Bosons and Fermions as Dislocations and Disclinations in the Spacetime Continuum

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We investigate the case for dislocations (translational displacements) and disclinations (rotational displacements) in the Spacetime Continuum corresponding to bosons and fermions respectively. The massless, spin-1 screw dislocation is identified with the photon, while edge dislocations correspond to bosons of spin-0, spin-1 and spin-2. Wedge disclinations are identified with quarks. We find that the twist disclination depends both on the space volume ℓ^3 of the disclination and on the length ℓ of the disclination. We identify the ℓ^3 twist disclination terms with the leptons, while the ℓ twist disclination which does not have a longitudinal (massive) component, is identified with the massless neutrino. We perform numerical calculations that show that the dominance of the ℓ and ℓ^3 twist disclination terms depend on the extent ℓ of the disclination: at low values of ℓ, the "weak interaction" term ℓ predominates up to about 10^{-18} m, which is the generally accepted range of the weak force, while at larger values of ℓ, the "electromagnetic interaction" term ℓ^3 predominates. The value of ℓ at which the two interactions in the total strain energy are equal is given by $\ell = 2.0 \times 10^{-18}$ m.

1 Introduction

Elementary quantum particles are classified into bosons and fermions based on integral and half-integral multiples of $\hbar$ respectively, where $\hbar$ is Planck's reduced constant. Bosons obey Bose-Einstein statistics while fermions obey Fermi-Dirac statistics and the Pauli Exclusion Principle. These determine the number of non-interacting indistinguishable particles that can occupy a given quantum state: there can only be one fermion per quantum state while there is no such restriction on bosons.

This is explained in quantum mechanics using the combined wavefunction of two indistinguishable particles when they are interchanged:

$$\begin{aligned} \text{Bosons:} \quad & \Psi(1,2) = \Psi(2,1) \\ \text{Fermions:} \quad & \Psi(1,2) = -\Psi(2,1)\,. \end{aligned} \tag{1}$$

Bosons commute and as seen from (1) above, only the symmetric part contributes, while fermions anticommute and only the antisymmetric part contributes. There have been attempts at a formal explanation of this phenomemon, the spin-statistics theorem, with Pauli's being one of the first [1]. Jabs [2] provides an overview of these and also offers his own attempt at an explanation.

However, as Feynman comments candidly [3, see p. 4-3],

> We apologize for the fact that we cannot give you an elementary explanation. An explanation has been worked out by Pauli from complicated arguments of quantum field theory and relativity. He has shown that the two must necessarily go together, but we have not been able to find a way of reproducing his arguments on an elementary level. It appears to be one of the few places in physics where there is a rule which can be stated very simply, but for which no one has found a simple and easy explanation. The explanation is deep down in relativistic quantum mechanics. This probably means that we do not have a complete understanding of the fundamental principle involved. For the moment, you will just have to take it as one of the rules of the world.

The question of a simple and easy explanation is still outstanding. Eq. (1) is still the easily understood explanation, even though it is based on the exchange properties of particles, rather than on how the statistics of the particles are related to their spin properties. At this point in time, it is an empirical description of the phenomenon.

2 Quantum particles from STC defects

Ideally, the simple and easy explanation should be a *physical* explanation to provide a complete understanding of the fundamental principles involved. The Elastodynamics of the Spacetime Continuum (*STCED*) [6,7] provides such an explanation, based on dislocations and disclinations in the spacetime continuum. Part of the current problem is that there is no understandable physical picture of the quantum level. *STCED* provides such a picture.

The first point to note is that based on their properties, bosons obey the superposition principle in a quantum state. In *STCED*, the location of quantum particles is given by their deformation displacement u^μ. Dislocations [7, see chapter 9] are translational displacements that commute, satisfy the superposition principle and behave as bosons. As shown in section §3-6 of [7], particles with spin-0, 1 and 2 are described by

$$u^{\mu;\nu} = \varepsilon^{\mu\nu}_{(0)} + \varepsilon^{\mu\nu}_{(2)} + \omega^{\mu\nu}_{(1)}\,, \tag{2}$$

i.e. a combination of spin-0 $\varepsilon^{\mu\nu}_{(0)}$ (mass as deformation particle aspect), spin-1 $\omega^{\mu\nu}_{(1)}$ (electromagnetism) and spin-2 $\varepsilon^{\mu\nu}_{(2)}$

(deformation wave aspect), where

$$\varepsilon^{\mu\nu} = \frac{1}{2}\left(u^{\mu;\nu} + u^{\nu;\mu}\right) = u^{(\mu;\nu)} \tag{3}$$

and

$$\omega^{\mu\nu} = \frac{1}{2}\left(u^{\mu;\nu} - u^{\nu;\mu}\right) = u^{[\mu;\nu]} \tag{4}$$

which are solutions of wave equations in terms of derivatives of the displacements $u^{\mu;\nu}$ as given in chapter 3 of [7].

Disclinations [7, see chapter 10], on the other hand, are rotational displacements that do not commute and that do not obey the superposition principle. You cannot have two rotational displacements in a given quantum state. Hence their number is restricted to one per quantum state. They behave as fermions.

Spinors represent spin one-half fermions. Dirac spinor fields represent electrons. Weyl spinors, derived from Dirac's four complex components spinor fields, are a pair of fields that have two complex components. Interestingly enough, "[u]sing just one element of the pair, one gets a theory of massless spin-one-half particles that is asymmetric under mirror reflection and ... found ... to describe the neutrino and its weak interactions" [4, p. 63].

"From the point of view of representation theory, Weyl spinors are the fundamental representations that occur when one studies the representations of rotations in four-dimensional space-time... spin-one-half particles are representation of the group SU(2) of transformations on two complex variables." [4, p. 63]. To clarify this statement, each rotation in three dimensions (an element of SO(3)) corresponds to two distinct elements of SU(2). Consequently, the SU(2) transformation properties of a particle are known as the particle's spin.

Hence, the unavoidable conclusion is that bosons are dislocations in the spacetime continuum, while fermions are disclinations in the spacetime continuum. Dislocations are translational displacements that commute, satisfy the superposition principle and behave as bosons. Disclinations, on the other hand, are rotational displacements that do not commute, do not obey the superposition principle and behave as fermions.

The equations in the following sections of this paper are derived in Millette [7]. The constants $\bar{\lambda}_0$ and $\bar{\mu}_0$ are the Lamé elastic constants of the spacetime continuum, where $\bar{\mu}_0$ is the shear modulus (the resistance of the continuum to *distortions*) and $\bar{\lambda}_0$ is expressed in terms of $\bar{\kappa}_0$, the bulk modulus (the resistance of the continuum to *dilatations*) according to

$$\bar{\lambda}_0 = \bar{\kappa}_0 - \bar{\mu}_0/2 \tag{5}$$

in a four-dimensional continuum.

3 Dislocations (bosons)

Two types of dislocations are considered in this paper: screw dislocations (see Fig. 1) and edge dislocations (see Fig. 2).

Fig. 1: A stationary screw dislocation in cartesian (x, y, z) and cylindrical polar (r, θ, z) coordinates.

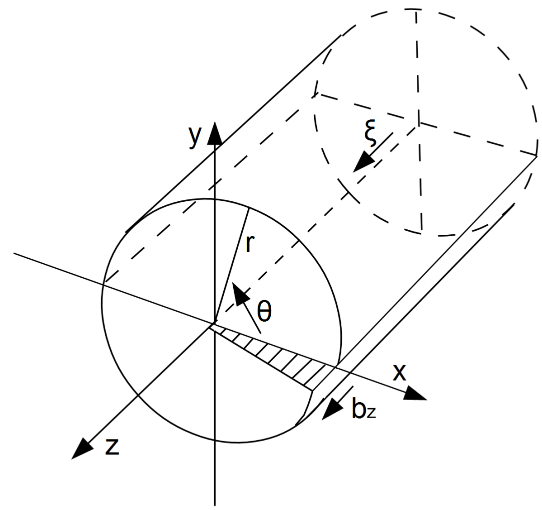
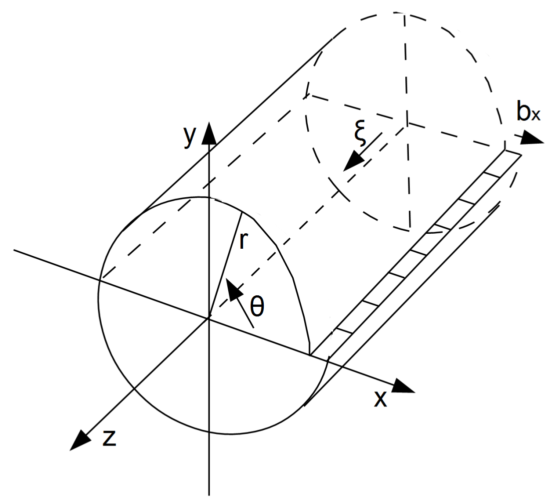


Fig. 2: A stationary edge dislocation in cartesian (x, y, z) and cylindrical polar (r, θ, z) coordinates.



Dislocations, due to their translational nature, are defects that are easier to analyze than disclinations.

3.1 Screw dislocation

The screw dislocation is analyzed in sections §9-2 and §15-1 of [7]. It is the first defect that we identified with the photon due to its being massless and of spin-1. Consequently, its longitudinal strain energy is zero

$$W^S_\parallel = 0. \tag{6}$$

Its transverse strain energy is given by [7, eq. (16.5)]

$$W_{\perp}^{S} = \frac{\bar{\mu}_0}{4\pi} b^2 \ell \ln \frac{\Lambda}{b_c}, \tag{7}$$

where b is the spacetime Burgers dislocation vector [9], ℓ is the length of the dislocation, b_c is the size of the core of the dislocation, of order b_0, the smallest spacetime Burgers dislocation vector [10], and Λ is a cut-off parameter corresponding to the radial extent of the dislocation, limited by the average distance to its nearest neighbours.

3.2 Edge dislocation

The edge dislocation is analyzed in sections §9-3, §9-5 and §15-2 of [7]. The longitudinal strain energy of the edge dislocation is given by [7, eq. (16.29)]

$$W_{\parallel}^{E} = \frac{\bar{\kappa}_0}{2\pi} \bar{\alpha}_0^2 \left(b_x^2 + b_y^2\right) \ell \ln \frac{\Lambda}{b_c} \tag{8}$$

where

$$\bar{\alpha}_0 = \frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0}, \tag{9}$$

ℓ is the length of the dislocation and as before, Λ is a cut-off parameter corresponding to the radial extent of the dislocation, limited by the average distance to its nearest neighbours. The edge dislocations are along the z-axis with Burgers vector b_x for the edge dislocation proper represented in Fig. 2, and a different edge dislocation with Burgers vector b_y which we call the gap dislocation. The transverse strain energy is given by [7, eq. (16.54)]

$$W_{\perp}^{E} = \frac{\bar{\mu}_0}{4\pi} \left(\bar{\alpha}_0^2 + 2\bar{\beta}_0^2\right)\left(b_x^2 + b_y^2\right) \ell \ln \frac{\Lambda}{b_c} \tag{10}$$

where

$$\bar{\beta}_0 = \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0}. \tag{11}$$

The total longitudinal (massive) dislocation strain energy $W_{\parallel}^{D}$ is given by (8)

$$W_{\parallel}^{D} = W_{\parallel}^{S} + W_{\parallel}^{E} = W_{\parallel}^{E}, \tag{12}$$

given that the screw dislocation longitudinal strain energy is zero, while the total transverse (massless) dislocation strain energy is given by the sum of the screw (along the z axis) and edge (in the $x - y$ plane) dislocation transverse strain energies

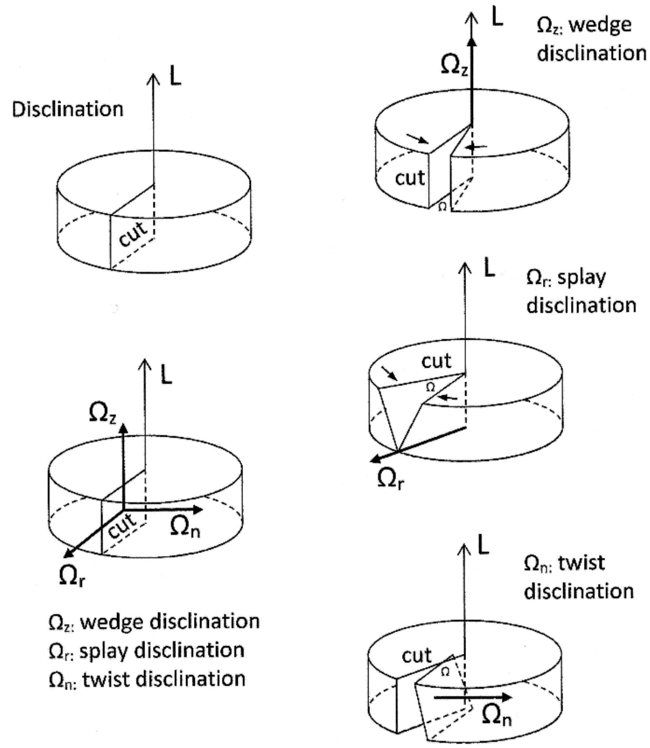
$$W_{\perp}^{D} = W_{\perp}^{S} + W_{\perp}^{E} \tag{13}$$

to give

$$W_{\perp}^{D} = \frac{\bar{\mu}_0}{4\pi} \left[b_z^2 + \left(\bar{\alpha}_0^2 + 2\bar{\beta}_0^2\right)\left(b_x^2 + b_y^2\right)\right] \ell \ln \frac{\Lambda}{b_c}. \tag{14}$$

It should be noted that as expected, the total longitudinal (massive) dislocation strain energy $W_{\parallel}^{D}$ involves the spacetime bulk modulus $\bar{\kappa}_0$, while the total transverse (massless) dislocation strain energy $W_{\perp}^{D}$ involves the spacetime shear modulus $\bar{\mu}_0$.

Fig. 3: Three types of disclinations: wedge (top), splay (middle), twist (bottom) [5, 7].



The total strain energy of dislocations

$$W^{D} = W_{\parallel}^{D} + W_{\perp}^{D} \tag{15}$$

provides the total energy of massive and massless bosons, with $W_{\parallel}^{D}$ corresponding to the longitudinal particle aspect of the bosons and $W_{\perp}^{D}$ corresponding to the wave aspect of the bosons. As seen in [11], the latter is associated with the wavefunction of the boson. The spin characteristics of these was considered previously in section 2, where they were seen to correspond to spin-0, spin-1 and spin-2 solutions.

4 Disclinations (fermions)

The different types of disclinations considered in this paper are given in Fig. 3. Disclinations are defects that are more difficult to analyze than dislocations, due to their rotational nature. This mirrors the case of fermions, which are more difficult to analyze than bosons.

4.1 Wedge disclination

The wedge disclination is analyzed in sections §10-6 and §15-3 of [7]. The longitudinal strain energy of the wedge disclination is given by [7, eq. (16.62)]

$$W_{\parallel}^{W} = \frac{\bar{\kappa}_0}{4\pi}\,\Omega_z^2\,\ell\left[\bar{\alpha}_0^2\left(2\Lambda^2\ln^2\Lambda - 2b_c^2\ln^2 b_c\right) + \right.$$
$$+\,\bar{\alpha}_0\bar{\gamma}_0\left(2\Lambda^2\ln\Lambda - 2b_c^2\ln b_c\right) + \tag{16}$$
$$\left. +\,\tfrac{1}{2}(\bar{\alpha}_0^2 + \bar{\gamma}_0^2)\left(\Lambda^2 - b_c^2\right)\right]$$

where Ω^{μ} is the spacetime Frank vector,

$$\bar{\gamma}_0 = \frac{\bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \tag{17}$$

and the other constants are as defined previously. In most cases $\Lambda \gg b_c$, and (16) reduces to

$$W_{\parallel}^{W} \simeq \frac{\bar{\kappa}_0}{2\pi}\,\Omega_z^2\,\ell\,\Lambda^2\left[\bar{\alpha}_0^2\,\ln^2\Lambda + \,\bar{\alpha}_0\bar{\gamma}_0\ln\Lambda + \,\tfrac{1}{4}(\bar{\alpha}_0^2 + \bar{\gamma}_0^2)\right] \tag{18}$$

which is rearranged as

$$W_{\parallel}^{W} \simeq \frac{\bar{\kappa}_0}{2\pi}\,\bar{\alpha}_0^2\,\Omega_z^2\,\ell\,\Lambda^2\left[\ln^2\Lambda + \frac{\bar{\gamma}_0}{\bar{\alpha}_0}\,\ln\Lambda + \frac{1}{4}\left(1 + \frac{\bar{\gamma}_0^2}{\bar{\alpha}_0^2}\right)\right]. \tag{19}$$

The transverse strain energy of the wedge disclination is given by [7, eq. (16.70)]

$$W_{\perp}^{W} = \frac{\bar{\mu}_0}{4\pi}\,\Omega_z^2\,\ell\left[\bar{\alpha}_0^2\left(\Lambda^2\ln^2\Lambda - b_c^2\ln^2 b_c\right) - \right.$$
$$-\left(\bar{\alpha}_0^2 - 3\bar{\alpha}_0\bar{\beta}_0\right)\left(\Lambda^2\ln\Lambda - b_c^2\ln b_c\right) + \tag{20}$$
$$\left. +\,\frac{1}{2}\left(\bar{\alpha}_0^2 - 3\bar{\alpha}_0\bar{\beta}_0 + \frac{3}{2}\,\bar{\beta}_0^2\right)\left(\Lambda^2 - b_c^2\right)\right].$$

In most cases $\Lambda \gg b_c$, and (20) reduces to

$$W_{\perp}^{W} \simeq \frac{\bar{\mu}_0}{4\pi}\,\Omega_z^2\,\ell\left[\bar{\alpha}_0^2\,\Lambda^2\ln^2\Lambda - \right.$$
$$-\left(\bar{\alpha}_0^2 - 3\bar{\alpha}_0\bar{\beta}_0\right)\Lambda^2\ln\Lambda + \tag{21}$$
$$\left. +\,\frac{1}{2}\left(\bar{\alpha}_0^2 - 3\bar{\alpha}_0\bar{\beta}_0 + \frac{3}{2}\,\bar{\beta}_0^2\right)\Lambda^2\right]$$

which is rearranged as

$$W_{\perp}^{W} \simeq \frac{\bar{\mu}_0}{4\pi}\,\bar{\alpha}_0^2\,\Omega_z^2\,\ell\,\Lambda^2\left[\ln^2\Lambda - \left(1 - 3\,\frac{\bar{\beta}_0}{\bar{\alpha}_0}\right)\ln\Lambda + \right.$$
$$\left. +\,\frac{1}{2}\left(1 - 3\,\frac{\bar{\beta}_0}{\bar{\alpha}_0} + \frac{3}{2}\,\frac{\bar{\beta}_0^2}{\bar{\alpha}_0^2}\right)\right]. \tag{22}$$

We first note that both the longitudinal strain energy $W_{\parallel}^{W}$ and the transverse strain energy $W_{\perp}^{W}$ are proportional to Λ^2 in the limit $\Lambda \gg b_c$. The parameter Λ is equivalent to the extent of the wedge disclination, and we find that as it becomes more extended, its strain energy is increasing parabolically. This behaviour is similar to that of quarks (confinement) which are fermions. In addition, as $\Lambda \to b_c$, the strain energy decreases and tends to 0, again in agreement with the behaviour of quarks (asymptotic freedom).

We thus identify wedge disclinations with quarks. The total strain energy of wedge disclinations

$$W^{W} = W_{\parallel}^{W} + W_{\perp}^{W} \tag{23}$$

provides the total energy of the quarks, with $W_{\parallel}^{W}$ corresponding to the longitudinal particle aspect of the quarks and $W_{\perp}^{W}$ corresponding to the wave aspect of the quarks. We note that the current classification of quarks include both ground and excited states – the current analysis needs to be extended to excited higher energy states.

We note also that the rest-mass energy density $\rho^{W}c^2$ of the wedge disclination (see [7, eq. (10.102)]) is proportional to $\ln r$ which also increases with increasing r, while the rest-mass energy density $\rho^{E}c^2$ of the edge dislocation and $\rho^{T}c^2$ of the twist disclination (see [7, eqs. (9.134) and (10.123)] respectively) are both proportional to $1/r^2$ which decreases with increasing r as expected of bosons and leptons.

4.2 Twist disclination

The twist disclination is analyzed in sections §10-7 and §15-4 of [7]. Note that as mentioned in that section, we do not differentiate between twist and splay disclinations in this subsection as twist disclination expressions include both splay disclinations and twist disclinations proper. Note also that the Frank vector $(\Omega_x, \Omega_y, \Omega_z)$ corresponds to the three axes (Ω_r, Ω_n, Ω_z) used in Fig. 3 for the splay, twist and wedge disclinations respectively.

The longitudinal strain energy of the twist disclination is given by [7, eq. (16.80)]

$$W_{\parallel}^{T} = \frac{\bar{\kappa}_0}{6\pi}\,\bar{\alpha}_0^2\left(\Omega_x^2 + \Omega_y^2\right)\ell^3\ln\frac{\Lambda}{b_c}\,. \tag{24}$$

One interesting aspect of this equation is that the twist disclination longitudinal strain energy $W_{\parallel}^{T}$ is proportional to the cube of the length of the disclination (ℓ^3), and we can't dispose of it by considering the strain energy per unit length of the disclination as done previously. We can say that the twist disclination longitudinal strain energy $W_{\parallel}^{T}$ is thus proportional to the space volume of the disclination, which is reasonable considering that disclinations are rotational deformations. It is also interesting to note that $W_{\parallel}^{T}$ has the familiar dependence $\ln\Lambda/b_c$ of dislocations, different from the functional dependence obtained for wedge disclinations in section 4.1. The form of this equation is similar to that of the longitudinal strain energy for the stationary edge dislocation (see [7, eq. (16.15)]) except for the factor $\ell^3/3$.

The transverse strain energy of the twist disclination is given by [8]

$$\begin{aligned} W_\perp^T &= \frac{\bar{\mu}_0}{2\pi}\,\frac{\ell^3}{3}\Big[\left(\Omega_x^2+\Omega_y^2\right)\left(\bar{\alpha}_0^2+\tfrac{1}{2}\bar{\beta}_0^2\right)+ \\ &+\;2\,\Omega_x\Omega_y\left(\bar{\alpha}_0^2-2\bar{\beta}_0^2\right)\Big]\ln\frac{\Lambda}{b_c}+ \\ &+\;\frac{\bar{\mu}_0}{2\pi}\,\ell\Big[\left(\Omega_x^2+\Omega_y^2\right)\Big(\bar{\alpha}_0^2\left(\Lambda^2\ln^2\Lambda-b_c^2\ln^2 b_c\right)+ \\ &+\;\bar{\alpha}_0\bar{\gamma}_0\left(\Lambda^2\ln\Lambda-b_c^2\ln b_c\right)-\frac{1}{2}\bar{\alpha}_0\bar{\gamma}_0\left(\Lambda^2-b_c^2\right)+ \\ &+\;2\bar{\beta}_0^2\ln\frac{\Lambda}{b_c}\Big)-2\,\Omega_x\Omega_y\Big(\bar{\alpha}_0\bar{\beta}_0\left(\Lambda^2\ln\Lambda-b_c^2\ln b_c\right)+ \\ &+\;\frac{1}{2}\bar{\beta}_0\bar{\gamma}_0\left(\Lambda^2-b_c^2\right)\Big)\Big]. \end{aligned} \tag{25}$$

In most cases $\Lambda \gg b_c$, and (25) reduces to

$$\begin{aligned} W_\perp^T &\simeq \frac{\bar{\mu}_0}{2\pi}\,\frac{\ell^3}{3}\Big[\left(\Omega_x^2+\Omega_y^2\right)\left(\bar{\alpha}_0^2+\tfrac{1}{2}\bar{\beta}_0^2\right)+ \\ &+\;2\,\Omega_x\Omega_y\left(\bar{\alpha}_0^2-2\bar{\beta}_0^2\right)\Big]\ln\frac{\Lambda}{b_c}+ \\ &+\;\frac{\bar{\mu}_0}{2\pi}\,\ell\,\Lambda^2\Big[\left(\Omega_x^2+\Omega_y^2\right)\Big(\bar{\alpha}_0^2\ln^2\Lambda+\bar{\alpha}_0\bar{\gamma}_0\ln\Lambda- \\ &-\;\frac{1}{2}\bar{\alpha}_0\bar{\gamma}_0\Big)-2\,\Omega_x\Omega_y\Big(\bar{\alpha}_0\bar{\beta}_0\ln\Lambda+\frac{1}{2}\bar{\beta}_0\bar{\gamma}_0\Big)\Big] \end{aligned} \tag{26}$$

which can be rearranged to give

$$\begin{aligned} W_\perp^T &\simeq \frac{\bar{\mu}_0}{2\pi}\,\bar{\alpha}_0^2\,\frac{\ell^3}{3}\Big[\left(\Omega_x^2+\Omega_y^2\right)\left(1+\frac{1}{2}\,\frac{\bar{\beta}_0^2}{\bar{\alpha}_0^2}\right)+ \\ &+2\,\Omega_x\Omega_y\left(1-2\,\frac{\bar{\beta}_0^2}{\bar{\alpha}_0^2}\right)\Big]\ln\frac{\Lambda}{b_c}+ \\ &+\frac{\bar{\mu}_0}{2\pi}\,\bar{\alpha}_0^2\,\ell\,\Lambda^2\Big[\left(\Omega_x^2+\Omega_y^2\right)\Big(\ln^2\Lambda+\frac{\bar{\gamma}_0}{\bar{\alpha}_0}\ln\Lambda- \\ &-\frac{1}{2}\,\frac{\bar{\gamma}_0}{\bar{\alpha}_0}\Big)-2\,\Omega_x\Omega_y\Big(\frac{\bar{\beta}_0}{\bar{\alpha}_0}\ln\Lambda+\frac{1}{2}\,\frac{\bar{\beta}_0\bar{\gamma}_0}{\bar{\alpha}_0^2}\Big)\Big]. \end{aligned} \tag{27}$$

As noted previously, $W_\parallel^T$ depends on the space volume ℓ^3 of the disclination and has a functional dependence of $\ln\Lambda/b_c$ as do the dislocations. The transverse strain energy $W_\perp^T$ depends on the space volume ℓ^3 of the disclination with a functional dependence of $\ln\Lambda/b_c$, but it also includes terms that have a dependence on the length ℓ of the disclination with a functional dependence similar to that of the wedge disclination including Λ^2 in the limit $\Lambda \gg b_c$. The difference in the case of the twist disclination is that its transverse strain energy $W_\perp^T$ combines ℓ^3 terms with the functional dependence $\ln\Lambda/b_c$ of dislocations, associated with the "electromagnetic interaction", and ℓ terms with the $\Lambda^2\ \ln^2\Lambda$ functional dependence of wedge disclinations, associated with the "strong interaction". This, as we will see in later sections, seems to be the peculiar nature of the weak interaction, and uniquely positions twist disclinations to represent leptons and neutrinos as participants in the weak interaction.

This leads us to thus separate the longitudinal strain energy of the twist disclination as

$$W_\parallel^T = W_\parallel^{\ell^3} + W_\parallel^{\ell} = W_\parallel^{\ell^3} \tag{28}$$

given that $W_\parallel^{\ell} = 0$, and the transverse strain energy of the twist disclination as

$$W_\perp^T = W_\perp^{\ell^3} + W_\perp^{\ell}\,. \tag{29}$$

We consider both ℓ^3 twist disclination and ℓ twist disclination terms in the next subsections.

4.2.1 ℓ^3 twist disclination

The longitudinal strain energy of the ℓ^3 twist disclination is thus given by the ℓ^3 terms of (24)

$$W_\parallel^{\ell^3} = \frac{\bar{\kappa}_0}{6\pi}\,\bar{\alpha}_0^2\left(\Omega_x^2+\Omega_y^2\right)\ell^3\ln\frac{\Lambda}{b_c}\,. \tag{30}$$

The transverse strain energy of the ℓ^3 twist disclination is given by the ℓ^3 terms of (25)

$$\begin{aligned} W_\perp^{\ell^3} &= \frac{\bar{\mu}_0}{2\pi}\,\frac{\ell^3}{3}\Big[\left(\Omega_x^2+\Omega_y^2\right)\left(\bar{\alpha}_0^2+\tfrac{1}{2}\bar{\beta}_0^2\right)+ \\ &+2\,\Omega_x\Omega_y\left(\bar{\alpha}_0^2-2\bar{\beta}_0^2\right)\Big]\ln\frac{\Lambda}{b_c}\,. \end{aligned} \tag{31}$$

In most cases $\Lambda \gg b_c$, and (31) is left unchanged due to its functional dependence on $\ln\Lambda/b_c$.

The total strain energy of the ℓ^3 twist disclination terms is given by

$$W^{\ell^3} = W_\parallel^{\ell^3} + W_\perp^{\ell^3}\,. \tag{32}$$

It is interesting to note that $W_\parallel^{\ell^3}$ of (30) and $W_\perp^{\ell^3}$ of (31) are proportional to $\ln\Lambda/b_c$, as are the screw dislocation (photon) and edge dislocation (bosons). This, and the results of the next subsection, leads us to identify the ℓ^3 twist disclination terms with the leptons (electron, muon, tau) fermions, where the heavier muon and tau are expected to be excited states of the electron.

4.2.2 ℓ twist disclination

The longitudinal strain energy of the ℓ twist disclination terms in this case is zero as mentioned previously

$$W_\parallel^{\ell} = 0\,. \tag{33}$$

The transverse strain energy of the ℓ twist disclination is thus also given by the ℓ terms of (25):

$$W_\perp^\ell = \frac{\bar{\mu}_0}{2\pi}\,\ell\Big[\Big(\Omega_x^2+\Omega_y^2\Big)\Big(\bar{\alpha}_0^2\,\Big(\Lambda^2\ln^2\Lambda - b_c^2\ln^2 b_c\Big) + \bar{\alpha}_0\bar{\gamma}_0\,\Big(\Lambda^2\ln\Lambda - b_c^2\ln b_c\Big) - \frac{1}{2}\bar{\alpha}_0\bar{\gamma}_0\Big(\Lambda^2 - b_c^2\Big) + 2\bar{\beta}_0^2\,\ln\frac{\Lambda}{b_c}\Big) - 2\,\Omega_x\Omega_y\Big(\bar{\alpha}_0\bar{\beta}_0\,\Big(\Lambda^2\ln\Lambda - b_c^2\ln b_c\Big) + \frac{1}{2}\bar{\beta}_0\bar{\gamma}_0\Big(\Lambda^2 - b_c^2\Big)\Big)\Big]. \tag{34}$$

In most cases $\Lambda \gg b_c$, and (34) reduces to

$$W_\perp^\ell = \frac{\bar{\mu}_0}{2\pi}\,\ell\,\Lambda^2\Big[\Big(\Omega_x^2+\Omega_y^2\Big)\Big(\bar{\alpha}_0^2\,\ln^2\Lambda + \bar{\alpha}_0\bar{\gamma}_0\,\ln\Lambda - \frac{1}{2}\bar{\alpha}_0\bar{\gamma}_0\Big) - 2\,\Omega_x\Omega_y\Big(\bar{\alpha}_0\bar{\beta}_0\ln\Lambda + \frac{1}{2}\bar{\beta}_0\bar{\gamma}_0\Big)\Big] \tag{35}$$

which can be rearranged to give

$$W_\perp^\ell = \frac{\bar{\mu}_0}{2\pi}\,\bar{\alpha}_0^2\,\ell\,\Lambda^2\Big[\Big(\Omega_x^2+\Omega_y^2\Big)\Big(\ln^2\Lambda + \frac{\bar{\gamma}_0}{\bar{\alpha}_0}\,\ln\Lambda - \frac{1}{2}\,\frac{\bar{\gamma}_0}{\bar{\alpha}_0}\Big) - 2\,\Omega_x\Omega_y\Big(\frac{\bar{\beta}_0}{\bar{\alpha}_0}\ln\Lambda + \frac{1}{2}\,\frac{\bar{\beta}_0\bar{\gamma}_0}{\bar{\alpha}_0^2}\Big)\Big]. \tag{36}$$

The total strain energy of the ℓ twist disclination is given by

$$W^\ell = W_\parallel^\ell + W_\perp^\ell = W_\perp^\ell \tag{37}$$

given that the ℓ twist disclination does not have a longitudinal (massive) component. Since the ℓ twist disclination is a massless fermion, this leads us to identify the ℓ twist disclination with the neutrino.

There is another aspect to the strain energy $W_\perp^T$ given by (25) that is important to note. As we have discussed, the ℓ^3 twist disclination terms and the $\ln\Lambda/b_c$ functional dependence as observed for the screw dislocation (photon) and edge dislocation (bosons) has led us to identify the ℓ^3 portion with the leptons (electron, muon, tau) fermions, where the heavier muon and tau are expected to be excited states of the electron. These are coupled with transverse ℓ twist disclination terms which are massless and which have a functional dependence similar to that of the wedge disclination, which has led us to identify the ℓ portion with the weakly interacting neutrino. If the muon and tau leptons are excited states of the electron derivable from (25), this would imply that the neutrino portion would also be specific to the muon and tau lepton excited states, thus leading to muon and tau neutrinos.

We will perform numerical calculations in the next section which will show that the dominance of the ℓ and ℓ^3 twist disclination terms depend on the extent ℓ of the disclination, with the ℓ "weak interaction" terms dominating for small values of ℓ and the ℓ^3 "electromagnetic interaction" terms dominating for larger values of ℓ. The ℓ twist disclination terms would correspond to weak interaction terms while the ℓ^3 twist disclination terms would correspond to electromagnetic interaction terms. The twist disclination represents the unification of both interactions under a single "electroweak interaction".

This analysis also shows why leptons (twist disclinations) are participants in the weak interaction but not the strong interaction, while quarks (wedge disclinations) are participants in the strong interaction but not the weak interaction.

It should be noted that even though the mass of the neutrino is currently estimated to be on the order of 10's of eV, this estimate is based on assuming neutrino oscillation between the currently known three lepton generations, to explain the anomalous solar neutrino problem. This is a weak explanation for that problem, which more than likely indicates that we do not yet fully understand solar astrophysics. One can only hope that a fourth generation of leptons will not be discovered! Until the anomaly is fully understood, we can consider the twist disclination physical model where the mass of the neutrino is zero to be at least a first approximation of the neutrino STC defect model.

4.3 Twist disclination sample numerical calculation

In this section, we give a sample numerical calculation that shows the lepton-neutrino connection for the twist disclination. We start by isolating the common strain energy elements that don't need to be calculated in the example. Starting from the longitudinal strain energy of the twist disclination (24) and making use of the relation $\bar{\kappa}_0 = 32\bar{\mu}_0$ [7, eq. (5.53)], (24) can be simplified to

$$W_\parallel^T = \frac{\bar{\mu}_0}{2\pi}\,\bar{\alpha}_0^2\,2\Omega^2\left[32\,\frac{\ell^3}{3}\,\ln\frac{\Lambda}{b_c}\right] \tag{38}$$

where an average Ω is used instead of Ω_x and Ω_y. Defining K as

$$K = \frac{\bar{\mu}_0}{2\pi}\,\bar{\alpha}_0^2\,2\Omega^2\,, \tag{39}$$

then (38) is written as

$$\frac{W_\parallel^T}{K} = 32\,\frac{\ell^3}{3}\,\ln\frac{\Lambda}{b_c}\,. \tag{40}$$

Similarly for the transverse strain energy of the twist disclination, starting from (27), the equation can be simplified to

$$W_\perp^T \simeq \frac{\bar{\mu}_0}{2\pi}\,\bar{\alpha}_0^2\,2\Omega^2\Big\{\Big[\frac{\ell^3}{3}\,\Big(1+\frac{1}{2}\,\frac{\bar{\beta}_0^2}{\bar{\alpha}_0^2} + 1 - 2\,\frac{\bar{\beta}_0^2}{\bar{\alpha}_0^2}\Big)\ln\frac{\Lambda}{b_c}\Big] + \Big[\ell\,\Lambda^2\Big(\ln^2\Lambda + \frac{\bar{\gamma}_0}{\bar{\alpha}_0}\,\ln\Lambda - \frac{1}{2}\,\frac{\bar{\gamma}_0}{\bar{\alpha}_0} - \frac{\bar{\beta}_0}{\bar{\alpha}_0}\ln\Lambda - \frac{1}{2}\,\frac{\bar{\beta}_0\bar{\gamma}_0}{\bar{\alpha}_0^2}\Big)\Big]\Big\}. \tag{41}$$

Using the definition of K from (39), this equation becomes

$$\begin{aligned} \frac{W_\perp^T}{K} &\simeq \frac{\ell^3}{3}\left(2-\frac{3}{2}\frac{\bar\beta_0^2}{\bar\alpha_0^2}\right)\ln\frac{\Lambda}{b_c}+ \\ &+\ \ell\,\Lambda^2\left(\ln^2\Lambda+\frac{\bar\gamma_0-\bar\beta_0}{\bar\alpha_0}\ \ln\Lambda-\frac{1}{2}\,\frac{\bar\gamma_0}{\bar\alpha_0}\left(1+\bar\beta_0\right)\right). \end{aligned} \tag{42}$$

Using the numerical values of the constants $\bar\alpha_0$, $\bar\beta_0$ and $\bar\gamma_0$ from [7, eqs. (19.14) and (19.35)], (42) becomes

$$\begin{aligned} \frac{W_\perp^T}{K} &\simeq \frac{\ell^3}{3}\ (1.565)\ \ln\frac{\Lambda}{b_c}+ \\ &+\ell\,\Lambda^2\left(\ln^2\Lambda-\ln\Lambda-0.62\right). \end{aligned} \tag{43}$$

For this sample numerical calculation, we use $b_c\sim10^{-35}$m of the order of the spacetime Burgers dislocation constant b_0, and the extent of the disclination $\Lambda\sim10^{-18}$ m of the order of the range of the weak force. Then

$$\frac{W_\parallel^T}{K}=\frac{32}{3}\,(39.1)\,\ell^3=417\,\ell^3\,. \tag{44}$$

and

$$\begin{aligned} \frac{W_\perp^T}{K} &\simeq 0.522\,(39.1)\,\ell^3+ \\ &+\Lambda^2\,(1714+41.4-0.62)\ \ell \end{aligned} \tag{45}$$

which becomes

$$\frac{W_\perp^T}{K}\simeq 20.4\,\ell^3+1755\,\Lambda^2\,\ell \tag{46}$$

and finally

$$\frac{W_\perp^T}{K}\simeq 20.4\,\ell^3+1.76\times10^{-33}\,\ell\,. \tag{47}$$

We consider various values of ℓ to analyze its effect on the strain energy. For $\ell=10^{-21}$ m,

$$\frac{W_\parallel^T}{K}=4.2\times10^{-61}\quad(\ell^3\text{ term}) \tag{48}$$

$$\frac{W_\perp^T}{K}=2.0\times10^{-62}+1.8\times10^{-54}\quad(\ell^3\text{ term}+\ell\text{ term}). \tag{49}$$

For $\ell=10^{-18}$ m,

$$\frac{W_\parallel^T}{K}=4.2\times10^{-52}\quad(\ell^3\text{ term}) \tag{50}$$

$$\frac{W_\perp^T}{K}=2.0\times10^{-53}+1.8\times10^{-51}\quad(\ell^3\text{ term}+\ell\text{ term}). \tag{51}$$

For $\ell=10^{-15}$ m,

$$\frac{W_\parallel^T}{K}=4.2\times10^{-43}\quad(\ell^3\text{ term}) \tag{52}$$

$$\frac{W_\perp^T}{K}=2.0\times10^{-44}+1.8\times10^{-48}\quad(\ell^3\text{ term}+\ell\text{ term}). \tag{53}$$

For $\ell=10^{-12}$ m,

$$\frac{W_\parallel^T}{K}=4.2\times10^{-34}\quad(\ell^3\text{ term}) \tag{54}$$

$$\frac{W_\perp^T}{K}=2.0\times10^{-35}+1.8\times10^{-45}\quad(\ell^3\text{ term}+\ell\text{ term}). \tag{55}$$

In the sums of $W_\perp^T/K$ above, the first term ℓ^3 represents the electromagnetic interaction, while the second term ℓ represents the weak interaction. Thus we find that at low values of ℓ, the weak force predominates up to about 10^{-18} m, which is the generally accepted range of the weak force. At larger values of ℓ, the electromagnetic force predominates. The value of ℓ at which the two interactions in the transverse strain energy are equal is given by

$$20.4\,\ell^3=1.76\times10^{-33}\,\ell\,, \tag{56}$$

from which we obtain

$$\ell=0.9\times10^{-17}\text{ m}\sim10^{-17}\text{ m}\,. \tag{57}$$

At that value of ℓ, the strain energies are given by

$$\frac{W_\parallel^T}{K}=3.0\times10^{-49} \tag{58}$$

$$\frac{W_\perp^T}{K}=3.1\times10^{-50}\,. \tag{59}$$

The longitudinal (massive) strain energy predominates over the transverse strain energy by a factor of 10.

Alternatively, including the longitudinal ℓ^3 strain energy in the calculation, the value of ℓ at which the two interactions in the total strain energy are equal is given by

$$417\,\ell^3+20.4\,\ell^3=1.76\times10^{-33}\,\ell\,, \tag{60}$$

from which we obtain

$$\ell=2.0\times10^{-18}\text{ m}\,. \tag{61}$$

At that value of ℓ, the strain energies are given by

$$\frac{W_\parallel^T}{K}=3.3\times10^{-51} \tag{62}$$

$$\frac{W_\perp^T}{K}=3.7\times10^{-51}\,. \tag{63}$$

The longitudinal (massive) strain energy and the transverse strain energy are then of the same order of magnitude.

5 Quantum particles and their associated spacetime defects

Table 1 provides a summary of the identification of quantum particles and their associated spacetime defects as shown in this paper.

STC defect	Type of particle	Particles
Screw dislocation	Massless boson	Photon
Edge dislocation	Massive boson	Spin-0 particle
		Spin-1 Proca eqn
		Spin-2 graviton
Wedge disclination	Massive fermion	Quarks
ℓ^3 Twist disclination	Massive fermion	Leptons
ℓ Twist disclination	Massless fermion	Neutrinos

Table 1: Identification of quantum particles and their associated defects.

6 Discussion and conclusion

In this paper, we have investigated the case for dislocations and disclinations in the Spacetime Continuum corresponding to bosons and fermions respectively. Dislocations are translational displacements that commute, satisfy the superposition principle and behave as bosons. Disclinations, on the other hand, are rotational displacements that do not commute, do not obey the superposition principle and behave as fermions, including having their number restricted to one per quantum state as it is not possible to have two rotational displacements in a given quantum state.

We have considered screw and edge dislocations. The massless, spin-1 screw dislocation is identified with the photon. The total strain energy of dislocations W^D corresponds to the total energy of massive and massless bosons, with $W^D_\parallel$ corresponding to the longitudinal particle aspect of the bosons and $W^D_\perp$ corresponding to the wave aspect of the bosons, with the latter being associated with the wavefunction of the boson. Their spin characteristics correspond to spin-0, spin-1 and spin-2 solutions.

We have considered wedge and twist disclinations, of which the splay disclination is a special case. Wedge disclinations are identified with quarks. The strain energy of wedge disclinations is proportional to Λ^2 in the limit $\Lambda \gg b_c$. The parameter Λ is equivalent to the extent of the wedge disclination, and we find that as it becomes more extended, its strain energy is increasing parabolically. This behaviour is similar to that of quarks (confinement) which are fermions. In addition, as $\Lambda \to b_c$, the strain energy decreases and tends to 0, again in agreement with the behaviour of quarks (asymptotic freedom). The total strain energy of wedge disclinations W^W thus corresponds to the total energy of the quarks, with $W^W_\parallel$ corresponding to the longitudinal particle aspect of the quarks and $W^W_\perp$ corresponding to the wave aspect of the quarks.

The twist disclination longitudinal strain energy $W^T_\parallel$ is found to be proportional to the cube of the length of the disclination (ℓ^3), and hence depends on the space volume ℓ^3 of the disclination with a functional dependence of $\ln \Lambda/b_c$ as do the dislocations. The transverse strain energy $W^T_\perp$ also depends on the space volume ℓ^3 of the disclination with a functional dependence of $\ln \Lambda/b_c$, but it also includes terms that have a dependence on the length ℓ of the disclination with a functional dependence similar to that of the wedge disclination including Λ^2 in the limit $\Lambda \gg b_c$.

We have considered both ℓ^3 twist disclination and ℓ twist disclination terms. We note that $W^{\ell^3}_\parallel$ and $W^{\ell^3}_\perp$ are proportional to $\ln \Lambda/b_c$, as are the screw dislocation (photon) and edge dislocation (bosons), which leads us to identify the ℓ^3 twist disclination terms with the leptons (electron, muon, tau) fermions, where the heavier muon and tau are expected to be excited states of the electron. Given that the ℓ twist disclination does not have a longitudinal (massive) component, it is a massless fermion and this leads us to identify the ℓ twist disclination with the neutrino. Thus the twist disclination transverse strain energy $W^T_\perp$ combines ℓ^3 terms with the functional dependence $\ln \Lambda/b_c$ of dislocations and ℓ terms with the functional dependence Λ^2 of wedge disclinations.

We have performed numerical calculations that show that the dominance of the ℓ and ℓ^3 twist disclination terms depend on the length ℓ of the disclination. We find that at low values of ℓ, the "weak interaction" term ℓ predominates up to about 10^{-18} m, which is the generally accepted range of the weak force. At larger values of ℓ, the "electromagnetic interaction" term ℓ^3 predominates. The value of ℓ at which the two interactions in the total strain energy are equal is given by $\ell = 2.0 \times 10^{-18}$ m. We conclude that in $W^T_\perp$, the ℓ twist disclination terms represent the weak interaction terms while the ℓ^3 twist disclination terms represent the electromagnetic interaction terms. The twist disclination hence represents the unification of both interactions under a single "electroweak interaction".

This analysis also shows why leptons (twist disclinations) are participants in the weak interaction but not the strong interaction (wedge disclinations). In addition, if the muon and tau leptons are excited states of the electron derivable from (25), this would imply that the neutrino portion would also be specific to the muon and tau lepton excited states, thus leading to muon and tau neutrinos. A summary of the identification of quantum particles and their associated spacetime defects as shown in this paper is provided in Table 1.

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References

1. Pauli W. The Connection Between Spin and Statistics. *Phys. Rev.*, 1940, v. 58, 716–722. Reprinted in Schwinger, J., ed. Selected Papers on Quantum Electrodynamics. Dover Publications, New York, 1958, pp 372–378.
2. Jabs A. Connecting Spin and Statistics in Quantum Mechanics. arXiv: quant-ph/0810.2300v4.
3. Feynman R. P., Leighton R. B., Sands M. Lectures on Physics, Volume III: Quantum Mechanics. Addison-Wesley Publishing Company, Reading, Massachusetts, 1965.
4. Woit P. Not Even Wrong; The Failure of String Theory and the Search for Unity in Physical Law. Basic Books, NewYork, 2006.
5. Kleinert H. Multivalued Fields in Condensed Matter, Electromagnetism, and Gravitation. World Scientific Publishing, Singapore, 2008.
6. Millette P. A. Elastodynamics of the Spacetime Continuum. *The Abraham Zelmanov Journal*, 2012, v. 5, 221–277.
7. Millette P. A. Elastodynamics of the Spacetime Continuum: A Spacetime Physics Theory of Gravitation, Electromagnetism and Quantum Physics. American Research Press, Rehoboth, NM, 2017.
8. Corrected eq. (16.87) of [7].
9. Millette P. A. Dislocations in the Spacetime Continuum: Framework for Quantum Physics. *Progress in Physics*, 2015, vol. 11 (4), 287–307.
10. Millette P. A. The Burgers Spacetime Dislocation Constant b_0 and the Derivation of Planck's Constant. *Progress in Physics*, 2015, vol. 11 (4), 313–316.
11. Millette P. A. Wave-Particle Duality in the Elastodynamics of the Spacetime Continuum (STCED). *Progress in Physics*, 2014, vol. 10 (4), 255–258.