Quantum Gravity Aspects of Global Scaling and the Seismic Profile of the Earth

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In this paper we derive a profile of the Earth's interior from our fractal model of matter as chain system of harmonic quantum oscillators. Model claims are verified by geophysical data. Global scaling as model of quantum gravity is discussed.

Introduction

The origin of gravity is a key topic in modern physics. The universality of free fall means that the gravity acceleration of a test body at a given location does not depend on its mass, physical state or chemical composition. This discovery, made four centuries ago by Galilei, is confirmed by modern empirical research with an accuracy of 10^{-11} - 10^{-12} [1–3]. A century ago Einstein supposed that gravity is indistinguishable from, and in fact the same thing as, acceleration. In fact, Earth's surface gravity acceleration can be derived from the orbital elements of any satellite, also from Moon's orbit:

$$g = \frac{4\pi^2 R^3}{(T \cdot r)^2} = \frac{4\pi^2 (384399000\,\mathrm{m})^3}{(2360591\,\mathrm{s} \cdot 6371000\,\mathrm{m})^2} = 9.83\ \mathrm{m\ s}^{-2},$$

where R is the semi-major axis of Moon's orbit, T is the orbital period of the Moon and r is the average radius of the Earth. No data about the mass or chemical composition of the Earth or the Moon is needed.

The 3rd law of Johannes Kepler describes the ratio R^3/T^2 as constant for a given orbital system. Kepler's discovery is confirmed by high accuracy radar and laser ranging of the movement of artificial satellites. The geocentric gravitational constant [4] equals:

$$\mu = 4\pi^2 R^3/T^2 = 3.986004418(8) \cdot 10^{14}\ \mathrm{m^3 s^{-2}}.$$

Kepler's 3rd law is of geometric origin and can be derived from Gauss's flux theorem in 3D-space within basic scale considerations. It applies to all conservative fields which decrease with the square of the distance and does not require the presence of mass.

The orbital elements R and T are directly measured, while $\mu = GM$ is an interpretation that provides mass as a source of gravity and the universality of the big G. Both postulates are essential in Newton's law of universal gravitation and in Einstein's general theory of relativity.

Nevertheless, coincidence and causality is not the same thing and Newton's hypothesis about mass as source of gravity could turn out to be a dispensable assumption.

In the case of mass as source of gravity, in accordance with Newton's shell theorem, a solid body with a spherically symmetric mass distribution should attract particles outside it as if its total mass were concentrated at its center. In contrast, the attraction exerted on a particle should decrease as the particle goes deeper into the body and it should become zero at the body's center.

A boat at the latitude 86.71 and longitude 61.29 on the surface of the Arctic Ocean would be at the location that is regarded as having the highest gravitational acceleration of 9.8337 m/s^2 on Earth. At higher or lower position to the center of the Earth, gravity should be of less intensity. This conclusion seems correct, if only mass is a source of gravity acceleration and if the big G is universal under any conditions and in all scales.

The Preliminary Reference Earth Model [5] affirms the decrease of the gravity acceleration with the depth. However, this hypothesis is still under discussion [6–8].

In 1981, Stacey, Tuck, Holding, Maher and Morris [9, 10] reported anomalous measures (larger values than expected) of the gravity acceleration in deep mines and boreholes. In [11] Frank Stacey writes: "Modern geophysical measurements indicate a 1% difference between values at 10 cm and 1 km (depth). If confirmed, this observation will open up a new range of physics". In fact, gravity is the only interaction that is not described yet by a quantum theory.

In [12] we have introduced a fractal model of matter as a chain system of harmonic quantum oscillators. The model statements are quite general, that opens a wide field of possible applications.

Already in [13] we could show that scale invariance is a fundamental characteristic of this model. On this background we proposed quantum scaling as model of particle mass generation [14] and we could show that particle rest masses coincide with the eigenstates of the system. This is valid not only for hadrons, but for mesons and leptons as well. Andreas Ries [15] demonstrated that this model allows for the prediction of the most abundant isotope of a given chemical element.

In the framework of our model, physical characteristics of celestial bodies can be understood as macroscopic quantized eigenstates in chain systems of oscillating protons and electrons [16]. This is also valid for accelerations. In [17] was shown that the surface gravity accelerations of the planets in the solar system correspond with attractor nodes of stability in chain systems of protons and electrons.

Our model allows us to see a connection between the stability of the solar system and the stability of electron and proton and consider global scaling as a forming factor of the solar system. This may be of cosmological significance.

In this paper we derive a profile of the Earth's interior from our fractal model of matter as chain system of harmonic quantum oscillators. Model claims are verified by geophysical data. Global scaling as model of quantum gravity is discussed.

Methods

In [13] we have shown that the set of natural frequencies of a chain system of similar harmonic oscillators can be described as set of finite continued fractions $\mathcal{F}$, which are natural logarithms:

$$\ln(\omega_{jk}/\omega_{00}) = n_{j0} + \cfrac{z}{n_{j1} + \cfrac{z}{n_{j2} + \ddots + \cfrac{z}{n_{jk}}}} = \tag{1}$$

$$= [z, n_{j0}; n_{j1}, n_{j2}, \ldots, n_{jk}] = \mathcal{F}$$

where ω_{jk} is the set of angular frequencies and ω_{00} is the fundamental frequency of the set. The denominators are integer: $n_{j0}, n_{j1}, n_{j2}, \ldots, n_{jk} \in \mathrm{Z}$, the cardinality $j \in \mathrm{N}$ of the set and the number $k \in \mathrm{N}$ of layers are finite. In the canonical form, the numerator z equals 1.

In the canonical form, for finite continued fractions, the distribution density of the eigenvalues reaches maxima near reciprocal integers 1, 1/2, 1/3, 1/4, ... which are the attractor points of the fractal set $\mathcal{F}$ of natural logarithms (fig. 1).

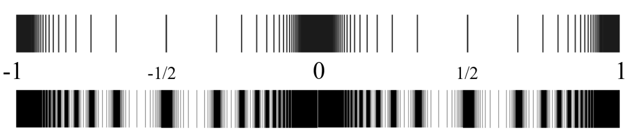

Fig. 1: The canonical form of $\mathcal{F}$ for $k = 1$ (above) and for $k = 2$ (below) in the range $-1 \leqslant \mathcal{F} \leqslant 1$.

Any finite continued fraction represents a rational number [18]. Therefore, all natural frequencies ω_{jk} in (1) are irrational, because for rational exponents the natural exponential function is transcendental [19]. This circumstance provides for high stability of eigenstates in a chain system of harmonic oscillators because it prevents resonance interaction between the elements of the system [20]. Already in 1987 we have applied continued fractions of the type $\mathcal{F}$ as criterion of stability in engineering [21, 22].

In the case of harmonic quantum oscillators, the continued fractions $\mathcal{F}$ define not only fractal sets of natural angular frequencies ω_{jk}, angular accelerations $a_{jk} = c \cdot \omega_{jk}$, oscillation periods $\tau_{jk} = 1/\omega_{jk}$ and wavelengths $\lambda_{jk} = c/\omega_{jk}$ of the chain system, but also fractal sets of energies $\mathrm{E}_{jk} = \hbar \cdot \omega_{jk}$ and masses $\mathrm{m}_{jk} = \mathrm{E}_{jk}/c^2$ which correspond with the eigenstates of the system. For this reason, we call the continued fraction $\mathcal{F}$ the "fundamental fractal" of eigenstates in chain systems of harmonic quantum oscillators.

In the canonical form ($z = 1$) of the fundamental fractal $\mathcal{F}$, shorter continued fractions correspond with more stable eigenstates of a chain system of harmonic oscillators. Therefore, integer logarithms represent the most stable eigenstates (main attractor nodes).

As the cardinality and number of layers of the continued fractions $\mathcal{F}$ are finite but not limited, in each point of the space-time occupied by the chain system of harmonic quantum oscillators the scalar $\mathcal{F}$ is defined. Consequently, any chain system of harmonic quantum oscillators can be seen as source of the scalar field $\mathcal{F}$, the fundamental field of the system. Figure 2 shows the linear 2D-projection of the first layer (k = 1) of the fundamental field $\mathcal{F}$ in the canonical form (z = 1) in the interval $-1 \leqslant \mathrm{F} \leqslant 1$.

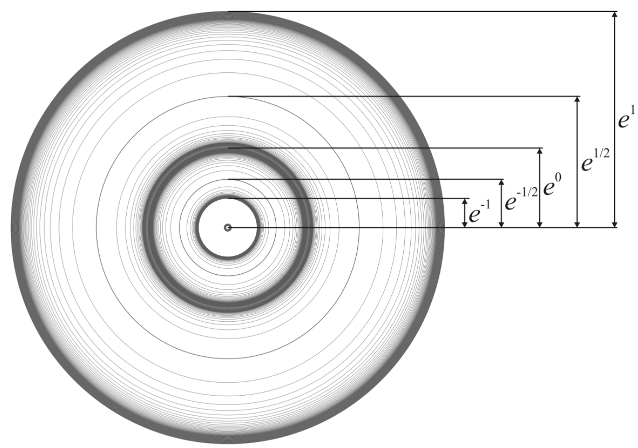

Fig. 2: The first layer ($k = 1$) of the linear 2D-projection of the fundamental field $\mathcal{F}$ in the canonical form ($z = 1$) in the range $-1 \leqslant \mathcal{F} \leqslant 1$.

The scalar potential difference $\Delta\mathcal{F}$ of sequent equipotential surfaces at a given layer k is defined by the difference of continued fractions (1). In the canonical form ($z = 1$):

$$\Delta\mathcal{F} = \mathcal{F}(\mathrm{j,k}) - \mathcal{F}(\mathrm{j+1,k}) =$$
$$= [n_{j0}; n_{j1}, n_{j2}, \ldots, n_{jk}] - [n_{j0}; n_{j1}, n_{j2}, \ldots, n_{j+1,k}]$$

Normal matter is formed by nucleons and electrons because they are exceptionally stable quantum oscillators. In the concept of isospin, proton and neutron are viewed as two states of the same quantum oscillator. Furthermore, they have similar rest masses. However, a free neutron decays into a proton, an electron and antineutrino within 15 minutes while the life-spans of the proton and electron top everything that is measurable, exceeding 10^{29} years [23].

These unique properties of the electron and proton predestinate their physical characteristics as fundamental units. Table 1 shows the basic set of electron and proton units that can be considered as a fundamental metrology (c is the speed of light in a vacuum, $\hbar$ is the Planck constant, k_B is the Boltzmann constant).

Property	Electron	Proton
rest mass m	$9.10938356(11) \cdot 10^{-31}$ kg	$1.672621898(21) \cdot 10^{-27}$ kg
energy $\mathrm{E} = \mathrm{m}c^2$	0.5109989461(31) MeV	938.2720813(58) MeV
angular frequency $\omega = \mathrm{E}/\hbar$	$7.76344071 \cdot 10^{20}$ Hz	$1.42548624 \cdot 10^{24}$ Hz
angular oscillation period $\tau = 1/\omega$	$1.28808867 \cdot 10^{-21}$ s	$7.01515 \cdot 10^{-25}$ s
angular wavelength $\lambda = c/\omega$	$3.8615926764(18) \cdot 10^{-13}$ m	$2.1030891 \cdot 10^{-16}$ m
angular acceleration $a = c\omega$	$2.327421 \cdot 10^{29}$ ms^{-2}	$4.2735 \cdot 10^{32}$ ms^{-2}

Table 1: The basic set of physical properties of the electron and proton. Data taken from Particle Data Group [23]. Frequencies, oscillation periods, accelerations and the proton wavelength are calculated.

In [16] was shown that the fundamental metrology (tab. 1) is completely compatible with Planck units [24]. Originally proposed in 1899 by Max Planck, these units are also known as natural units, because the origin of their definition comes only from properties of nature and not from any human construct. Max Planck wrote [27] that these units, "regardless of any particular bodies or substances, retain their importance for all times and for all cultures, including alien and non-human, and can therefore be called natural units of measurement". Planck units reflect the characteristics of space-time.

In [12, 14] we have introduced a fractal model of matter as a chain system of oscillating protons and electrons. We hypothesize that scale invariance of the fundamental field $\mathcal{F}$ calibrated on the physical properties of the proton and electron (tab. 1) is a universal characteristic of organized matter and criterion of stability. This hypothesis we have called 'global scaling' [16, 26, 27].

Results

The proton-to-electron mass ratio is approximately 1836, so that the mass contribution of the proton to normal matter is very high, for example in the hydrogen atom (protium) it is $1 - 1/1836 = 99.95$ percent. Consequently, the mass contribution of the electron is only 0.05 percent. In heavier atoms which contain neutrons, the electron contribution to atomic mass is even smaller. Therefore, in this paper we investigate a fractal model of matter as chain system of oscillating protons and derive a profile of the Earth's interior from it.

As figure 1 shows, in an attractor node of the layer $k = 0$, the potential difference on the layer $k = 1$ changes its signature and compression of the equipotential density is changed to decompression. The same is valid in attractor nodes of the layer $k = 1$. There the potential difference on the layer $k = 2$ changes its signature. Therefore, we expect that near the attractor nodes of $\mathcal{F}$ the dramatic increase of the field strength and the change of compression to decompression of the equipotential density in the attractor nodes should lead to measurable consequences. This should be valid at least for the main attractor nodes on the layer $k = 0$.

Figure 3 shows the $\mathcal{F}$ calibrated on the angular Compton wavelength of the proton in the canonical ($z = 1$) linear 2D-projection for $k = 1$ in the interval $[49; \infty] \leqslant \mathcal{F} \leqslant [52; -4]$. At the graphic's left side the corresponding radii in km are indicated. The radial distribution of equipotential nodes represents the expected 2D-profile of the Earth's interior.

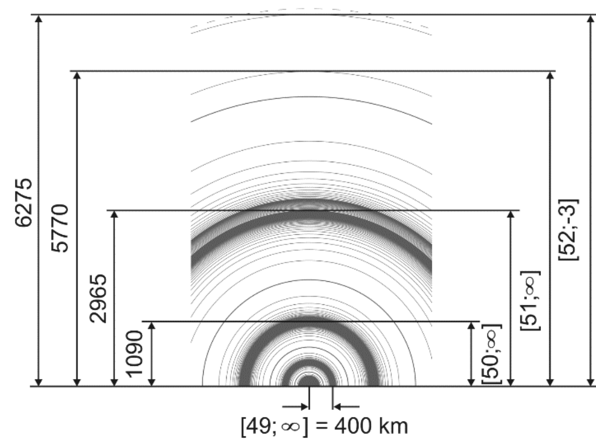

Fig. 3: The fundamental field $\mathcal{F}$ calibrated on the proton in the canonical ($z = 1$) linear 2D-projection for $k = 1$ in the interval $[49; \infty] \leqslant \mathcal{F} \leqslant [52; -4]$. Radius in km (left side). The dotted line at the top indicates the Earth surface that coincides with the significant subnode $[44; 4] = 6372$ km of the $\mathcal{F}$ calibrated on the electron.

The propagation speed of a seismic compression wave depends on the density and elasticity of the medium and should therefore correspond with zones of compression and decompression near the main nodes of the fundamental field $\mathcal{F}$.

In accordance with both empirical models of the Earth interior PREM [5] and IASP91 [28], the crust-mantle boundary (Mohorovicic discontinuity, 'Moho') is in between 35 and 90 km depth from the Earth surface, where seismic P-waves jump in speed abruptly from 6 to 8 km/s. In our model, the Moho corresponds with the compression zone before the significant subnode $[52; -4] = 6275$ km of the $\mathcal{F}$ calibrated on the proton.

Detailed seismic studies have shown that the speed of P-waves (longitudinal pressure waves) in the mantle increases rather rapidly from about 9 to 11 km/s at depths between 400 and 700 km, marking a layer called the transition zone. This zone separates the upper mantle from the lower mantle. In our model, the transition zone corresponds with the compression zone before the significant subnode [52; -3] = 5770 km of the $\mathcal{F}$ calibrated on the proton.

As they travel more deeply into the mantle, P-waves increase their speed from 8 km/s at the Moho to about 13 km/s at a depth of 2900 km. Though, once P-waves penetrate below 2900 km, their velocity suddenly drops from 13 km/s back down to about 8 km/s. This dramatic reduction in speed after a depth of 2900 km defnes the boundary between the Earth's mantle and the core. The outer core seems liquid, because seismic S-waves (transversal shear waves) do not pass this boundary. In contrast, the innermost part of the core within a radius of 1210 km seems solid. Reaching the inner core, P-waves again jump to a velocity of 11 km/s.

Both models PREM and IASP91 identify these boundaries with the radius of the liquid core (3480 km) and the radius of the inner solid core (1210 km). These estimations correspond with the compression zones before the main attractors [51; ∞] and [50; ∞] of the proton $\mathcal{F}$ and confirm that P-waves increase their velocity in the compression zone before the attractor. Then in the decompression zone after the attractor, they decrease the velocity. This coincidence is a strong confirmation of global scaling and demonstrates that the current dimension and structure of the Earth interior is not casual, but an essential criterion of its stability.

It is a notable circumstance that P-waves reach cosmic velocities. In fact, at the Moho, P-waves jump to velocities near 8 km/s that is in the range of the first cosmic velocity a rocket must have to reach a circular orbit around the Earth. In the transition zone that separates the upper mantle from the lower mantle, P-waves jump to 11 km/s that is in the range of the second cosmic velocity a rocket must have to escape the Earth gravity acceleration. Through the lower mantle, the P-wave reach 13 km/s at the core-mantle boundary that is in the range of velocities a rocket launched from Earth must have to escape the solar system.

This similarity seems not by case: cosmic escape velocities do not depend on the mass of the object escaping the Earth. The velocity a rocket must have to reach a circular orbit around the Earth depends only on the gravity acceleration g and the radius r of the departure orbit. It is notable that no data about the mass of the Earth is needed. In the case of departure from the Earth surface:

$$v_{circular} = \sqrt{(gr)} = \sqrt{9.8\,\mathrm{m/s^2} \cdot 6371000\,\mathrm{m}} = 7.9 \text{ km/s}.$$

The second cosmic velocity a rocket must have to escape the Earth gravity acceleration is $\sqrt{2}$ times higher:

$$v_{escape} = \sqrt{2} \cdot v_{circular} = 11.2 \text{ km/s}.$$

Conversely, an object that falls under the attraction of the Earth surface gravity acceleration g from infinity, starting with zero velocity, will strike the Earth surface with a velocity equal to its escape velocity.

In accordance with our model (fig. 3), the inner core of the Earth should have a substructure that origins from the attractor node [49; ∞] = 400 km of the $\mathcal{F}$ calibrated on the proton (fig. 4). In fact, the seismological exploration of the Earth's inner core has revealed unexpected structural complexities. There is a clear hemispherical dichotomy in anisotropy and also evidence of a subcore with a radius 300–600 km [29]. Considering that the radius of the Sun coincides with the main attractor node [49; ∞] of the $\mathcal{F}$ calibrated on the electron:

$$\ln\left(\frac{r_{\text{Sun}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{6.96407 \cdot 10^{8} \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 48.945$$

we can write down the equation for the ratio of the radii:

$$\frac{r_{\text{Sun}}}{r_{\text{Earth subcore}}} = \frac{\lambda_{\text{electron}}}{\lambda_{\text{proton}}}.$$

Already in [16] we have shown that the minimum and maximum values of the Earth's radius approximate the significant node [44; 4] of the $\mathcal{F}$ calibrated on the electron:

$$\ln\left(\frac{r_{\text{Earth max}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{6.384 \cdot 10^{3} \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 44.252,$$

$$\ln\left(\frac{r_{\text{Earth min}}}{\lambda_{\text{electron}}}\right) = \ln\left(\frac{6.353 \cdot 10^{3} \text{ m}}{3.8615926764 \cdot 10^{-13} \text{ m}}\right) = 44.247.$$

Figure 3 shows the node [44; 4] of the electron $\mathcal{F}$ as dotted line in the top of the graphic.

Conclusion

In the framework of our model of matter as chain system of harmonic quantum oscillators, the fractal fundamental field $\mathcal{F}$ affects any type of physical interaction, including the gravitational. Fundamental particles like electron and proton are not the ultimate sources, but stability nodes of the fundamental field of any chain system of harmonic quantum oscillators. The spatial and temporal distribution of these stability nodes is determined by the ratio of fundamental constants. Already Paul Dirac [30] mentioned that "... whether a thing is constant or not does not have any absolute meaning unless that quantity is dimensionless".

Applying our fractal model of matter to the analysis of gravimetric and seismic characteristics of the Earth we did show that it corresponds well with established empirical models of the Earth interior. We interpret this correspondence as evidence of the fractality, scale invariance and macroscopic quantization of Earth's gravity field.

We presume that gravity is a scale-invariant attractor effect of stability nodes in chain systems of oscillating protons and electrons. May be this hypothesis could become a bridge that connects the island of gravity research with the continent of quantum physics.

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