Elimination of Anomalies Reported for $b \to s\ell\ell$ and $b \to c\ell\bar{\nu}_\ell$ Semi-Leptonic Decay Ratios R(K,K*) and R(D,D*) when the Lepton Families Represent Discrete Symmetry Binary Subgroups 2T, 2O, 2I of SU(2)

Franklin Potter

Sciencegems.com, 8642 Marvale Drive, Huntington Beach, CA 92646 USA. E-mail: frank11hb@yahoo.com

The large discrepancies between the measured and predicted values of B meson decay ratios R(K) and R(D) could indicate lepton flavor universality violation and new physics beyond the Standard Model. I propose that the only new physics is that each lepton family represents a different discrete symmetry binary subgroup of SU(2) and that lepton flavor mixing exists because the 3 families act collectively to achieve SU(2) symmetry. Successful calculations of the neutrino mixing angles and of the measured ratios R(K,K*) and R(D, D*) by using those mixing angles confirm that the 3 lepton families represent the 3 binary subgroups 2T, 2O, and 2I.

1 Introduction

Perhaps the hottest research topic today in particle physics is whether the door to new physics (NP) has been pried ajar by the Belle, BaBar, and LHCb reports of significant discrepancies from the Standard Model (SM) predicted values in the B meson semi-leptonic decay ratios. In particular, rare $b \to s\ell\ell$ and $b \to c\ell\bar{\nu}_\ell$ decays are now known to exhibit significant deviations from the SM predictions for both their branching ratios and their angular distributions [1]. One possible interpretation of these results would be the violation of lepton flavor universality (LFUV) with regard to the weak interaction.

Over the past two decades these deviations from the SM predicted values have triggered a variety of models of NP, such as Z' models with gauged L_μ - L_τ, models with leptoquarks, models with compositeness, etc. For a complete list of the great variety of proposed NP models, see [2].

I claim that the only NP required is to properly identify the lepton and quark family symmetries. Previously, I have shown [3] that their EW flavor states actually represent 3 specific discrete symmetry subgroups of SU(2). In better words, the true reason for lepton mixing is the collective action of the 3 lepton families with their discrete symmetries to mimic the SU(2) weak isospin eigenstates $\pm\frac{1}{2}$ demanded by the SM gauge interaction bosons representing $SU(2)_W \times U(1)_Y$. The correct statement that the mixing angles represent a mismatch between the EW flavor states and their mass states is the consequence of but not the reason for the mixing. I explain below how this collective action is achieved by the 3 specific discrete symmetry binary subgroups of SU(2), known as 2T, 2O, and 2I, for the electron, muon, and tau families, respectively. The immediate results are the correct mixing angles and the correct ratios of branching ratios for b quark semi-leptonic decays.

Section 2 is a brief review of the recent experimental results for B meson semi-leptonic decays. Section 3 explains how the lepton mixing angles are derived from the generators of the 3 discrete symmetry subgroups of SU(2), or equivalently the group of unit quaternions Q. Section 4 includes a derivation of the electroweak (EW) boson states $W^\pm$, Z^0, and γ as well as the Weinberg angle. Finally, in Section 5, I calculate the ratios for the semi-leptonic b decays $b \to s\ell\ell$ and $b \to c\ell\bar{\nu}_\ell$ using alternative EW boson state assignments. In order to do so, one requires the appropriate discrete symmetry eigenstates for the leptons, quarks, and EW bosons, which I have discussed in the literature [3,4] and at conferences [5,6].

2 The B meson decays

The ratio of branching ratios has been used extensively to summarize both the theoretical and the experimental results because almost all the hadronic uncertainties are eliminated. For example, these four ratios for B meson decays exhibit large discrepancies of more than 2.5σ from their SM predictions [1]:

$$R(K)^{SM} = \frac{\mathcal{B}(B \to K\mu^+\mu^-)}{\mathcal{B}(B \to Ke^+e^-)} = 1.00 \pm O(1\%), \tag{1}$$

$$R(K^*)^{SM} = \frac{\mathcal{B}(B \to K^*\mu^+\mu^-)}{\mathcal{B}(B \to K^*e^+e^-)} = 1.00 \pm O(1\%), \tag{2}$$

$$R(D)^{SM} = \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\ell\nu_\ell)} = 0.298 \pm 0.003, \tag{3}$$

$$R(D^*)^{SM} = \frac{\mathcal{B}(B \to D^*\tau\nu_\tau)}{\mathcal{B}(B \to D^*\ell\nu_\ell)} = 0.255 \pm 0.004, \tag{4}$$

valid over a broad range of q^2 values.

LHCb has recently reported [7]

$$R(K)^{exp} = 0.745 \pm 0.090 \pm 0.036 \tag{5}$$

$$R(K^*)^{exp} = 0.685 \pm 0.113 \pm 0.047 \tag{6}$$

in the di-lepton invariant mass range 1 GeV^2 < q^2 < 6 GeV^2, exhibiting significant deviations from the SM predictions.

For muonic decays [8]

$$R(D)^{exp} = 0.407 \pm 0.039 \pm 0.024. \tag{7}$$

Table 1: Exact angle contributions by the U_2 generators of 2T, 2O, and 2I. Note that $\phi = (1+\sqrt{5})/2$, and Angle = arccosine (Factor), which is twice the projection angle to the k-axis.

Family	Group	U_1	U_2	U_3	Factor	Angle	Angle/2
ν_e, e^-	2T	j	$-\frac{i}{2}-\frac{j}{2}+\frac{k}{\sqrt{2}}$	i	-0.26422	105.3204°	52.660°
ν_μ, μ^-	2O	j	$-\frac{i}{2}-\frac{j}{\sqrt{2}}+\frac{k}{2}$	i	+0.80116	36.7581°	18.379°
ν_τ, τ^-	2I	j	$-\frac{i}{2}-\frac{\phi j}{2}+\frac{\phi^{-1} k}{2}$	i	-0.53695	122.4764°	61.238°

$$R(D^*)^{exp} = 0.336 \pm 0.027 \pm 0.030. \quad (8)$$

I propose that the values of the ratios R(K), R(K*), R(D), and R(D*), all can be expressed in terms of the lepton mixing angles, without venturing outside the realm of the SM local interaction symmetry group $SU(3)_C \times SU(2)_W \times U(1)_Y$. For example, I derive in Section 5 how using the lepton family mixing angles predicts

$$R(K) = \frac{Cos\,\theta_{23}}{Cos\,\theta_{13}} = \frac{Cos\,42.859^\circ}{Cos\,8.578^\circ} = 0.74127\ R(K)^{SM}, \quad (9)$$

$$R(D) = \frac{Cos\,\theta_{33}}{Cos\,\theta_{23}} = \frac{Cos\,0.000^\circ}{Cos\,42.859^\circ} = 1.36420\ R(D)^{SM}, \quad (10)$$

which agree with the experimental values 0.745 ±0.090 ± 0.036 and 0.407±0.039±0.024, respectively.

Why does this procedure work? Because the $W^\pm$ and Z^0 bosons have discrete symmetry properties, too, and are eigenstates of the binary product group 2I × 2I'. In the traditional way of thinking, such an alternative way to express $W^\pm$ and Z^0 comes as a big surprise!

3 Brief review of neutrino mixing

In 2013 I derived [3] the exact lepton mixing angles for the neutrino PMNS mixing matrix by first assigning the three lepton families to three special discrete symmetry binary subgroups of the unitary quaternion group Q, which is equivalent to the SU(2) group used for the two electroweak (EW) isospin flavor states $\pm\frac{1}{2}$ in each lepton and quark family. I provide a brief review of that lepton mixing angle derivation here.

The group Q of unitary quaternions has these discrete symmetry subgroups:

$$2T, 2O, 2I, D_{2n}, C_{2n}, C_n \quad (n\,odd). \quad (11)$$

If I assume that leptons are 3-D entities at the Planck scale, then only 2T, 2O, and 2I, are useful for identifying them. So I assigned these 3 finite binary subgroups to the electron family (ν_e, e^-), to the muon family (ν_μ, μ^-), and to the tau family (ν_τ, τ^-), respectively.

These 3 binary subgroups each have the 3 quaternion generators U_1, U_2, and U_3 as given in Table 1. Notice that for each group only two of the three generators, U_1 = j, and U_3 = i, are the same as for SU(2), which has the three quaternion generators j, k, i. Their other generator, U_2, is different for each binary subgroup and different from each other. By demanding that the three U_2 generators collectively act as the k-generator of SU(2), their linear superposition provides three equations for three unknown factors. Their normalized factors, the corresponding angles calculated by their inverse cosine projections to the k-axis, and the physical rotation angles, are quantities all listed in Table 1.

Defining the lepton mixing angles by $\theta_{ij} = |\,\theta_i - \theta_j\,|$ produces the three neutrino PMNS mixing angles

$$\theta_{12} = 34.281^\circ \quad \text{vs} \quad 33.56^\circ \pm 0.77^\circ\ (exp) \quad (12)$$

$$\theta_{23} = 42.859^\circ \quad \text{vs} \quad 41.6^\circ \pm 1.5^\circ\ (exp) \quad (13)$$

$$\theta_{13} = 8.578^\circ \quad \text{vs} \quad 8.46^\circ \pm 0.15^\circ\ (exp), \quad (14)$$

with their absolute values agreeing with the experimental values. Note that I have no mixing among the charged lepton flavor states, unitarity of the PMNS mixing matrix, a normal mass state hierarchy, and no additional neutrino states beyond those in the three known lepton families.

Therefore, I claim that the three lepton families represent the three chosen discrete symmetry binary subgroups 2T, 2O, 2I, and that they act collectively to mimic the SU(2) symmetry required for the isospin flavor states of the EW component of the SM.

4 Electroweak boson states W^+, Z^0, W^-, γ

The SM local gauge group SU(2) × U(1) has four EW interaction bosons W^+, Z^0, W^-, γ, which can be derived from the four quaternion generators i, j, k, b, with the first three generators for SU(2) or Q and the generator b for U(1) [or, equivalently, for the 2-element inversion group I_2]. These four generators required for the EW boson operations on the lepton flavor states must be able to perform the discrete rotations of the binary subgroups 2T, 2O, and 2I, in order to go from one lepton flavor state $\pm\frac{1}{2}$ to the other in each family. Of course, the Lie groups SU(2), or Q, are capable of doing these discrete rotations because they include all possible operations.

But there exists a smaller group with discrete symmetry that can provide the essential operations. One might expect that the largest group 2I of binary icosahedral operations by itself would be able to perform the required rotations in the normal space $C^2 = R^4$. However, some operations in the binary octahedral group 2O for the muon family would be omitted, so one finds that the product group 2I × 2I' is necessary, where 2I' provides certain "reciprocal" operations, as they are called.

In a 2014 paper [9], by using 2I × 2I', I derived the Weinberg angle, i.e., the weak mixing angle, using $U_2 \times U'_2$ to predict

$$\theta_W = 30^\circ \;\text{ vs }\; 28.4^\circ \pm 0.5^\circ \;\; (exp). \tag{15}$$

The discrepancy between the measured and the theoretical values of the Weinberg angle could be indicating that the 30° value applies at the Planck scale.

One now defines the four EW boson states in terms of the 2I × 2I' weak isospin states by these four relations:

$$|W^+> = |+\tfrac{1}{2}> |+\tfrac{1}{2}> \tag{16}$$

$$|Z^0> = \left(|+\tfrac{1}{2}> |-\tfrac{1}{2}> + |-\tfrac{1}{2}> |+\tfrac{1}{2}>\right) / \sqrt{2} \tag{17}$$

$$|W^-> = |-\tfrac{1}{2}> |-\tfrac{1}{2}> \tag{18}$$

$$|\gamma> = \left(|+\tfrac{1}{2}> |-\tfrac{1}{2}> - |-\tfrac{1}{2}> |+\tfrac{1}{2}>\right) / \sqrt{2}. \tag{19}$$

where the upper state $+\frac{1}{2}$ for 2I is the tau neutrino flavor state ν_τ and the lower state $-\frac{1}{2}$ is the τ^- state. The tau family antiparticle states representing the 2I' discrete symmetry group have the upper and lower states τ^+ and $\bar{\nu}_\tau$.

One would expect that these four EW boson state identifications in terms of 2I × 2I' eigenstates would be important for understanding their decays into leptons and quarks. Indeed, unless one uses these particular identifications, the B meson decays will have large discrepancies with the SM predictions and remain a challenge for the SM traditional approach, paricularly for the semi-leptonic decays

$$b \;\rightarrow\; s\,\ell^+\ell^- \text{ and } b \rightarrow c\ell\bar{\nu}_\ell, \tag{20}$$

precisely the decays for R(K) and R(D).

Therefore, I can re-define the EW boson states in terms of the tau lepton family flavor states for calculation purposes and determine the consequences for the b semi-leptonic decays:

$$|W^+> = |\nu_\tau> |\tau^+> \tag{21}$$

$$|Z^0> = (|\nu_\tau> |\bar{\nu}_\tau> + |\tau^-> |\tau^+>) / \sqrt{2} \tag{22}$$

$$|W^-> = |\tau^-> |\bar{\nu}_\tau> \tag{23}$$

$$|\gamma> = (|\nu_\tau> |\bar{\nu}_\tau> - |\tau^-> |\tau^+>) / \sqrt{2}. \tag{24}$$

That these assignments work well in determining the ratios R(K) and R(D) is discussed in the next section.

5 b → $s\ell\ell$ and b → $c\ell\bar{\nu}_\ell$

The traditional way to handle these decays would be to examine the Wilson coefficients [10] and determine which ones are possibly responsible for the discrepancies of the experimental results from the SM predictions.

However, now that I have proposed explicit expressions for the EW bosons in terms of the tau family flavor states, I can calculate directly the decay ratios reported in the literature. For the decay b → $s\ell\ell$ in which R(K) is expressed in terms of the ratio of the branching ratios of $Z^0 \rightarrow \mu^-\mu^+$ and $Z^0 \rightarrow e^-e^+$ in Eq. 1, the semi-leptonic B meson decays require the Z^0 decays expressed as

$$|\tau^-> |\tau^+> \rightarrow |\mu^-> |\mu^+> \tag{25}$$

$$|\tau^-> |\tau^+> \rightarrow |e^-> |e^+>, \tag{26}$$

with each decay being proportional to the cosine of the specific lepton mixing angle between families, i.e., one predicts their ratio

$$R(K) = \frac{\cos\theta_{23}}{\cos\theta_{13}} = \frac{0.73303}{0.98888} = 0.74127, \tag{27}$$

which is the measured value of R(K) = 0.745 ±0.090 ±0.36.

The R(K*) ratio has the same Z^0 decays, so the prediction is the same,

$$R(K^*) = \frac{\cos\theta_{23}}{\cos\theta_{13}} = \frac{0.73303}{0.98888} = 0.74127, \tag{28}$$

which is within the measured value of R(K) = 0.685 ±0.113 ±0.47 with its large uncertainties.

In order to use the same procedure for b→ $c\ell\bar{\nu}_\ell$, which involves the W^- decay, the three W^- decays are expressed as

$$|\tau^-> |\bar{\nu}_\tau> \rightarrow |\tau^-> |\bar{\nu}_\tau> \tag{29}$$

$$|\tau^-> |\bar{\nu}_\tau> \rightarrow |\mu^-> |\bar{\nu}_\mu> \tag{30}$$

$$|\tau^-> |\bar{\nu}_\tau> \rightarrow |e^-> |\bar{\nu}_e>, \tag{31}$$

again with each decay being proportional to the cosine of the lepton mixing angle. For example, taking the ratio of the first two, one obtains the factors

$$R(D)^\mu = \frac{\cos\theta_{33}}{\cos\theta_{23}} = \frac{1}{0.73303} = 1.364, \tag{32}$$

and the ratio of the first and third produces

$$R(D)^e = \frac{\cos\theta_{33}}{\cos\theta_{13}} = \frac{1}{0.98888} = 1.011. \tag{33}$$

Either or both of these factors multiplies the SM predicted value in order to achieve the measured values of R(D) and R(D*). The W^- decay to the muon family alone produces

$$R(D)^\mu = 1.364 \,\text{x}\, 0.298 = 0.408, \tag{34}$$

$$R(D^*)^\mu = 1.364 \,\text{x}\, 0.255 = 0.348. \quad (35)$$

both predicted values matching the experimental values 0.407 ±0.039 ±0.024 and 0.336 ±0.027 ±0.030, respectively, for purely muonic decays.

And for the other product, the one involving the tau family states decaying to the electron family only, the predicted results are

$$R(D)^e = 1.011 \,\text{x}\, 0.298 = 0.301, \quad (36)$$

$$R(D^*)^e = 1.011 \,\text{x}\, 0.255 = 0.258. \quad (37)$$

Therefore, if there is a significant electron family contribution to the R(D*) decay channel, that would lower the total predicted R(D*) value for those reports that average both the muon and electron contributions.

6 Summary

There is no evidence in these semi-leptonic decays for lepton flavor violation. The lepton mixing angles are used to successfully calculate the B meson ratios R(K), R(K*), R(D), and R(D*), which involve ratios of the semi-leptonic b quark decays $b \to s\ell\ell$ and $b \to c\ell\bar{\nu}_\ell$. No discrepancies between the predicted values and the experimental values exist when the lepton families are expressed in terms of the 3 discrete symmetry binary subgroups 2T, 2O, and 2I of SU(2) and the EW boson states are expressed in terms of the discrete symmetry product group 2I × 2I'. The predicted values agree with the experimental values for all four ratios when expressed in terms of the appropriate mixing angles.

The key idea is that the lepton mixing angles exist because the 3 binary subgroups identifying the 3 lepton family discrete symmetries are acting collectively to achieve the SU(2) Lie symmetry of the EW part of the SM. One immediate consequence is that the EW boson states W^+, Z^0, W^-, γ can be expressed in terms of the discrete symmetry product group 2I × 2I', a real surprise. With these discrete symmetry groups, I calculate the neutrino mixing angles, the Weinberg angle, and the four B meson ratios, all in agreement with the experimental values.

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