relativistic particle energy $\sqrt{m^2c^4 + c^2p^2}$; define the de Broglie waves for the electron and proton as periodic undulations within the Planck vacuum in the vicinity of the electron and proton cores; and easily explain the double-slit electron-diffraction thought experiment.

1 Force transformation

The electron and proton cores, $(-e_*, m_e)$ and (e_*, m_p), exert the two-term coupling forces [1]

$$\pm\left(\frac{e_*^2}{r^2} - \frac{mc^2}{r}\right) \tag{1}$$

on the Planck vacuum (PV) negating-energy continuum, where the plus and minus signs refer to the electron and proton respectively and mc^2 represents the rest energy of either particle. The bare charge e_* is assumed to be a massless point charge. The massive particle cores, however, possess a small spherical extension due to the zero-point formation of their derived masses [2].

The coupling force vanishes

$$\frac{e_*^2}{r_c^2} - \frac{mc^2}{r_c} = 0 \tag{2}$$

at the Compton radius r_c $(= e_*^2/mc^2)$ of either particle, leading to the Compton relations

$$r_c \cdot mc^2 = e_*^2 \qquad \longrightarrow \qquad r_e m_e c^2 = r_p m_p c^2 = e_*^2 \tag{3}$$

for the electron $(r_e m_e)$ and proton $(r_p m_p)$, and the (reduced) Planck constant $\hbar = e_*^2/c$. It is noted that (1) is a force acting between a free-space particle and the vacuum state – it is not a free-space/free-space force as are the Coulomb and Newton forces. The Compton relations and $\hbar = e_*^2/c$ are used throughout the following calculations.

The vanishing force (2) can be expressed as a tensor 4-force difference. In the primed rest frame of the particle where these static forces apply, this vanishing force difference $\Delta F'_\mu$ is $(\mu = 1, 2, 3, 4)$

$$\Delta F'_\mu = \left[\mathbf{0}, i\left(\frac{e_*^2}{r_c^2} - \frac{mc^2}{r_c}\right)\right] = [0, 0, 0, i\,0] \tag{4}$$

where $i = \sqrt{-1}$. Thus the vanishing of the component $\Delta F'_4 = 0$ in (4) can be thought of as the source of the Compton relations in (3).

The force difference in the laboratory frame (in which the rest frame travels at velocity v along the z-axis) [3]

$$\Delta F_\mu = a_{\mu\nu}\Delta F'_\nu = 0_\mu \tag{5}$$

follows from the tensor nature of (4) and the Lorentz transformation $x_\mu = a_{\mu\nu}\, x'_\nu$, where $x_\mu = (x, y, z, ict)$,

$$a_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & -i\beta\gamma \\ 0 & 0 & i\beta\gamma & \gamma \end{pmatrix} \tag{6}$$

and $\mu, \nu = (1, 2, 3, 4)$. Thus (5) yields

$$\begin{aligned} \Delta F_\mu &= \left[0, 0, \beta\gamma\left(\frac{e_*^2}{r_c^2} - \frac{mc^2}{r_c}\right), i\,\gamma\left(\frac{e_*^2}{r_c^2} - \frac{mc^2}{r_c}\right)\right] \\ &= \left[0, 0, \frac{1}{r_c}\left(\frac{e_*^2}{r_d} - c \cdot m\gamma v\right), \frac{i}{r_c}\left(\frac{e_*^2}{r_L} - c \cdot m\gamma c\right)\right] \\ &= [0, 0, 0, i\,0] \end{aligned} \tag{7}$$

where

$$r_d = \frac{r_c}{\beta\gamma} \qquad \text{and} \qquad r_L = \frac{r_c}{\gamma} \tag{8}$$

are the de Broglie radii for the space and time directions respectively; and where $\beta = v/c < 1$ and $\gamma = 1/\sqrt{1-\beta^2}$.

The force difference $\Delta F_3 = 0$ in (7) gives the de Broglie relation

$$r_d \cdot cp = e_*^2 \qquad \text{or} \qquad r_d = \frac{\hbar}{p} \tag{9}$$

in the space direction, where $p = m\gamma v$ is the relativistic particle momentum. The force difference $\Delta F_4 = 0$ gives the de Broglie relation

$$r_L \cdot E = e_*^2 \qquad \text{or} \qquad r_L = \frac{\hbar}{m\gamma c} \tag{10}$$

in the time direction, where $E = m\gamma c^2$ is the total relativistic particle energy.

The momentum and energy in equations (9) and (10) are derived from nothing more than the vanishing of the Lorentz transformation of (2), whose results can be taken a step further:

$$\begin{aligned} E &= \frac{e_*^2}{r_L} = \frac{e_*^2\gamma}{r_c} = mc^2\gamma \\ &= mc^2\left(1 + \frac{\beta^2}{1-\beta^2}\right)^{1/2} \\ &= \left(m^2c^4 + c^2p^2\right)^{1/2} \end{aligned} \quad (11)$$

showing that this well known equation has its source in the two-term particle/PV coupling force.

2 Conclusions and comments

The vast accumulation of electron diffraction data leaves no doubt that the electron and proton possess a wave nature. If the corresponding waves are roughly expressed in terms of planewaves, then it is reasonable to assign $2\pi r_d$ and $2\pi r_L$ as the wavelengths in the space and time directions respectively. As a first approximation then, the electron and proton de Broglie waves are planewaves propagating within the PV continuum.

The Synge primitive (or planewave) quantization of spacetime [4, p.106] is an independent calculation that parallels the ideas of the previous paragraph. That quantization divides the space and time axes of the Minkowski spacetime diagram into equal segments, where the space and time segments are r_d and r_L respectively (Synge actually multiplies these two segments by 2π which defines a phase space). The particle/PV coupling of the previous section provides the physical explanation for that quantization in terms of the coupling force (1).

Although the implied mathematics of the two previous paragraphs involves planewaves (which are global), the PV wave phenomenon must be a local property associated with the particle/PV interaction in the neighborhood of the particle cores $(-e_*, m_e)$ and (e_*, m_p), with characteristic (radian) frequencies defined by

$$\omega_c = \frac{e_*^2/r_c}{\hbar} = \frac{c}{r_c} \quad (12)$$

with

$$\omega_L = \frac{e_*^2/r_L}{\hbar} = \gamma\omega_c \quad \text{and} \quad \omega_d = \frac{e_*^2/r_d}{\hbar} = \beta\gamma\omega_c \quad (13)$$

for each particle. Then (11) yields

$$\omega_L^2 = \omega_c^2 + \omega_d^2\,. \quad (14)$$

The preceding results offer a simple explanation for the double-slit thought experiment [5, p.85]. Consider a collimated beam of monoenergetic electrons that is directed at an opaque wall containing two narrow, parallel, and closely spaced slits A and B, with a detection screen at some distance beyond the slits. Being a particle (although with a wave-like nature), the electron cannot go through both slits at the same time. Now consider the two experiments: (1) with slit A open and slit B closed; and (2) with both slits A and B open. Assume that the slits are narrower than one de Broglie wavelength ($2\pi r_d$) and that their separation distance is several wavelengths.

If the electrons are particle-like with no wave-like qualities, the screen would show a bell-shaped excitation curve in case (1) and two superimposed bell-shaped curves in case (2). But for case (2), however, the *overwhelming* diffraction evidence demands a well defined oscillatory excitation curve on the screen — because the particle exhibits a definite wave-particle nature. Since the electron must go through A or B, but not both, this result is difficult to understand [5, p.85] with present-day physics. But if the free-space particle is accompanied by a PV de Broglie wave, the diffraction of that wave through A and B, and its interaction with the particle core, easily explains the oscillatory curve on the detection screen.

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