On the Quantum-Relativistic Behavior of Moving Particles

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The *zitterbewegung* of massless elementary electrical charges consists of two distinct vacuum induced fluctuations. The first, random loops (spin) at the light speed (co-moving frame) [1], is attributed to absorptions and emissions of zero-point radiation at the Compton's rate (stochastic electrodynamics). It will be shown that the second (de Broglie) emerges because such radiation, just passing but tangled for a while (rest mass), doesn't submit to the ordinary motion of bodies; its light speed is ensured by truncations and restoration of the translational motion (inertia). Synchronized with absorption-emission, kinetic energy becomes vibrational energy (x-ray), and vice versa. The implied works are due to back and forth self-stresses (contractions) triggered by imminent violations of the light speed limit (loops at the light speed plus ordinary motion) implicit in the improper de Broglie phase velocity. Time spent to preserve the normal motility of the tangled radiation is observed only in the fixed frame (time dilation).

1 Introduction

Due to permanent interactions with the Planck's vacuum [2–4], massless elementary electrical charges (MEEC) are induced to move along quantum-relativistic paths at the speed of the interacting radiation, independently of the observed ordinary motion of particles, as implicit in the approach originating the concept of *zitterbewegung* [5]. In such approach, it was considered a particle (an electron) of rest mass m_0, which, therefore, must be attributed (respecting the peculiarities of the interaction) to the mass-equivalent of the zero-point energy absorbed (incident momentum) and emitted (reaction momentum) by MEEC. It means that MEEC, on average, retain zero-point radiation; a boson giving the rest mass.

In the particular case of free particles, the argued paths are continual random "jumps" (diffusion of probability) among trajectories belonging to the ensemble dictated by the Dirac equation [6]. Theoretical results indicate that such trajectories are curvilinear, over which particles are found at the light speed, which agrees with experimental facts. Indeed, if they are seen as random loops of electrical current in the co-moving frame (a charge e moving at the light speed c over a spherical shell of average radius r_c), then we find that the corresponding magnetic moment,

$$\mu_z = IA = \frac{ec}{2\pi r_c}\pi r_c^2 = \frac{ecr_c}{2}\,, \tag{1}$$

matches the observed magnetic moment of spin-1/2 particles,

$$\mu_z \approx \frac{e\hbar}{2m_0}\,, \tag{2}$$

if $2\pi r_c = \lambda_c$, where $\lambda_c = h/m_0 c$ is the Compton's wavelength.

Alternatively, if an electron can be found over circles at the light speed (co-moving frame), then its momentum components should fluctuate like $p' = m_0\dot{q}' = m_0 c\cos(\omega' t' + \phi_{q'})$, where $\phi_{q'}$ are random phases. It implies the coordinates

$$q' = \frac{c}{\omega_c}\sin(\omega' t' + \phi_{q'})\,, \tag{3}$$

where c/ω' is the radius of the loops of current (fluctuations with spherical shape). Inserting the corresponding variances (averaging over random phases),

$$\Delta p'^2 = \frac{1}{2}(m_0 c)^2, \quad \Delta q'^2 = \frac{1}{2}\frac{c^2}{\omega'^2}\,, \tag{4}$$

into the minimum uncertainty relation, $\Delta p'\Delta q' = \hbar/2$, yields

$$\omega' = \frac{m_0 c^2}{\hbar}\,, \quad r_c = \frac{c}{\omega'} = \frac{\lambda_c}{2\pi}\,, \tag{5}$$

that is, the Compton's angular frequency ($\omega' = \omega_c$).

Considering the center of mass of the fluctuations (vibrations) at the origin of the co-moving frame ($\mathbf{x}' = 0$), it implies that a free particle moving in the x-direction of the fixed frame will be seen as a material wave of wave number $\mathbf{k} = (k, 0, 0)$. Phase invariance, considering special relativity, i.e.

$$\omega' t' - \mathbf{k}'\cdot\mathbf{x}' = \omega t - \mathbf{k}\cdot\mathbf{x}, \tag{6}$$

implies

$$t' = \frac{\omega}{\omega_c}\left(t - \frac{x}{v_p}\right), \quad v_p = \frac{\omega}{k}\,, \tag{7}$$

where v_p is the phase velocity. Comparing the Eq. (7) with the Lorentz transformation

$$t' = \gamma\left(t - \frac{v}{c^2}\,x\right) \tag{8}$$

one gets the parameters of the material wave [7]:

$$\omega = \gamma\,\frac{m_0 c^2}{\hbar}\,, \quad k = \gamma\,\frac{m_0 v}{\hbar}\,, \quad v_p = \frac{\omega}{k} = \frac{c^2}{v}\,, \tag{9}$$

from which we can see that v_p is a violation of the natural speed of electromagnetic waves. This fact makes v_p meaningless in the context of the special relativity, which is reinforced by the existence of a group velocity (transport of matter) coinciding with the particle velocity, i.e.

$$v_g = \frac{\partial \omega}{\partial k} = \frac{\partial E}{\partial p} = v\,. \tag{10}$$

Technically, the concept of group velocity requires that the resultant material wave be a superposition of waves of different frequencies, which agrees with the successful concept of wave packet [8]. However, a wave packet implies a set of phase velocities. As the phase velocity of the resultant material wave is a violation of the natural speed of radiation, then we should expect that the phase velocities of the constituent waves also are speed violations (at least mostly).

Here, is it wise keep in mind that such speed violations, being in full agreement with the concepts expressed by equations (6), (8) and (10), cannot be meaningless. In effect, notice that an evolution at the phase velocity ($x = v_p t$) implies that time "stops" in the co-moving frame ($t' = 0$). Emphasizing, in this particular situation, time is computed only in the fixed frame. Remarkably, despite of being an improper evolution, it agrees with the ultimate meaning of time dilation.

Until now, we have seen that single frequency (ω_c) fluctuations of spherical shape ($r_c = c/\omega_c$) become multi-frequency fluctuations in the fixed frame [9], which manifest as a wave packet (material wave). The emergence of multiple angular frequencies implies that the translational motion cause a break of the spherical shape of the fluctuations, given that for each emerging angular frequency there must correspond a different radius ($\omega_i r_i = c$, where c is invariant). Coincidentally, this agrees with length contraction, i.e., according to the theory of special relativity, in the fixed frame the fluctuations must present an ellipsoidal shape.

The above argumentation implies that the phase velocity v_p is a statistical quantity; given that all frequencies implied in the wave packet do not exist simultaneously but in the elapsed time of an ordinary measurement (much greater than $2\pi/\omega_c$).

Physically, contractions of the vacuum induced fluctuations requires back and forth forces, whose resultant, at least on average, must be zero. Moreover, these forces — defined only in the fixed frame — do not have the same nature of the electromagnetic forces (from the Planck's vacuum) responsible by the fluctuations in the co-moving frame.

The search for forces triggered by the translational motion must begin noting that the speed violation v_p is dominant in the Lorentz transformations (LT), i.e.

$$x' = \gamma\,(x - vt)\,, \quad t' = \gamma\left(t - \frac{x}{v_p}\right), \quad \gamma = \left(1 - \frac{v}{v_p}\right)^{-\frac{1}{2}}, \tag{11}$$

which suggests that LT — to account for the light speed limit in both reference frames — just consider imminent velocity violations when the linear translational motion takes place. In other words, the emerging vibrations, whose *statistical superposition* gives v_p, must relate to a mechanism ensuring the speed limit of the zero-point radiation (ZPR) tangled for a moment by MEEC (co-moving frame), given that the relative velocity is lower than c.

Let us analyze, heuristically, the complete motion. From the equations (1) to (5) and the presence of the Planck's vacuum, it is implicit that in the co-moving frame MEEC are found over circular trajectories at the speed of the "impregnating" zero-point radiation (ZPR). Therefore, it be expected the occurrence of all sort of violations of the light speed limit when the ordinary translational motion is added. However, resulting velocities for MEEC — imbued with the properties of radiation — either greater or smaller than c are forbidden by the well-known Maxwell's relation $\mu\varepsilon c^2 = 1$. So, it is plausible to think that the vibrations implied in the wave packet — related to radii contraction of the fluctuations — arise to avoid any possible speed violation of the tangled ZPR, which would result from the simple combination of random orbits at light speed with the observed motion of matter.

In the next sections, based on well-known physical facts, it will be presented some evidences that the periodical motion induced by the Planck's vacuum combined with the ordinary motion of particles implies the appearance of periodical back and forth self-stresses, which are imposed by the *normal motility* of the *tangled radiation*. Here, it must be emphasized the following: First, *tangled radiation* is ZPR continually imprisoned during an infinitesimal time (less than $2\pi/\omega_c$) by MEEC. Second, *normal motility* relates to evolutions of free radiation; assumed to be extensible to the *tangled radiation*, given the massless nature of the "host".

2 The need for periodical longitudinal self-stresses

The energy carried by the material wave is the vibrational energy, $E = \hbar\omega$, which must be the energy of the particle, $E = \gamma m_0 c^2$. Therefore,

$$\omega = \frac{m_0 c^2}{\hbar} + \frac{(\gamma - 1) m_0 c^2}{\hbar} = \omega_c + \omega_T\,, \tag{12}$$

where the Compton's frequency (ω_c) expresses the rate at which zero-point energy is going in and out of the MEEC (on average remaining as rest energy), and ω_T accounts for all vibrations implied in the wave packet; likewise that v_p represents all corresponding phase velocities (one at a time).

Given the statistical nature of the wave packet (in the sense of the quantum superposition), it implies that particles can present, at a given time, only kinetic energy, or only vibrational energy, or a mix of them; all these possibilities occurring, in accordance with energy conservation, at a very high rate (synchronized with ω_c).

Coincidentally, for $v \ll c$, ω_T is the *maximum* frequency emitted by electrons in a x-ray apparatus (Duane-Hunt formula, $\hbar\omega_{max} = eV = m_0 v^2/2$), which does not contradict the fact that electrons can collide presenting frequencies different from ω_{max}. In effect, these other frequencies can be built into the well-known wavelength spread of x-ray data; the complementary energy (kinetic) simply warm the target.

The above facts suggest that kinetic energy becomes vibrational energy, and vice-versa, but the sum of them, at any time, is $(\gamma - 1)m_0c^2$ or $\hbar\omega_T$, as required by energy conservation. Inexorably, such changes of the kinetic energy imply positive and negative works on the particle. Nonetheless, if one takes into account that the MEEC-ZPR *electromagnetic* interaction is completely resolved, in the sense that it yields well-defined rest energy (mass), spin and Compton's parameters, then there must be another reason for the emergence of vibrations triggered by the translational motion. Only remains to appeal to the dynamics allowed by the tangled ZPR, which, in view of the above, only can be attributed to periodical back and forth self-stresses, whose sole purpose is to ensure its light speed limit; an imposition of the hindmost nature of radiation.

Here, it should be pointed up that these self-stresses — ensuring the normal motility of the tangled radiation — cannot be interpreted in the same sense of Poincaré stresses [10], which were postulated in order to guarantee the stability of the Abraham-Lorentz model for the electron. In effect, the semi-classical electron stability should be understood as an electromagnetic pressure balance involving the Planck's vacuum, as proposed by Casimir [11].

3 The Zitterbewegung and self-stresses

A formal account for the two kind of vacuum induced fluctuations, as exposed elsewhere, can be seen in the quantum-relativistic approach of the *zitterbewegung* [12], although not working the properties of the Planck's vacuum of explicit way; that is, using the recipes of the stochastic electrodynamics. This is possible because Lorentz transformations as well as quantum equations takes into account non-localized statistical features of the wave packet (the ultimate product of the matter-vacuum interaction). Hence, the following results, despite of evidencing the co-moving loops of electrical current (spin), should be interpreted statistically [13].

Inserting the Dirac Hamiltonian, $H = c\alpha_j p_j + \beta m_0 c^2$, into the Heisenberg picture of quantum mechanics and considering that the matrices α_j and β commute with momentum (p_j) and position (x_j) operators, one gets

$$\frac{dp_j}{dt} = \frac{i}{\hbar}\left[H, p_j\right] = 0\,, \quad \frac{dx_j}{dt} = \frac{i}{\hbar}\left[H, x_j\right] = c\alpha_j\,, \tag{13}$$

where the first implies that H and p_j commute (constants of the motion), and the second, in the full sense of the operation

$$c\alpha_j\psi = \pm c\psi\,, \tag{14}$$

where ψ represents a four-component spinor, means that "a measurement of a component of the velocity of a free electron is certain to lead to the result $\pm c$" [5, p. 262], which is not the ordinary velocity of free particles, but that of the tangled ZPR.

The result (14) means that — on average, everywhere, in all directions and with equal probability — electrons go forth and back at the light speed; an expected behavior, considering the main properties of the interacting ZPR (homogeneity, isotropy, randomness and Lorentz invariant spectral density).

Whenever the electron is on a permitted Dirac trajectory, despite of being temporarily, it must obey the parameters of such trajectory. As the trajectories are curvilinear, then there are accelerations. In fact, they are given by $\ddot{x}_j = (i/\hbar)[H, \dot{x}_j]$, where $\dot{x}_j = c\alpha_j$, which corresponds to the equations

$$\ddot{x}_j = \frac{2i\left(H\dot{x}_j - c^2 p_j\right)}{\hbar}\,, \quad \ddot{x}_j = \frac{2i\left(c^2 p_j - \dot{x}_j H\right)}{\hbar}\,, \tag{15}$$

since $Hc\alpha_j + c\alpha_j H = 2cp_j$. Integrating, yields respectively

$$\dot{x}_j = c^2 p_j H^{-1} + \eta_j e^{i2Ht/\hbar}; \quad \dot{x}_j = c^2 p_j H^{-1} + \eta'_j e^{-i2Ht/\hbar}; \tag{16}$$

where the operators η and η' (constants of integration) must take into account that these *components* must match, periodically, the tangential velocity ($c\alpha_j$), as implicit in Eq. (14), which implies that $\eta = \eta' = c\alpha_j - c^2 p_j H^{-1}$. Moreover, on average the velocity must be the observed one ($c^2 p_j H^{-1}$). Therefore, the velocity operator becomes

$$\dot{x}_j = c^2 p_j H^{-1} + \left(c\alpha_j - c^2 p_j H^{-1}\right)\cos\left(2Ht/\hbar\right)\,, \tag{17}$$

from which, considering the same above conditions, one gets the position operator

$$x_j(t) = c^2 p_j H^{-1} t + \frac{\left(\hbar c\alpha_j H^{-1} - \hbar c^2 p_j H^{-2}\right)}{2}\sin\left(\frac{2H}{\hbar}t\right). \tag{18}$$

Notice, for $p_j = 0$ the operators (17) and (18) violate the minimum uncertainty relation ($m_0\Delta\dot{x}_j\Delta x_j = \hbar/2$) by a factor 2 (the eigenvalues of α_j are unitary and $H \to m_0c^2$). This happens because the Dirac Hamiltonian takes into account matter and antimatter, whose energy gap is $2H$ [14, p. 949]. For only one kind of particle (e.g. free electrons in the two slit experiment, where is not verified the presence of positrons), it suffices to ignore the factor 2 in the equations (15).

Regardless of the comment made in the previous paragraph, the statistical components of the velocity of an electron (moving at the speed of light), as expressed by Eq. (17), show that — in order to maintain the speed imposed by the tangled radiation — the translational velocity $c^2 p_j H^{-1}$ is periodically subtracted and added, depending on the sign of c. Indeed, apart intermediary values, for forward evolutions of the local motion ($+c$), the translational motion is completely subtracted, and for backward evolutions ($-c$), it is completely restored, as can be seen from the allowed values of the cosine and the Eq. (14). Clearly, synchronized with absorptions and emission of zero-point energy (rest energy), the kinetic energy changes at the Compton's rate (considering only one kind of particle). As truncations and restorations of the translational motion behaves as vibrations, then kinetic energy is being transformed into vibrational energy, and vice versa. These positive and negative works, necessarily, imply back and forth forces (zero, on average). However, as there are no external forces — other than those yielding the well-defined evolutions in the co-moving frame — then such works must be assigned to periodical longitudinal self-stresses (PLSS), which are imposed by the very motility of radiation, as inferred in the preceding paragraph.

From the position operators (18) — statistical coordinates defined in the fixed frame — we can verify the following: First, they do not explicit a set of vibrations composing the wave packet, but the motion of the resulting material wave, whose statistical frequency is $\omega = H/\hbar$. Second, for $p_j = 0$, these coordinates agree with the proposed equations (3); evolutions with spherical shape in the co-moving frame. Third, in the fixed frame ($p_j \neq 0$), the amplitude of the vibration (enclosed difference of operators) suffers a contraction in the direction of the motion; evolutions with ellipsoidal shape.

4 Final remarks

Fluctuations with spherical shape (co-moving frame) becoming fluctuations with ellipsoidal shape (fixed frame) explains the emergence of all vibrations implied in the wave packet, but in the sense that a motion with constant tangential velocity (light speed) over an ellipsoid implies an infinite number of angular frequencies. The wave packet is a statistical concept; it simply expresses the fact that during the time of an ordinary measurement the particle can be found at any position on the ellipsoidal surface; each one corresponding to a given angular frequency (particle states). This is the fundamental feature of quantum superposition.

As "self-impulses", in principle, cannot be observed in the co-moving frame, then the corresponding time intervals also not. From another point of view, the strength of self-stresses depends of the relative velocity, but an observer in the co-moving frame cannot decide about the constant velocity of such frame (principle of relativity); therefore, also cannot decide about self-stresses (and its duration). This is implicit in the LT, as can be seen inserting the improper evolution $x = (c^2/v)t$ (triggering a given self-stress), which gives $t' = 0$. In short, the time spent to preserve the "integrity" of the tangled ZPR is computed only in the fixed frame, which is in full agreement with the cumulative time dilation.

The vibrational energy corresponding to self-stresses only are emitted as radiation under non-uniform decelerations (as in a x-ray apparatus). Contrasting with thermal excitations (external forces), PLSS only imply restrictions to the mobility of vacuum induced fluctuations (without external forces); so, radiationless.

The corresponding back and forth strains (restrictions to the translational motion) explain the non-cumulative length contraction.

Newton's inertia relates to the de Broglie periodicity [15]; that is, the periodicity of the wave packet, whose corresponding vibrations come from PLSS; "opposing forces".

To finalize, truncations of the ordinary motion followed by complete restoration of the kinetic energy, as implicit in the Eq. (17), is in full agreement with the observed energy conservation (first Newton's law).

5 Conclusion

In the light of the foregoing, the quantum relativistic behavior of particles emerge because the ZPR, continually entrapped by MEEC during the time of an absorption-emission of zero-point energy, does not submit to the ordinary motion of bodies. From another point of view, the quantum of the Higgs field (Higgs boson) does not move with the observed velocities of the corresponding particle; its light speed is ensured by conservative periodical truncations and restorations of the ordinary motion, whose momentum dependent strength (amplitude of emerging vibrations) explain why inertia (mass) increases with the particle velocity.

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