A Symplectic Cosmological Model

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In this paper, we use the Lie algebra of the dual Poincaré dynamical group which when acted upon by its coadjoint, displays energy momentum and spin as pure geometrical quantities. When extended to the full group, one obtains negative mass species in accordance with our *Janus Cosmological Model* and the twin universe model conjectured by A. Sakharov. Within a *5D Kaluza space*, the theory leads to a new matter/antimatter duality implying negative energy photons emitted on the negative domain of this twin Universe. This accounts for the dark matter and dark energy which are thereof impossibe to detect in our domain. Finally, we show that shifting to a Hermitean space-time with an associated complex dynamic group yields imaginary energy, imaginary energy and imaginary charges all embedded in a symplectic (complex) framework which remains open to wide investigations.

Notations

Space time indices: $m, n = 0, 1, 2, 3$.
Space-time signature: -2.
Einstein's constant: $\varkappa$.

Introduction

Symplectic geometry relies on symplectic manifolds. Those are said symplectic when they are endowed with a so-called *symplectic form* that allows for the measurement of sizes of 2-dimensional objects. In Riemannian geometry, the metric tensor probes lengths and angles, whereas the symplectic form measures areas.

The term symplectic was first coined by *H. Weyl* in 1939 as a substitute to rather confusing (line) complex groups and/ or Abelian linear groups. The relativistic symplectic mechanics [1] was primarly developed by the french mathematician J. M. Souriau from dynamic groups theory. It provides a new description of energy, momentum and spin only in terms of pure geometrical quantities. This arises from two objects: n-dimensional space and its isometry group.

In what follows, we briefly describe its properties which we apply to a particular cosmological model featuring two types of masses and energies comparable to the *twin Universe* originally conjectured by A. Zakharov.

1 The Janus Cosmological Model (JCM)

The main mathematical tool used here is the so-called "*momentum map*" which is inferred from the *co-adjoint* action of the group on the dual of its *Lie algebra*. (The coadjoint of the Lie group is the dual of the adjoint representation.) Applying the technique of this coadjoint action leads to the appearance of generalized linear and angular momenta: {energy E, 3-momentum p, spin s}. The action of the group corresponds to

$$M' = L\,M\,{}^{\mathrm{T}}M + N\,{}^{\mathrm{T}}P\,{}^{\mathrm{T}}L - L\,P\,{}^{\mathrm{T}}N, \tag{1}$$

$$P' = L\,P, \tag{2}$$

where P is the generalized energy-momentum 4-vector

$$\begin{pmatrix} E \\ p_1 \\ p_2 \\ p_3 \end{pmatrix}, \tag{3}$$

L is here the element of the *Lorentz group* and N is the *boost 4-vector*. In the classical treatment, one merely considers the *restricted Poincaré Group* which is formed with orthochronous components L_0. Hence, the *full Poincaré Group* can be written as

$$\begin{pmatrix} \lambda\,L_0 & N \\ 0 & 1 \end{pmatrix} \tag{4}$$

with $\lambda = \pm 1$.

We then obtain two kinds of matters an two kinds of photons with each an opposite mass and energy. This copes with the *Janus Cosmological Model* (JCM) we developed earlier [2–4]. Such a model involves particles with opposite masses and energy. However, as shown by H. Bondi [5], the field equations cannot sustain this duality due to the subsequent and unmanageable "*run away*" effect. In short, General Relativity deals with positive masses that are attractive, while negative masses would exhibit repelling forces. Therefore, if one considers a couple ($+m$, $-m$), the negative mass escapes and is "chased" by the positive one while at the same time experiencing a uniform acceleration.

This issue can be evaded by considering a bi-metric (our *JCModel*) within a single manifold $\mathfrak{M}_4$ equipped with two metric tensors ${}^{(+)}g_{\mu\nu}$ and ${}^{(-)}g_{\mu\nu}$, which define two field equations [5]:

$${}^{(+)}R_{\mu\nu} - \frac{1}{2}\,{}^{(+)}g_{\mu\nu}\,{}^{(+)}R = \varkappa\left[{}^{(+)}T_{\mu\nu} + \left(\frac{{}^{(-)}g}{{}^{(+)}g}\right)^{1/2}{}^{(-)}T_{\mu\nu}\right], \tag{5}$$

$${}^{(-)}R_{\mu\nu} - \frac{1}{2}\,{}^{(-)}g_{\mu\nu}\,{}^{(-)}R = \varkappa\left[{}^{(-)}T_{\mu\nu} + \left(\frac{{}^{(+)}g}{{}^{(+)}g}\right)^{1/2}{}^{(+)}T_{\mu\nu}\right]. \tag{6}$$

Those time dependent and time independent solutions fit the observational data.

2 Extension to a wider geometrical framework

We now turn consider an extension of the group to a five dimensional scheme so as to obtain an *isometry group* which acts on the basic *Kaluza space-time*

$$\begin{pmatrix} \lambda\mu & 0 & \phi \\ 0 & \lambda L_0 & N \\ 0 & 0 & 1 \end{pmatrix} \tag{7}$$

with $\mu = \pm 1$ and $\lambda = \pm 1$.

By extending to the fifth dimension, the *Noether theorem* induces an additional conserved scalar quantity which is readily identified with the electric charge q.

The $\mu = -1$ implies both the inversion of this charge and the inversion of the fifth dimension, which is just the geometrical expression of the matter-antimatter duality as primarly shown by J. M. Souriau [6]. Therefore the physics ruled by the dynamical group (7) exhibits straightforwardly the matter-antimatter symmetry in the two domains with opposed mass and energy. If we now add p-Kaluza-like dimensions, we obtain the metric under the form:

$$ds^2 = dx_0^2 - dx_1^2 - dx_2^2 - dx_3^2 - d\xi_1^2 - d\xi_2^2 - \ldots - d\xi_p^2 \,. \tag{8}$$

This can be coupled to an isometry group

$$\begin{pmatrix} \lambda\mu & 0 & \ldots & 0 & 0 & \phi_1 \\ 0 & \lambda\mu & \ldots & 0 & 0 & \phi_2 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \lambda\mu & 0 & \phi_p \\ 0 & 0 & \ldots & 0 & \lambda\mu & N \\ 0 & 0 & \ldots & 0 & 0 & 1 \end{pmatrix} \tag{9}$$

with $\mu = \pm 1$ and $\lambda = \pm 1$.

The electric charge is just one of the quantum charges. Here again, the ($\mu = -1$) terms reflect the C-symmetry: they account for the classical matter-antimatter representation. The ($\mu = -1$; $\lambda = -1$) correspond to the PT-symmetry classically associated with the "Feynman antimatter" which is no longer indentified with the "C-antimatter". This is due to the presence of the time reversal T inducing both the mass and energy inversion. In other words, the group representation (9) which is the basis of the *JC Model*, provides two distinct types of antimatters:

— The C-type corresponding to Dirac's antimatter.

— The PT-type corresponding to Feynman's antimatter.

3 Remark about Andrei Sakharov' scheme

In classical cosmology a severe problem remains, due to the absence of observation of primordial antimatter. In 1967, Sakharov suggested that the Universe comprises two domains: the actual Universe and its twin Universe, each connected through a singularity [8–10]. Both are CPT-symmetrical. Since the mass inversion goes with T-symmetry, our *JC Model* [3,4] corresponds to such CPT-symmetry. The so-called *twin matter* becomes nothing but a copy of ordinary particles with opposite masses and charges. If, as suggested by Sakharov, positive masses are synthetized by positive energy quarks faster than the synthesis of negative masses from positive energy antiquarks, then in the *positive energy domain* we find:

— Remnant positive masses matter.

— The equivalent (ratio 3/1) of positive energy antiquarks.

— Positive energy photons.

In analogy to Sakharov's ideas, the negative energy domain would be thus composed of:

— Remnant negative masses matter.

— The equivalent (ratio 3/1) of negative energy quarks.

— Negative energy photons.

As shown in [3,4], the negative material suitably replaces both dark matter and so-called dark energy. Accordingly, by emitting negative energy photons, the remnant negative masses matter are genuinely invisible.

4 Remark about the Quantum Theory of Fields (QFT)

In QFT the time reversal operator is a complex operator which can be *linear* and *unitary*, as well as antilinear and anti-unitary. If chosen linear and unitary, this operator implies the existence of negative energy states, which are *à priori* banned by QFT. In Weinberg [7], we quote: "*In order to avoid this disastrous conclusion, we are forced to conclude that T is anti-linear and anti-unitary*". On page 104, Weinberg also writes: "*no examples are known of particles that furnish unconventional representation of inversions, so these possibilities will not be pursued further here*". Actually, this was true until the discovery of the acceleration of the expanding universe which implies the action of a negative pressure. As a pressure is likened to an energy density, this new phenomenon implies in turn the existence of negative energy states and as a result, it questions QFT by itself. In the same manner, it also raises some questions as to the validity of the so-called CPT theorem and the vacuum instability. Indeed, classically, one considers that a particle may loose energy through the emission of a photon, so that such a process would lead to negative energy states. But if we consider that a negative mass particle emits negative energy photons, this process would lead to stable zero energy state.

5 Extension of the method to a complex field

If one replaces the Minkowski coordinates $\{x_0, x_1, x_2, x_3\}$ with complex coordinates we may form the *Hermitean Riemann* *metric*:

$$ds^2 = dx_0{}^*dx_0 - dx_1{}^*dx_1 - dx_2{}^*dx_2 - dx_3{}^*dx_3\,. \tag{10}$$

This metric is defined on a *Hermitean manifold*.

Lest us now consider the real matrix G

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{11}$$

and the complex Lorentz group defined as

$$^*LGL = G\,, \tag{12}$$

$^*L$ stands for the *adjoint* of L.

One can then easily show that the *complex Poincaré group*

$$\begin{pmatrix} L & N \\ 0 & 1 \end{pmatrix} \tag{13}$$

is an isometry group of such a Hermitean space and can be considered as a dynamic group. Surprizingly, all classical (matrix) calculations can be extended to such complex framework, by simply substituting the matrices $^*A$ to the transpose matrices $^{\mathrm{T}}A$.

As a result, the complex momentum obeys the law:

$$M' = L\,M\,{}^*L + N\,{}^*P\,{}^*L - L\,P\,{}^*N, \tag{14}$$

$$P' = L\,P, \tag{15}$$

where $^*P$ is the complex energy momentum 4-vector. This extended physics grants the mass a complex nature implying the possible existence of purely real masses $\pm m$ and purely imaginary masses: $\pm(-1)^{1/2}\,m$. At the same time, such masses can exchange imaginary photons whose energies are: $\pm(-1)^{1/2}\,E$.

Conclusion

J. M. Souriau gave the first purely geometrical interpretation of all classsical physics features, namely — energy, momenta, and spin. When extended to higher dimensions it provides a geometrical interpretation of the matter-antimatter duality. In addition, one can notice that the complex approach of space definition yields complex physical quantities. The physical meaning of these complex quantities should demand further scrutiny and as such remains a new open field of investigations.

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