On a 4th Rank Tensor Gravitational Field Theory

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In an earlier publication, we showed that a slightly varying cosmological term is a necessary ingredient to restore the true tensor nature of the gravitational field produced by neutral matter. As a result, this term induces a background field filling the entire vacuum. The global energy-momentum tensor of matter and its gravity field is proved to be intrinsically conserved like the Einstein tensor, once it has been identified with the Rosenfeld-Belinfante symmetric tensor. Within the GR representation in the absence of matter, the remnant field never vanishes and we showed that it represents the lower horizon state of the Lorentzian space-time vacuum. In what follows, we work out a 4th rank tensor theory of gravity which formally leads to have the background field superimposed onto the large scale structure of space-time classically described by the de Sitter Universe with a cosmological constant. Our 4th rank tensor theory thus substantiates the recent investigations which would adopt the de Sitter Space-time as a mathematical frame more general that the Minkowski space.

Introduction

By introducing a space-time variable term Ξ that supersedes the so-called cosmological constant Λ in Einstein's field equations, we formally showed that the gravity field of a (neutral) massive source is no longer described by an *ill-defined pseudo-tensor*, but it is represented by a *true canonical tensor* [1]. As a result, the physical space should be always filled with a *homogeneous vacuum background field* [2] which is described by a tensor on the r.h.s. of the Einstein's "source free" equations. Inspection shows that the matter-gravity tensor must be identified with the *Rosenfeld-Belinfante symmetric tensor* [3, 4], thus complying with the intrinsic conservation property of the *Einstein tensor* as it should be. Regarding the *vacuum background field*, it was shown to be a *space-time contraction* unveiling a low horizon state, arising from the geodesics incompleteness postulate [5]. Conversely, it is desirable to analyze the background field nature in the larger scale. To this effect, we suggest here a 4th rank tensor theory based on the full Riemann curvature, and which suitably generalizes the Einstein-Ricci 2nd rank tensor formulation. Unlike many attempts of the kind, our mathematical approach does not trivially entail Einstein GR theory. In fact, due to its peculiar formulation, it leads to view the usual Einstein equations as merely initial conditions following the Cauchy problem.

As will turn out, such a broader theory clearly grants the background field a sound macroscopic meaning. When matter is absent, it closely follows the pattern of the *constant curvature space-time* described by the de Sitter metric when the term Ξ is reduced to the cosmological constant Λ.

In this way, the vacuum background field may be regarded as an intrinsic property of the basic physical structure of our Universe.

Notations

Space-time Greek indices run from $\alpha = \beta$: 0, 1, 2, 3, while spatial Latin indices run from $a = b$: 1, 2, 3. The space-time signature is -2. In the present text, $\varkappa$ is Einstein's constant $8\pi G/c^4 = 8\pi G$ with $c = 1$.

1 The background field and the gravitational field tensor (reminder)

In a pseudo-Riemannian manifold V_4, let us first set the following tensor densities

$$\mathfrak{g}^{\alpha\beta} = \sqrt{-g}\, g^{\alpha\beta}\,, \tag{1.1}$$

$$\mathfrak{G}^{\alpha\beta} = \sqrt{-g}\, G^{\alpha\beta} \quad \text{(Einstein tensor density)}, \tag{1.2}$$

$$\mathfrak{G}^{\alpha}_{\beta} = \sqrt{-g}\, G^{\alpha}_{\beta}\,, \tag{1.2bis}$$

$$\mathfrak{R}^{\alpha\beta} = \sqrt{-g}\, R^{\alpha\beta} \quad \text{(Ricci tensor density)}. \tag{1.2ter}$$

In density notations, the usual field equations with a massive source then read

$$\mathfrak{G}^{\alpha\beta} = \mathfrak{R}^{\alpha\beta} - \frac{1}{2}\, g^{\alpha\beta}\mathfrak{R} - g^{\alpha\beta}\Lambda\sqrt{-g} = \varkappa\, \mathfrak{T}^{\alpha\beta}, \tag{1.3}$$

where

$$\mathfrak{T}^{\alpha\beta} = \sqrt{-g}\, T^{\alpha\beta}$$

while Λ is the so-called cosmological constant.

However, unlike the Einstein tensor $G^{\alpha\beta}$ which is *conceptually conserved*, the conditions

$$\partial_\alpha \mathfrak{T}^{\alpha}_{\beta} = 0 \tag{1.4}$$

are never satisfied in a general coordinates system [6]. To cure this problem, we have demonstrated once more the conservation condition

$$\partial_\alpha \left[(\mathfrak{T}^{\alpha}_{\beta})_{\text{matter}} + (\mathfrak{t}^{\beta}_{\alpha})_{\text{gravity}} \right] = 0\,, \tag{1.5}$$

but where $(\mathfrak{t}_{\alpha}^{\beta})_{\text{gravity}}$ is no longer a pseudo-tensor density.

To achieve this, we introduced a space-time varying term Ξ in place of the cosmological constant Λ, and whose scalar density is denoted by

$$\zeta = \Xi \sqrt{-g}\,. \tag{1.6}$$

Its variation is given by

$$\zeta = \sqrt{-g}\,\nabla_a \kappa^a = \partial_a\left(\sqrt{-g}\,\kappa^a\right) \tag{1.7}$$

and the term

$$\zeta = \sqrt{-g}\,\nabla_a \kappa^a \tag{1.8}$$

is related to the *vacuum volume expansion scalar* $\theta = \nabla_a \theta^a$ (see [7] for detail).

Such a form allows to maintain the original *Einstein Lagrangian density* as

$$\mathfrak{L}_E = \sqrt{-g}\, g^{\alpha\beta}\left[\left\{{}^{\nu}{}_{\alpha\beta}\right\}\left\{{}^{\lambda}{}_{\lambda\nu}\right\} - \left\{{}^{\lambda}{}_{\alpha\nu}\right\}\left\{{}^{\nu}{}_{\beta\lambda}\right\}\right], \tag{1.9}$$

the latter expression being used to derive the new canonical gravity tensor attached to a mass:

$$(\mathfrak{t}_{\beta}^{\alpha})_{\text{gravity}} = \frac{1}{2\varkappa}\Big[\left\{{}^{\alpha}{}_{\gamma\mu}\right\}\partial_\beta \mathfrak{g}^{\gamma\mu} - \left\{{}^{\gamma}{}_{\gamma\mu}\right\}\partial_\beta \mathfrak{g}^{\mu\alpha} - \delta_\beta^\alpha(\mathfrak{L}_E - \zeta)\Big], \tag{1.10}$$

ζ can be regarded as a *Lagrangian density* characterizing a specific *vacuum background field* which pre-exists in the absence of matter. Close inspection of equation (1.10) shows that local gravitational field of matter is just a mere "excited state" of the background field. Sufficiently far from the massive source, $(\mathfrak{t}_{\beta}^{\alpha})_{\text{gravity}} \to (\mathfrak{t}_{\beta}^{\alpha})_{\text{background}}$.

2 Symmetrization of the gravity tensor

The tensor density (1.10) includes the *Einstein-Dirac pseudo-tensor density* [8] which is not symmetric.

Symmetrizing the canonical tensor $(\Theta_{\beta}^{\alpha})_{\text{gravity}}$ extracted from $(\mathfrak{t}_{\beta}^{\alpha})_{\text{gravity}} = \sqrt{-g}\,(\Theta_{\beta}^{\alpha})_{\text{gravity}}$ is equivalent to identifying it with the *Belinfante-Rosenfeld tensor*:

$$(t^{\gamma\beta})_{\text{gravity}} = (\Theta^{\gamma\beta})_{\text{gravity}} + \nabla_\alpha \Upsilon^{\gamma\beta\alpha} \tag{2.1}$$

with

$$\Upsilon^{\gamma\beta\alpha} = \frac{1}{2}\left(S^{\gamma\beta\alpha} + S^{\beta\gamma\alpha} - S^{\alpha\beta\gamma}\right), \tag{2.2}$$

where the antisymmetric tensor $S^{\alpha\beta\gamma}$ is the contribution of the *intrinsic angular momentum*. Now, we check that:

$$\nabla_\alpha(\Theta_{\beta}^{\alpha})_{\text{gravity}} = \nabla_\alpha(t_{\beta}^{\alpha})_{\text{gravity}} = 0\,. \tag{2.3}$$

Far from matter $(t^{\alpha\beta})_{\text{gravity}} \to (t^{\alpha\beta})_{\text{background}}$ and $\Upsilon^{\alpha\beta\gamma} = 0$. By essence, $(t^{\alpha\beta})_{\text{background}}$ is thus symmetric.

The field equations with a (neutral) massive source together with its gravity tensor can now be explicitly written down

$$G^{\alpha\beta} = R^{\alpha\beta} - \frac{1}{2}\, g^{\alpha\beta} R = \varkappa (T^{\alpha\beta})_{\text{global}}\,, \tag{2.4}$$

where

$$(T^{\alpha\beta})_{\text{global}} = (T^{\alpha\beta})_{\text{matter}} + (t^{\alpha\beta})_{\text{gravity}} \tag{2.5}$$

with, for example $(T^{\alpha\beta})_{\text{matter}} = \rho u^\alpha u^\beta$ (here ρ is the homogeneous mass density).

3 The 4th rank theory of the gravitational field

3.1 The new field equations

We now state that the *true gravitational field equations with a source* are the 4th rank tensor equations

$$G_{\beta\gamma\ \mu}^{\ \ \alpha} = \varkappa\, T_{\beta\gamma\ \mu}^{\ \ \alpha}\,, \tag{3.1}$$

where

$$G_{\beta\gamma\ \mu}^{\ \ \alpha} = R_{\beta\gamma\ \mu}^{\ \ \alpha} - \frac{1}{2}\, R\left(\delta_\gamma^\alpha g_{\beta\mu} - \delta_\mu^\alpha g_{\beta\gamma}\right) \tag{3.1bis}$$

and

$$T_{\beta\gamma\ \mu}^{\ \ \alpha} = \delta_\gamma^\alpha (T_{\beta\mu})_{\text{global}} - \delta_\mu^\alpha (T_{\beta\gamma})_{\text{global}} \tag{3.2}$$

is the generalized energy-momentum tensor.

Our assumption can be legitimized by the following considerations. From Bianchi's second identities [9]

$$(s)_{\alpha\beta\gamma} \nabla_\alpha R_{\beta\gamma\lambda\mu} = 0\,, \tag{3.3}$$

where (s) denotes the cyclic sum, we easily infer [10]

$$\nabla_\alpha R_{\beta\gamma\ \mu}^{\ \ \alpha} = \nabla_\gamma R_{\beta\mu} - \nabla_\mu R_{\beta\gamma}\,, \tag{3.4}$$

hence

$$\nabla_\alpha G_{\beta\gamma\ \mu}^{\ \ \alpha} = \nabla_\gamma R_{\beta\mu} - \nabla_\mu R_{\beta\gamma} - \frac{1}{2}\,\nabla_\alpha R\left(\delta_\gamma^\alpha g_{\beta\mu} - \delta_\mu^\alpha g_{\beta\gamma}\right) \tag{3.5}$$

i.e.

$$\nabla_\alpha G_{\beta\gamma\ \mu}^{\ \ \alpha} = \nabla_\gamma R_{\beta\mu} - \nabla_\mu R_{\beta\gamma} - \frac{1}{2}\,\nabla_\gamma R g_{\beta\mu} + \frac{1}{2}\,\nabla_\mu R g_{\beta\gamma}\,. \tag{3.5bis}$$

The right hand side equation is obviously zero, therefore:

$$\nabla_\alpha G_{\beta\gamma\ \mu}^{\ \ \alpha} = 0\,. \tag{3.6}$$

The tensor

$$G_{\beta\gamma\ \mu}^{\ \ \alpha} = \delta_\gamma^\alpha R_{\beta\mu} - \delta_\mu^\alpha R_{\beta\gamma} - \frac{1}{2}\, R\left(\delta_\gamma^\alpha g_{\beta\mu} - \delta_\mu^\alpha g_{\beta\gamma}\right) \tag{3.6bis}$$

is thus intrinsically conserved as is the case for the Einstein-Ricci tensor $G_{\beta\mu}$, and we call it the *Einstein 4th rank tensor*.

In addition, we also have:

$$\nabla_\alpha T_{\beta\gamma\ \mu}^{\ \ \alpha} = \nabla_\alpha\left[\delta_\gamma^\alpha (T_{\beta\mu})_{\text{global}} - \delta_\mu^\alpha (T_{\beta\gamma})_{\text{global}}\right] = 0\,. \tag{3.7}$$

Proof:

$$\delta_\gamma^\alpha (T_{\beta\mu})_{\text{global}} = \delta_\gamma^\nu g_{\beta\nu} (T_\mu^\alpha)_{\text{global}} = g_{\beta\gamma} (T_\mu^\alpha)_{\text{global}} \tag{3.8}$$

and since $\nabla_\alpha (T_\mu^\alpha)_{\text{global}} = 0$ according to our initial demonstration, then $\nabla_\alpha\left[\delta_\gamma^\alpha (T_{\beta\mu})_{\text{global}}\right] = 0$. The same reasoning holds for $\delta_\mu^\alpha (T_{\beta\gamma})_{\text{global}}$

$$\delta_\mu^\alpha (T_{\beta\gamma})_{\text{global}} = \delta_\mu^\nu g_{\beta\nu} (T_\gamma^\alpha)_{\text{global}} = g_{\beta\mu} (T_\gamma^\alpha)_{\text{global}} \tag{3.8bis}$$

which finally yields (3.7).

Equations (3.6) and (3.7) tell us that the conservation conditions are fully satisfied by the system:

$$G_{\beta\gamma\ \mu}^{\ \ \alpha} = \varkappa\, T_{\beta\gamma\ \mu}^{\ \ \alpha}\,. \tag{3.9}$$

Hence, $T_{\beta\gamma\ \mu}^{\ \ \alpha}$ is confirmed to be the appropriate generalization of the energy-momentum 2nd rank tensor $(T_{\gamma\mu})_{\text{global}}$.

How do the Einstein second rank tensor equations fit in the theory?

3.2 Some hypothesis on the Cauchy problem

Let us consider again (3.1bis) and (3.2)

$$G_{\beta\gamma\ \mu}^{\ \ \alpha} = \delta^{\alpha}_{\gamma} R_{\beta\mu} - \delta^{\alpha}_{\mu} R_{\beta\gamma} - \frac{1}{2}\, R\left(\delta^{\alpha}_{\gamma}\, g_{\beta\mu} - \delta^{\alpha}_{\mu}\, g_{\beta\gamma}\right),$$

$$T_{\beta\gamma\ \mu}^{\ \ \alpha} = \delta^{\alpha}_{\gamma}(T_{\beta\mu})_{\text{global}} - \delta^{\alpha}_{\mu}(T_{\beta\gamma})_{\text{global}}\,,$$

and by subtraction we have:

$$\delta^{\alpha}_{\gamma}\left[G_{\beta\mu} - \varkappa(T_{\beta\mu})_{\text{global}}\right] - \delta^{\alpha}_{\mu}\left[G_{\beta\gamma} - \varkappa(T_{\beta\gamma})_{\text{global}}\right] = 0 \tag{3.10}$$

i.e.

$$P_{\beta\mu} - P_{\beta\gamma} = 0\,. \tag{3.10bis}$$

where $\boldsymbol{P} = \boldsymbol{G} - \varkappa\boldsymbol{T} = 0$ are the Einstein equations with a source which read in mixed indices as:

$$P^{\alpha}_{\beta} = 0\,. \tag{3.11}$$

Both relations (3.10bis) and (3.11) then strongly suggest that the Einstein equations $\boldsymbol{P} = 0$ can be regarded as mere initial conditions on a spacelike hypersurface Σ defined on V_4. To see this, consider Σ on which is given $P^{\alpha}_{\beta} = 0$, we must show that upon the above conditions, $\boldsymbol{P} = 0$ also holds beyond Σ [11].

For $\beta = 0$ and α reduced to spatial indices $i, k = 1, 2, 3$, equation (3.10bis) can be expressed by

$$P_{0\mu} = P_{0\gamma} \tag{3.12}$$

and (3.11) becomes:

$$g_{00}P^{i0} = -2g^{i0}P_{00} - g^{ik}P_{k0} \tag{3.12bis}$$

Now, if the hypersurface Σ admits the local equation $x^0 = 0$, we have $g_{00} \neq 0$ which means that $\boldsymbol{P} = 0$ would also hold beyond Σ.

On the hypersurface Σ, the zero initial data require that the system (3.12)–(3.12bis) admits nothing but the zero solution leading to $\boldsymbol{P} = 0$ as well. This is what we wanted to show.

In relation with (3.12), one may regard the equations

$$G_{\beta 0\ \mu}^{\ \ \alpha} - \varkappa\left[\delta^{\alpha}_{0}(T_{\beta\mu})_{\text{global}} - \delta^{\alpha}_{\mu}(T_{\beta 0})_{\text{global}}\right] = 0 \tag{3.13}$$

as *constraint equations for the initial data at the time* $x^0 = 0$ which are usually set in the Cauchy problem. For a particular example see [12].

3.3 Newton's law

Let us consider the massive tensor classically expressed by

$$(T^{\alpha\beta})_{\text{global}} = \rho u^{\alpha}u^{\beta} + (t^{\alpha\beta})_{\text{gravity}} \tag{3.14}$$

which becomes here

$$T_{\beta\gamma\ \mu}^{\ \ \alpha} = \delta^{\alpha}_{\gamma}\left[\rho\, u_{\beta}u_{\mu} + (t_{\beta\mu})_{\text{gravity}}\right] - \\ - \delta^{\alpha}_{\mu}\left[\rho\, u_{\beta}u_{\gamma} + (t_{\beta\gamma})_{\text{gravity}}\right]. \tag{3.15}$$

When the spatial 3-velocities are low and the gravitational field is weak, the static case corresponds to the Newton's law for which $u_0 = 1$ in an orthonormal basis. In the framework of our theory, this translates to:

$$G_{0i\ 0}^{\ \ i} = \varkappa\, T_{0i\ 0}^{\ \ i} \tag{3.16}$$

Explicitly: the left hand side is easily shown to reduce to:

$$G_{0i\ 0}^{\ \ i} = R_{00} - \frac{1}{2}\, R g_{00}\,. \tag{3.17}$$

In the same way, the right hand side of (3.16) reduces to:

$$T_{0i\ 0}^{\ \ i} = (\rho + t_{\text{gravity}}). \tag{3.17bis}$$

As usual, we can re-write the field equations as

$$R^0_0 = \varkappa\left[(\rho + t_{\text{gravity}}) - \frac{1}{2}\,\delta^0_0(\rho + t_{\text{gravity}})\right] \tag{3.18}$$

which eventually yields with the explicit value of the Einstein's constant

$$R^0_0 = 4\pi G(\rho + t_{\text{gravity}})\,, \tag{3.19}$$

where G is Newton's constant.

We then retrieve the *Poisson equation* which is also expressed by:

$$\Delta\psi = 4\pi G\rho'. \tag{3.19bis}$$

We have set: $\rho' = \rho + t_{\text{gravity}}$ because we consider a stationary gravity field (in a general case, the gravity field is "dragged" along with the mass and $\rho' = \rho + t_{\text{gravity}}$ no longer holds). With the metric approximation:

$$g_{00} = 1 + 2\psi\,, \tag{3.20}$$

where ψ is the Newton's gravitational potential

$$\psi = -G\int\frac{\rho'}{R}\,dV\,, \tag{3.21}$$

while R is here the distance from the observer to the volume element dV. Integration is performed over a volume V which comprises both the bare mass and its (stationary) gravitational field.

4 The background field in our Universe

We now come to the persistent field appearing in the 2nd rank tensor field equations when matter is absent. These are

$$G^{\beta\gamma} = R^{\beta\gamma} - \frac{1}{2}\, g^{\beta\gamma} R = \varkappa\,(t^{\beta\gamma})_{\text{background}} \tag{4.1}$$

with

$$(t_{\alpha\beta})_{\text{background}} = \frac{\Xi}{2\varkappa}\, g_{\alpha\beta}\,. \tag{4.2}$$

Expressed in the framework of the 4th rank tensor theory, this yields:

$$G_{\beta\gamma}{}^{\alpha}{}_{\mu} = R_{\beta\gamma}{}^{\alpha}{}_{\mu} - \frac{1}{2}\, R\left(\delta^{\alpha}_{\gamma}\, g_{\beta\mu} - \delta^{\alpha}_{\mu}\, g_{\beta\gamma}\right) = \frac{\Xi}{2}\left(\delta^{\alpha}_{\gamma}\, g_{\beta\mu} - \delta^{\alpha}_{\mu}\, g_{\beta\gamma}\right). \tag{4.3}$$

In virtue of $\nabla_{\alpha} G_{\beta\gamma}{}^{\alpha}{}_{\mu} = 0$, the r.h.s. is conserved:

$$\nabla_{\alpha}\left[\frac{\Xi}{2}\left(\delta^{\alpha}_{\gamma}\, g_{\beta\mu} - \delta^{\alpha}_{\mu}\, g_{\beta\gamma}\right)\right] = 0\,. \tag{4.3bis}$$

The latter equation is worthy of attention, for the term Ξ never happens to be a constant as could be (ambiguously) the case for $\nabla_{\alpha} G^{\alpha\beta} = \nabla_{\alpha} \frac{\Xi}{2}\, g^{\alpha\beta}$.

This lends support to the fact that only a 4th rank tensor theory can strictly describe a metric with a variable cosmological term. Therefore, after interchanging α with β, we find:

$$G_{\alpha\beta\gamma\mu} = \frac{\Xi}{2}\left(g_{\alpha\gamma}\, g_{\beta\mu} - g_{\alpha\mu}\, g_{\beta\gamma}\right). \tag{4.4}$$

The latter equations constitute here the *4th rank tensor background field equations* which characterize the *fundamental structure of physical space-time*.

They adequately generalize the Einstein space endowed with the cosmological constant Λ defined as:

$$G_{\beta\gamma} = R_{\beta\gamma} = \Lambda\, g_{\beta\gamma}\,. \tag{4.5}$$

For a specific value of Ξ, we retrieve the space-time of constant curvature [13], which characterizes the de Sitter Universe when $3\Lambda = R$ [14]:

$$R_{\alpha\beta\gamma\mu} = \frac{R}{12}\left(g_{\alpha\gamma}\, g_{\beta\mu} - g_{\alpha\mu}\, g_{\beta\gamma}\right). \tag{4.6}$$

Finally, let us emphasize a major point. In a Universe devoid of matter described by equations (4.4), the Weyl conformal trace-free tensor $C_{\alpha\beta\gamma\mu}$ never vanishes, in contrast to the de Sitter model equipped with curvature (4.6). However, the Weyl tensor being that part of the curvature which is not determined locally by the matter distribution, there is no reason why it should disappear in an "empty" model of space-time. Hence, our approach of a Universe with a pervasive background field proves to be physically consistent for it preserves the Weyl tensor, whatever its content.

So, as expected from our 2nd rank tensor field equations (4.1), the case $G^{\alpha}_{\beta\gamma\ \mu} = 0$ will never occur.

Conclusion

Our 4 th rank tensor gravitational field theory appears to be the appropriate extension of the 2nd order Einstein-Ricci formulation.

However, it should be noted that the presented theory does not use the well-known *Bel-Robinson tensor* [15] which gave birth to the very thorough paper of *R. Debever* on *Super Energy* [16].

The presented remarkably simple theory is partly inspired from a lecture given by *A. Lichnérowicz* in a Paris seminar dedicated to linearized field quantization solutions prior to their global formulation [17]. We have however substantially modified this theory allowing for a clearer physical significance of the vacuum background field on the very large scale structure of space-time.

Indeed, when matter is absent, the intrinsic curvature of space-time is modeled by the background field through its variable term Ξ, just as de Sitter's empty Universe does with its cosmological constant Λ arbitrarily introduced.

Such a close similarity with the de Sitter curvature should not come as a surprise. The de Sitter metric recently saw some revived interest among several physicists [18–20]. They conjectured that the laws of physics are invariant under the symmetry group of de Sitter space (*maximally symmetric* space-time), rather than the Poincaré group of special relativity. The full Poincare group is the semi-direct product of translations $\mathbf{T}$ with the Lorentz group $\mathbf{L} = SO(3,1)$: $\mathbf{P} = \mathbf{L} \otimes \mathbf{T}$. The latter acts transitively on the *Minkowski space* $\mathbf{M}$ which is homogeneous under $\mathbf{P}$.

In the framework of a generalized group where translations mix up non trivially with rotations, the requirements of homogeneity and isotropy lead ipso facto to the de Sitter Universe with a uniform scalar curvature. More specifically, the de Sitter space whose metric is induced from the pseudo-Euclidean metric $(+1,-1,-1,-1,-1)$ has a specific group of motion which is the pseudo-orthogonal group $SO(4,1)$ [21]. Then, de Sitter group obviously involves an additional length parameter l which is related to the (positive) cosmological term by:

$$\Lambda = \frac{3}{l^2}\,.$$

The *Poincaré group* "contracts" to the *Galilean group* for low velocities.

Analogously the *de Sitter group* "contracts" to the *Poincaré group* for short distance kinematics, when the order of magnitude of all translations are small compared to the de Sitter radius. (See: Wigner and Inönü, for the group contraction concept [22]). These distances are probed by high energies meaning that quantum effects must be taken into account. In that case, when we have $\Lambda \to \infty$, this would correspond to $\Lambda_P = 3/l_P^2$, where l_P is the Planck length. If $\Lambda \to 0$, however, the underlying space-time would reduce to the Minkowski space.

From the fundamental vacuum field equations (4.4), the variable term Ξ would represent a fluctuation between two appropriate values of Λ wherein the de Sitter space-time can be fully represented. In this view, the 4th rank tensor field equations are to the de Sitter space-time, what the 2nd rank tensor field equations are to the Minkowski space.

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