The Impact Crater Size-Frequency Distribution on Pluto Follows a Truncated Pareto Distribution: Results from a First Data Set Based on the Recent New Horizons' Flyby

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Recently it could be shown (Scholkmann, *Prog. in Phys.*, 2016, v. 12(1), 26-29) that the impact crater size-frequency distribution of Pluto (based on an analysis of first images obtained by the recent New Horizons' flyby) follows a power law ($\alpha = 2.4926 \pm 0.3309$) in the interval of diameter (D) values ranging from 3.75 ± 1.14 km to the largest determined value of 37.77 km. A reanalysis of this data set revealed that the whole crater SFD (i.e., with values in the interval of 1.2–37.7 km) can be described by a truncated Pareto distribution.

1 Introduction

The recent flyby of NASA's New Horizons spacecraft allowed high-resolution images of Pluto's surface morphology to be obtained, thus enabling a first determination of the impact crater size-frequency distribution (SFD) of a specific region, i.e., covering parts of Pluto's regions *Sputnik Planum*, *Al-Idrisi Montes* and *Voyager Terra* [1].

The first analysis of the crater SFD used a power law of the type $p(x) \sim x^{-\alpha}$ to model the data. In the present paper we show the results of an extended analysis. The inverse power law scaling is known as the Pareto distribution $p(x) \sim x^{-(c+1)}$. In the present paper we tested the hypothesis that an upper truncated Pareto distribution (i.e., a Pareto distribution in which the probability range is limited rather than infinite) can improve the modelling of the empirical crater SFD presented in [1].

We review the properties of the Pareto and the truncated Pareto distributions in Section 2, and report in Section 3 the results of applying the truncated Pareto distribution to the novel Pluto crater SFD data set.

2 From the Pareto to the truncated Pareto distribution

In the follwing we report the definitions of the probability density function (PDF), the distribution function (DF), the survival function (S) and the maximum likelihood estimator (MLE) for the two distributions analyzed here. The sample is made by crater diameter (D) values ($n = 83$) denoted by x_i.

2.1 The Pareto distribution

The Pareto PDF is given by

$$f(x; a, c) = ca^c x^{-(c+1)}, \tag{1}$$

with $c > 0$; the Pareto DF is defined as

$$F(x; a, c) = 1 - a^c x^{-c}, \tag{2}$$

and the survival function is given by

$$S(x; a, c) = 1 - F(x; a, c). \tag{3}$$

The parameter values can be estimated by applying the MLE:

$$a = \min(x_i), \tag{4a}$$

$$\frac{1}{c} = \left(\frac{1}{n}\right) \sum_{i=1}^{n} \ln\left(\frac{x_i}{\tilde{a}}\right). \tag{4b}$$

More details can be found in [2].

2.2 The truncated Pareto distribution

An upper truncated Pareto random variable is defined in the interval $[a, b]$, and the PDF is given by

$$f_T(x; a, b, c) = \frac{ca^c x^{-(c+1)}}{1 - \left(\frac{a}{b}\right)^c}; \tag{5}$$

and the truncated DF is defined as

$$F_T(x; a, b, c) = \frac{1 - \left(\frac{a}{x}\right)^c}{1 - \left(\frac{a}{b}\right)^c}. \tag{6}$$

The MLE determines the parameters according to

$$a = \min(x_i), \tag{7a}$$

$$b = \max(x_i), \tag{7b}$$

$$0 = \frac{n}{\tilde{c}} + \frac{n\left(\frac{a}{b}\right)^{\tilde{c}} \ln\left(\frac{a}{b}\right)}{1 - \left(\frac{a}{b}\right)^{\tilde{c}}} - \sum_{i=1}^{n} [\ln x_i - \ln a], \tag{7c}$$

where the value of $\tilde{c}$ can be found using Brent's method to find a root of a nonlinear function, i.e., by applying the FORTRAN subroutine ZBRENT [3]. More details can be found in [4].

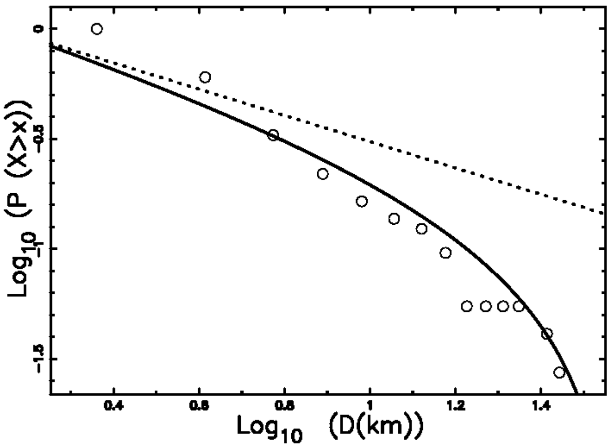

Fig. 1: Survival function (S) in a log-log plot for crater size in D = [1.38 km, 37.77 km]. Empty circles: empirical data, full line: S of the truncated Pareto PDF, dotted line: S of the Pareto PDF. The K-S test for the truncated Pareto gave $P_{KS} = 0.128$ and $d_{max} = 0.134$. The K-S test for the Pareto gave $P_{KS} = 0.0075$ and $d_{max} = 0.192$.

3 Data analysis and results

For statistical testing the Kolmogorov–Smirnov (K-S) test [5–7] was employed which does not require data binning. The K-S test, as implemented by the FORTRAN subroutine KSONE [3], finds the maximum distance (d_{max}) between the theoretical and the empirical DF as well the significance level P_{KS} (see equations 14.3.5 and 14.3.9 in [3]). A value of $P_{KS} \geqslant 0.1$ assures that the fit is acceptable.

When using the impact crater SFD data of Pluto with D = [3.75 km, 37.77 km] the Pareto PDF gave $c = 1.5299$ and thus $\alpha = 2.5299$ (similar to the value $\alpha = 2.4926$ reported by [1]), and the K-S test gave $P_{KS} = 0.866$ and $d_{max} = 0.091$. Figure 1 shows the empirical and and the two fitted distributions when the interval of crater size values is enlarged so that all D values are included in the fitting, i.e., D = [1.2 km, 37.77 km]. The truncated Pareto distribution describes the empirical crater SFD quite accurately over the whole interval of D values available.

4 Conclusions

The distribution of crater diameters of planets is commonly modeled by a power law. A small modification of the "simple" PDF by a truncated Pareto PDF (as given by equation (5)) allows the dichotomy of the infinite rather than finite range of existence to be avoided and provides better K-S test statistics with respect to the Pareto PDF (i.e., a "simple" power law), see captions of Figure 1.

In conclusion, we were able to show that the empirical impact crater SFD of Pluto (using a first data set based on recent New Horizons flyby) closely agrees with a truncated Pareto distribution. Applying the same modelling approach to an extended data set of Pluto's crater values is warranted to confirm our results – a task to be done as soon as new images of the New Horizon spacecraft are available.

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