QED Mass Renormalization, Vacuum Polarization and Self-Energies in the Elastodynamics of the Spacetime Continuum (STCED)

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In this paper, we consider the explanation of the Quantum Electrodynamics (QED) phenomena of self-energy, vacuum polarization and mass renormalization provided by the Elastodynamics of the Spacetime Continuum (*STCED*). We note that QED only deals with the *wave* aspect of wave-particle objects, and hence QED only deals with the distortion transverse strain energy $W_\perp$, while the dilatation massive longitudinal strain energy term $W_\parallel$ is not considered. Hence there is no possibility of properly deriving the mass, as QED uses an incomplete description of particle energies at the quantum level. Comparison of QED mass renormalization with *STCED* strain energy shows that the interaction of the particle with the medium or the field, δm, is the transverse strain energy present in the spacetime continuum (or vacuum), essentially a field energy. We provide the strain energy equivalence for QED mass renormalization and self-energies for bosons, quarks and leptons.

1 Introduction

In this paper, we consider the explanation of the Quantum Electrodynamics (QED) phenomena of self-energy, vacuum polarization and mass renormalization provided by the Elastodynamics of the Spacetime Continuum (*STCED*) [1–11]. QED is the well-known relativistic quantum field theory of electromagnetic dynamics (electrodynamics) in which charged particle interactions are described by the exchange of (virtual) photons. QED is a perturbative theory of the electromagnetic quantum vacuum [12], and the virtual particles are introduced as an interpretation of the propagators which appear in the perturbation expansion of vacuum expectation values represented by Feynman diagrams.

In *STCED*, energy propagates in the spacetime continuum (*STC*) as wave-like deformations which can be decomposed into *dilatations* and *distortions*. *Dilatations* involve an invariant change in volume of the spacetime continuum which is the source of the associated rest-mass energy density of the deformation. On the other hand, *distortions* correspond to a change of shape (shearing) of the spacetime continuum without a change in volume and are thus massless. Thus the deformations propagate in the continuum by longitudinal (*dilatation*) and transverse (*distortion*) wave displacements.

This provides a natural explanation for wave-particle duality, with the massless transverse mode corresponding to the wave aspects of the deformations and the massive longitudinal mode corresponding to the particle aspects of the deformations. The rest-mass energy density of the longitudinal mode is given by [1, see Eq.(32)]

$$\rho c^2 = 4\bar{\kappa}_0 \varepsilon \tag{1}$$

where ρ is the rest-mass density, c is the speed of light, $\bar{\kappa}_0$ is the bulk modulus of the *STC* (the resistance of the spacetime continuum to *dilatations*), and ε is the volume dilatation

$$\varepsilon = \varepsilon^{\alpha}{}_{\alpha} \tag{2}$$

which is the trace of the *STC* strain tensor obtained by contraction. The volume dilatation ε is defined as the change in volume per original volume $\Delta V/V$ [13, see pp. 149–152] and is an invariant of the strain tensor, as is the rest-mass energy density. Hence

$$mc^2 = 4\bar{\kappa}_0\,\Delta V \tag{3}$$

where m is the mass of the deformation and ΔV is the dilatation change in the spacetime continuum's volume corresponding to mass m. This demonstrates that mass is not independent of the spacetime continuum, but rather mass is part of the spacetime continuum fabric itself.

In *STCED*, $\bar{\lambda}_0$ and $\bar{\mu}_0$ are the Lamé elastic constants of the spacetime continuum: $\bar{\mu}_0$ is the shear modulus (the resistance of the spacetime continuum to *distortions*) and $\bar{\lambda}_0$ is expressed in terms of $\bar{\kappa}_0$, the bulk modulus:

$$\bar{\lambda}_0 = \bar{\kappa}_0 - \bar{\mu}_0/2 \tag{4}$$

in a four-dimensional continuum.

2 Energy in the spacetime continuum

In *STCED*, energy is stored in the spacetime continuum as strain energy [5]. As seen in [1, see Section 8.1], the strain energy density of the spacetime continuum is separated into two terms: the first one expresses the dilatation energy density (the mass longitudinal term) while the second one expresses the distortion energy density (the massless transverse term):

$$\mathcal{E} = \mathcal{E}_\parallel + \mathcal{E}_\perp \tag{5}$$

where

$$\mathcal{E}_\parallel = \frac{1}{2}\,\bar{\kappa}_0 \varepsilon^2 \equiv \frac{1}{32\bar{\kappa}_0}\,\rho^2 c^4\,, \tag{6}$$

ρ is the rest-mass density of the deformation, and

$$\mathcal{E}_{\perp} = \bar{\mu}_0 e^{\alpha\beta} e_{\alpha\beta} = \frac{1}{4\bar{\mu}_0} t^{\alpha\beta} t_{\alpha\beta} , \tag{7}$$

with the strain distortion

$$e^{\alpha\beta} = \varepsilon^{\alpha\beta} - e_s g^{\alpha\beta} \tag{8}$$

and the strain dilatation $e_s = \frac{1}{4}\varepsilon^{\alpha}{}_{\alpha}$. Similarly for the stress distortion $t^{\alpha\beta}$ and the stress dilatation t_s. Then the dilatation (massive) strain energy density of the deformation is given by the longitudinal strain energy density (6) and the distortion (massless) strain energy density of the deformation is given by the transverse strain energy density (7).

The strain energy W of the deformation is obtained by integrating (5) over the volume V of the deformation to give

$$W = W_{\parallel} + W_{\perp} \tag{9}$$

where $W_{\parallel}$ is the (massive) longitudinal strain energy of the deformation given by

$$W_{\parallel} = \int_V \mathcal{E}_{\parallel} \, \mathrm{d}V \tag{10}$$

and $W_{\perp}$ is the (massless) transverse distortion strain energy of the deformation given by

$$W_{\perp} = \int_V \mathcal{E}_{\perp} \, \mathrm{d}V \tag{11}$$

where the volume element $\mathrm{d}V$ in cylindrical polar coordinates is given by $r\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}z$ for a stationary deformation.

3 Quantum particles from STC defects

In [8, 10, 11], we show that quantum particles can be represented as defects in the spacetime continuum, specifically dislocations and disclinations. *Dislocations* are translational deformations, while *disclinations* are rotational deformations. In particular, we consider the simplest quantum particle defect given by the edge dislocation [10].

The strain energy density of a stationary edge dislocation is given by

$$W^E = W^E_{\parallel} + W^E_{\perp} . \tag{12}$$

The longitudinal strain energy of the edge dislocation $W^E_{\parallel}$ is given by [10, eq. (8)]

$$W^E_{\parallel} = \frac{\bar{\kappa}_0}{2\pi}\, \bar{\alpha}_0^2\, b^2\, \ell\, \log\frac{\Lambda}{b_c} \tag{13}$$

where

$$\bar{\alpha}_0 = \frac{\bar{\mu}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} , \tag{14}$$

ℓ is the length of the dislocation, b_c is the size of the core of the dislocation, of order b_0, the smallest spacetime Burgers dislocation vector [9] and Λ is a cut-off parameter corresponding to the radial extent of the dislocation, limited by the average distance to its nearest neighbours. In (13), the edge dislocation is along the z-axis with Burgers vector b along the x-axis.

The transverse strain energy $W^E_{\perp}$ is given by [10, eq. (10)]

$$W^E_{\perp} = \frac{\bar{\mu}_0}{4\pi}\left(\bar{\alpha}_0^2 + 2\bar{\beta}_0^2\right) b^2\, \ell\, \log\frac{\Lambda}{b_c} \tag{15}$$

where

$$\bar{\beta}_0 = \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \tag{16}$$

and the other parameters are as defined previously.

4 QED mass renormalization

The basic Feynman diagrams can be seen to represent screw dislocations as photons, edge dislocations as bosons, twist and wedge disclinations as fermions [10], and their interactions. The interaction of defects results from the overlap of the defects' strain energy densities. In QED, the exchange of virtual particles in interactions can be seen to be a perturbation expansion representation of the forces resulting from the overlap of the strain energy densities of the dislocations and disclinations.

Similarly, the phenomena of self-energy and vacuum polarization can be understood to result from the strain energy densities of individual defects. QED again represents this situation as a perturbation expansion of an interaction of a photon with the vacuum (photon self-energy also known as vacuum polarization) or of a particle such as an electron with its field (self-energy). In *STCED*, the perturbative expansions are replaced by finite analytical expressions for the strain energy density of individual screw dislocations as photons, edge dislocations as bosons, twist and wedge disclinations as fermions [10].

Quantum Mechanics and QED only deal with the transverse component of spacetime continuum deformations as they are only concerned with the *wave* aspect of wave-particle duality (see [14] for a discussion of this topic). The energy terms used in QED thus correspond to the transverse strain energy $W^E_{\perp}$. Hence there is no equivalent dilatation massive longitudinal strain energy term ($W^E_{\parallel}$) used in QED, and no possibility of properly deriving the mass from the theory, as QED uses an incomplete description of particle energies at the quantum level.

The mass term used in the QED equations is external to and not derived from quantum equations. It is thus found to not correspond to the actual mass of the particle and is characterized instead as the bare mass m_0 [15]. To this mass is added the interaction of the particle with the medium or the field, δm, the result of which m_{qm} is "renormalized" (the value of m_0 and the field corrections are infinite) and replaced with the actual experimental mass m according to

$$m_{qm} = m_0 + \delta m \rightarrow m . \tag{17}$$

Comparing this equation with (12), we find that

$$
\begin{aligned}
m &= W^E \\
m_0 &= W^E_\parallel = \frac{\bar{\kappa}_0}{2\pi}\,\bar{\alpha}_0^2\, b^2\,\ell\,\log\frac{\Lambda}{b_c} \\
\delta m &= W^E_\perp = \frac{\bar{\mu}_0}{4\pi}\left(\bar{\alpha}_0^2 + 2\bar{\beta}_0^2\right) b^2\,\ell\,\log\frac{\Lambda}{b_c}\,.
\end{aligned}
\tag{18}
$$

The interaction of the particle with the medium or the field, δm, is the transverse strain energy present in the spacetime continuum (or vacuum), essentially a field energy.

We note that the bare mass (*i.e.* the massive longitudinal strain energy) and the field correction (*i.e.* the transverse strain energy) are both finite in this approach and there is no need for the subtraction of infinities as both terms are well-behaved. If integrated over all of spacetime, they would be divergent, with the divergence being logarithmic in nature. However, contrary to QED, the strain energies are bounded by the density of defects present in the spacetime continuum, which results in an upperbound to the integral of half the average distance between defects. As mentioned by Hirth [16], this has little impact on the accuracy of the results due to the logarithmic dependence. Hence including the longitudinal dilatation mass density term as derived in *STCED* along with the transverse distortion energy density term in the strain energy density provides the expression for the mass m and eliminates the need for mass renormalization as the theory is developed with the correct mass term.

Eq. (18) applies to massive bosons as shown in [10]. For electrons, we have

$$
W^{\ell^3} = W^{\ell^3}_\parallel + W^{\ell^3}_\perp\,, \tag{19}
$$

where the defect in this case is the ℓ^3 twist disclination [10] and where (18) is replaced with the following:

$$
\begin{aligned}
m &= W^{\ell^3} \\
m_0 &= W^{\ell^3}_\parallel = \frac{\bar{\kappa}_0}{6\pi}\,\bar{\alpha}_0^2\left(\Omega_x^2 + \Omega_y^2\right)\ell^3\log\frac{\Lambda}{b_c} \\
\delta m &= W^{\ell^3}_\perp = \frac{\bar{\mu}_0}{2\pi}\,\frac{\ell^3}{3}\Big[\left(\Omega_x^2+\Omega_y^2\right)\left(\bar{\alpha}_0^2 + \tfrac{1}{2}\bar{\beta}_0^2\right) + \\
&\qquad + 2\,\Omega_x\Omega_y\left(\bar{\alpha}_0^2 - 2\bar{\beta}_0^2\right)\Big]\log\frac{\Lambda}{b_c}
\end{aligned}
\tag{20}
$$

where Ω^μ is the spacetime Frank vector. The same considerations as seen previously for bosons apply to (20) due to the logarithmic dependence of the expressions.

For quarks, we have

$$
W^W = W^W_\parallel + W^W_\perp \tag{21}
$$

where the defect in this case is the wedge disclination [10]. In most cases $\Lambda \gg b_c$, and we have

$$
\begin{aligned}
m &= W^W \\
m_0 &= W^W_\parallel \simeq \frac{\bar{\kappa}_0}{2\pi}\,\Omega_z^2\,\ell\,\Lambda^2\Big[\bar{\alpha}_0^2\,\log^2\Lambda + \\
&\qquad + \bar{\alpha}_0\bar{\gamma}_0\log\Lambda + \tfrac{1}{4}(\bar{\alpha}_0^2 + \bar{\gamma}_0^2)\Big] \\
\delta m &= W^W_\perp \simeq \frac{\bar{\mu}_0}{4\pi}\,\Omega_z^2\,\ell\,\Lambda^2\Big[\bar{\alpha}_0^2\,\log^2\Lambda - \\
&\qquad - \left(\bar{\alpha}_0^2 - 3\bar{\alpha}_0\bar{\beta}_0\right)\log\Lambda + \\
&\qquad + \tfrac{1}{2}\left(\bar{\alpha}_0^2 - 3\bar{\alpha}_0\bar{\beta}_0 + \tfrac{3}{2}\bar{\beta}_0^2\right)\Big]
\end{aligned}
\tag{22}
$$

where

$$
\bar{\gamma}_0 = \frac{\bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0}\,. \tag{23}
$$

In this case, both the longitudinal strain energy $W^W_\parallel$ and the transverse strain energy $W^W_\perp$ are proportional to Λ^2 in the limit $\Lambda \gg b_c$. The parameter Λ is equivalent to the extent of the wedge disclination, and we find that as it becomes more extended, its strain energy is increasing parabolically. This behaviour is similar to that of quarks (confinement). In addition, as shown in [10, see eqs. (16) and (20)], as $\Lambda \to b_c$, the strain energy decreases and tends to 0, again in agreement with the behaviour of quarks (asymptotic freedom).

5 Dislocation self-energy and QED self-energies

The dislocation self-energy is related to the dislocation self-force. The dislocation self-force arises from the force on an element in a dislocation caused by other segments of the *same* dislocation line. This process provides an explanation for the QED self-energies without the need to resort to the emission/absorption of virtual particles. It can be understood, and is particular to, dislocation dynamics as dislocations are defects that extend in the spacetime continuum [16, see p. 131]. Self-energy of a straight-dislocation segment of length L is given by [16, see p. 161]:

$$
\begin{aligned}
W_{self} = \frac{\bar{\mu}_0}{4\pi}&\left((\mathbf{b}\cdot\boldsymbol{\xi})^2 + \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0}\,|(\mathbf{b}\times\boldsymbol{\xi})|^2\right)\times \\
&\times L\left(\ln\frac{L}{b} - 1\right)
\end{aligned}
\tag{24}
$$

where there is no interaction between two elements of the segment when they are within $\pm b$, or equivalently

$$
W_{self} = \frac{\bar{\mu}_0}{4\pi}\left((\mathbf{b}\cdot\boldsymbol{\xi})^2 + \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0}\,|(\mathbf{b}\times\boldsymbol{\xi})|^2\right) L\,\ln\frac{L}{eb} \tag{25}
$$

where $e = 2.71828\ldots$. These equations provide analytic expressions for the non-perturbative calculation of quantum self energies and interaction energies, and eliminate the need for the virtual particle perturbative approach.

In particular, the pure screw (photon) self-energy

$$W^S_{self} = \frac{\bar{\mu}_0}{4\pi} (\mathbf{b} \cdot \boldsymbol{\xi})^2 L \left(\ln \frac{L}{b} - 1 \right) \tag{26}$$

and the pure edge (boson) self-energy

$$W^E_{self} = \frac{\bar{\mu}_0}{4\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} |(\mathbf{b} \times \boldsymbol{\xi})|^2 L \left(\ln \frac{L}{b} - 1 \right) \tag{27}$$

are obtained from (25), while (25) is also the appropriate equation to use for the dual wave-particle "system".

We can relate (27) to (12) and (18) by evaluating W^E from (12) using (13) and (15):

$$W^E = \frac{b^2}{4\pi} \left[2\bar{\kappa}_0 \bar{\alpha}_0^2 + \bar{\mu}_0 \left(\bar{\alpha}_0^2 + 2\bar{\beta}_0^2 \right) \right] \ell \log \frac{\Lambda}{b_c} . \tag{28}$$

Substituting for $\bar{\kappa}_0$ from (4), for $\bar{\alpha}_0$ from (14) and for $\bar{\beta}_0$ from (16), the factor in square brackets in the above equation becomes

$$[] = \frac{\bar{\mu}_0}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \left(4\bar{\mu}_0^2 + 6\bar{\mu}_0\bar{\lambda}_0 + 2\bar{\lambda}_0^2 \right) \tag{29}$$

which can be factored as

$$[] = \frac{2\bar{\mu}_0}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} (2\bar{\mu}_0 + \bar{\lambda}_0)(\bar{\mu}_0 + \bar{\lambda}_0) . \tag{30}$$

Substituting back into (28), we obtain

$$W^E_{self} = \frac{1}{2} W^E = \frac{\bar{\mu}_0}{4\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} b^2 \ell \log \frac{\Lambda}{b_c} . \tag{31}$$

As noted in [17, see p. 178], the self-energy and the interaction energies are described by the same equations in the non-singular theory, except that the self-energy is half of the interaction energy. We thus see that the above result (28) is essentially the same as (27) from Hirth [16, see p. 161] except that the log factors are slightly different, but similar in intent ($\log \Lambda/b_c$ compared to $\log \ell/eb$).

Dislocation self energies are thus found to be similar in structure to Quantum Electrodynamics self energies. They are also divergent if integrated over all of spacetime, with the divergence being logarithmic in nature. However, contrary to QED, dislocation self energies are bounded by the density of dislocations present in the spacetime continuum, which results in an upperbound to the integral of half the average distance between dislocations.

For a dislocation loop, as each element $d\mathbf{l}$ of the dislocation loop is acted upon by the forces caused by the stress of the other elements of the dislocation loop, the work done against these corresponds to the self-energy of the dislocation loop. The self-energy of a dislocation loop can be calculated from Eq. (4-44) of [16, see p. 110] to give

$$\begin{aligned} W_{self} = \frac{\bar{\mu}_0}{8\pi} \oint_{C_1=C} \oint_{C_2=C} \frac{(\mathbf{b} \cdot d\mathbf{l_1})(\mathbf{b} \cdot d\mathbf{l_2})}{R} + \\ + \frac{\bar{\mu}_0}{4\pi} \frac{\bar{\mu}_0 + \bar{\lambda}_0}{2\bar{\mu}_0 + \bar{\lambda}_0} \oint_{C_1=C} \oint_{C_2=C} \frac{(\mathbf{b} \times d\mathbf{l_1}) \cdot \mathbf{T} \cdot (\mathbf{b} \times d\mathbf{l_2})}{R} \end{aligned} \tag{32}$$

where $\mathbf{T}$ is as defined in Eq. (4-44) of [16, see p. 110].

The photon self-energy also known as vacuum polarization is obtained from the strain energy density of screw dislocations. The longitudinal strain energy of the screw dislocation $W^S_\parallel = 0$ as given by [10, eq. (6)] *i.e.* the photon is massless. The photon self-energy is given by half the transverse strain energy of the screw dislocation $W^S_\perp$ given by [10, eq. (7)]

$$W^S_{self} = \frac{1}{2} W^S_\perp = \frac{\bar{\mu}_0}{8\pi} b^2 \ell \log \frac{\Lambda}{b_c} \tag{33}$$

which again includes the $\log \Lambda/b_c$ factor. Comparing this expression with (26) and with (32), we find that (26) is likely off by a factor of 2, being proportional to $1/8\pi$ as per Hirth's (32) and (33), not $1/4\pi$ as given in Hirth's (24) and Hirth's (26).

6 Disclination self-energy and QED self-energies

From dislocation self-energies, we can calculate the photon self-energy (also known as the vacuum polarization) and, in the general case, the boson self-energy.

The fermion self-energies are calculated from the corresponding disclination self-energies, with the lepton self-energy calculated from the interaction energy W^{ℓ^3} of the ℓ^3 twist disclination, the neutrino self-energy calculated from the interaction energy W^ℓ of the ℓ twist disclination and the quark self-energy calculated from the interaction energy W^W of the wedge disclination, using the result that self-energy is half of the interaction energy as seen previously in Section 5.

6.1 The ℓ^3 twist disclination self-energy and lepton self-energies

The lepton (electron) self-energy is calculated from the interaction energy W^{ℓ^3} of the ℓ^3 twist disclination by evaluating W^{ℓ^3} from (19) using $W^{\ell^3}_\parallel$ and $W^{\ell^3}_\perp$ from (20):

$$\begin{aligned} W^{\ell^3} &= \frac{\bar{\kappa}_0}{6\pi} \bar{\alpha}_0^2 \left(\Omega_x^2 + \Omega_y^2 \right) \ell^3 \log \frac{\Lambda}{b_c} + \\ &+ \frac{\bar{\mu}_0}{2\pi} \frac{\ell^3}{3} \Big[\left(\Omega_x^2 + \Omega_y^2 \right) \left(\bar{\alpha}_0^2 + \tfrac{1}{2} \bar{\beta}_0^2 \right) + \\ &+ 2\, \Omega_x \Omega_y \left(\bar{\alpha}_0^2 - 2\bar{\beta}_0^2 \right) \Big] \log \frac{\Lambda}{b_c} . \end{aligned} \tag{34}$$

Substituting for $\bar{\kappa}_0$ from (4), for $\bar{\alpha}_0$ from (14) and for $\bar{\beta}_0$ from (16), (34) becomes

$$\begin{aligned} W^{\ell^3} &= \frac{\ell^3}{6\pi} \frac{\bar{\mu}_0}{(2\bar{\mu}_0 + \bar{\lambda}_0)^2} \times \\ &\times \Big[\left(\Omega_x^2 + \Omega_y^2 \right) \left(2\bar{\mu}_0^2 + 2\bar{\mu}_0\bar{\lambda}_0 + \tfrac{1}{2}\bar{\lambda}_0^2 \right) - \\ &- 2\, \Omega_x \Omega_y \left(\bar{\mu}_0^2 + 4\bar{\mu}_0\bar{\lambda}_0 + 2\bar{\lambda}_0^2 \right) \Big] \log \frac{\Lambda}{b_c} \end{aligned} \tag{35}$$

which can be factored as

$$W^{\ell^3} = \frac{\ell^3}{12\pi}\frac{\bar{\mu}_0}{(2\bar{\mu}_0+\bar{\lambda}_0)^2}\left\{\left(\Omega_x^2+\Omega_y^2\right)\left(2\bar{\mu}_0+\bar{\lambda}_0\right)^2 - \right. \\ \left. - 4\Omega_x\Omega_y\left[\left(\bar{\mu}_0+\bar{\lambda}_0\right)\left(\bar{\mu}_0+2\bar{\lambda}_0\right)+\bar{\mu}_0\bar{\lambda}_0\right]\right\}\log\frac{\Lambda}{b_c}. \tag{36}$$

The lepton self-energy is then given by

$$W^{\ell^3}_{self} = \frac{1}{2}W^{\ell^3} = \frac{\bar{\mu}_0}{24\pi}\left\{\left(\Omega_x^2+\Omega_y^2\right) - \right. \\ \left. -4\,\Omega_x\Omega_y\frac{\left(\bar{\mu}_0+\bar{\lambda}_0\right)\left(\bar{\mu}_0+2\bar{\lambda}_0\right)+\bar{\mu}_0\bar{\lambda}_0}{(2\bar{\mu}_0+\bar{\lambda}_0)^2}\right\}\ell^3\,\log\frac{\Lambda}{b_c}, \tag{37}$$

where we have used the result that self-energy is half of the interaction energy as seen previously in Section 5.

6.2 The ℓ twist disclination self-energy and the neutrino self-energy

The neutrino self-energy is calculated from the strain energy W^ℓ of the ℓ twist disclination. The longitudinal strain energy of the ℓ twist disclination $W^\ell_\parallel = 0$ as given by [10, eq. 33)] *i.e.* the neutrino is massless. In most cases $\Lambda \gg b_c$, and the strain energy W^ℓ of the ℓ twist disclination is given by the transverse strain energy $W^\ell = W^\ell_\perp$ given by [10, eq. (35)]:

$$W^\ell = \frac{\bar{\mu}_0}{2\pi}\,\ell\,\Lambda^2\left[\left(\Omega_x^2+\Omega_y^2\right)\left(\bar{\alpha}_0^2\,\log^2\Lambda+\bar{\alpha}_0\bar{\gamma}_0\,\log\Lambda - \right.\right. \\ \left.\left. -\tfrac{1}{2}\,\bar{\alpha}_0\bar{\gamma}_0\right) - 2\,\Omega_x\Omega_y\left(\bar{\alpha}_0\bar{\beta}_0\log\Lambda+\tfrac{1}{2}\bar{\beta}_0\bar{\gamma}_0\right)\right]. \tag{38}$$

Substituting for $\bar{\alpha}_0$ from (14), for $\bar{\beta}_0$ from (16) and for $\bar{\gamma}_0$ from (23), (38) becomes

$$W^\ell = \frac{\bar{\mu}_0}{2\pi}\frac{\ell\,\Lambda^2}{(2\bar{\mu}_0+\bar{\lambda}_0)^2}\left\{\left(\Omega_x^2+\Omega_y^2\right)\left[\bar{\mu}_0^2\,\log^2\Lambda + \right.\right. \\ \left. +\bar{\mu}_0\bar{\lambda}_0\left(\log\Lambda-\tfrac{1}{2}\right)\right] - \\ \left. -2\,\Omega_x\Omega_y\left[\bar{\mu}_0\left(\bar{\mu}_0+\bar{\lambda}_0\right)\log\Lambda+\tfrac{1}{2}\,\bar{\lambda}_0\left(\bar{\mu}_0+\bar{\lambda}_0\right)\right]\right\}. \tag{39}$$

The neutrino self-energy is then given by

$$W^\ell_{self} = \frac{1}{2}W^\ell = \frac{\bar{\mu}_0}{4\pi}\frac{\ell\,\Lambda^2}{(2\bar{\mu}_0+\bar{\lambda}_0)^2}\times \\ \times\left\{\left(\Omega_x^2+\Omega_y^2\right)\left[\bar{\mu}_0^2\,\log^2\Lambda+\bar{\mu}_0\bar{\lambda}_0\left(\log\Lambda-\tfrac{1}{2}\right)\right] - \right. \\ \left. -2\,\Omega_x\Omega_y\left(\bar{\mu}_0+\bar{\lambda}_0\right)\left(\bar{\mu}_0\log\Lambda+\tfrac{1}{2}\,\bar{\lambda}_0\right)\right\} \tag{40}$$

where we have used the result that self-energy is half of the interaction energy as seen previously in Section 5.

6.3 The wedge disclination self-energy and quark self-energies

The quark self-energy is calculated from the interaction energy W^W of the wedge disclination by evaluating W^W from (21) using $W^W_\parallel$ and $W^W_\perp$ from (22). In most cases $\Lambda \gg b_c$, and we have

$$W^W \simeq \frac{\bar{\kappa}_0}{2\pi}\,\Omega_z^2\,\ell\,\Lambda^2\left[\bar{\alpha}_0^2\,\log^2\Lambda + \right. \\ \left. +\bar{\alpha}_0\bar{\gamma}_0\log\Lambda+\tfrac{1}{4}(\bar{\alpha}_0^2+\bar{\gamma}_0^2)\right] + \\ +\frac{\bar{\mu}_0}{4\pi}\,\Omega_z^2\,\ell\,\Lambda^2\left[\bar{\alpha}_0^2\,\log^2\Lambda - \right. \\ -\left(\bar{\alpha}_0^2-3\bar{\alpha}_0\bar{\beta}_0\right)\log\Lambda + \\ \left. +\tfrac{1}{2}\left(\bar{\alpha}_0^2-3\bar{\alpha}_0\bar{\beta}_0+\tfrac{3}{2}\,\bar{\beta}_0^2\right)\right]. \tag{41}$$

Substituting for $\bar{\kappa}_0$ from (4), for $\bar{\alpha}_0$ from (14) for $\bar{\beta}_0$ from (16) and for $\bar{\gamma}_0$ from (23), (41) becomes

$$W^W \simeq \frac{\Omega_z^2}{2\pi}\frac{\ell\,\Lambda^2}{(2\bar{\mu}_0+\bar{\lambda}_0)^2}\left[\bar{\mu}_0^2\left(\bar{\mu}_0+\bar{\lambda}_0\right)\log^2\Lambda + \right. \\ +\bar{\mu}_0\left(\bar{\mu}_0^2+2\bar{\mu}_0\bar{\lambda}_0+\bar{\lambda}_0^2\right)\log\Lambda + \\ \left. +\tfrac{1}{4}\,\bar{\lambda}_0\left(\bar{\mu}_0^2+2\bar{\mu}_0\bar{\lambda}_0+\bar{\lambda}_0^2\right)\right] \tag{42}$$

which can be factored as

$$W^W \simeq \frac{\Omega_z^2}{2\pi}\frac{\ell\,\Lambda^2}{(2\bar{\mu}_0+\bar{\lambda}_0)^2}\left[\bar{\mu}_0^2\left(\bar{\mu}_0+\bar{\lambda}_0\right)\log^2\Lambda + \right. \\ \left. +\left(\bar{\mu}_0+\bar{\lambda}_0\right)^2\left(\bar{\mu}_0\log\Lambda+\tfrac{1}{4}\,\bar{\lambda}_0\right)\right]. \tag{43}$$

The quark self-energy is then given by

$$W^W_{self} = \frac{1}{2}W^W \simeq \frac{\Omega_z^2}{4\pi}\frac{(\bar{\mu}_0+\bar{\lambda}_0)^2}{(2\bar{\mu}_0+\bar{\lambda}_0)^2}\,\ell\,\Lambda^2\times \\ \times\left[\frac{\bar{\mu}_0^2}{\bar{\mu}_0+\bar{\lambda}_0}\log^2\Lambda+\bar{\mu}_0\log\Lambda+\tfrac{1}{4}\,\bar{\lambda}_0\right] \tag{44}$$

where we have used the result that self-energy is half of the interaction energy as seen previously in Section 5.

7 Discussion and conclusion

In this paper, we have considered how the Elastodynamics of the Spacetime Continuum (*STCED*) explains the Quantum Electrodynamics (QED) phenomena of self-energy, vacuum polarization and mass renormalization. We have noted that QED only deals with the *wave* aspect of wave-particle objects, and hence QED only deals with the distortion transverse strain energy $W^E_\perp$, while the dilatation massive longitudinal strain energy term $W^E_\parallel$ is not considered. Hence there is no possibility of properly deriving the mass, as QED uses an incomplete description of particle energies at the quantum level.

Comparison of mass renormalization with *STCED* strain energy shows that the interaction of the particle with the medium or the field, δm, is the transverse strain energy present in the spacetime continuum (or vacuum), essentially a field energy. We provide the strain energy equivalence for QED mass renormalization for bosons, leptons and quarks.

Both the bare mass (*i.e.* the massive longitudinal strain energy) and the field correction (*i.e.* the transverse strain energy) are finite in this approach and there is no need for the subtraction of infinities as both terms are well-behaved. Contrary to QED, the strain energies are bounded by the density of defects present in the spacetime continuum, which results in an upperbound to the integral of half the average distance between defects. Hence including the longitudinal dilatation mass density term as derived in *STCED* along with the transverse distortion energy density term in the strain energy density provides the expression for the mass m and eliminates the need for mass renormalization as the theory is developed with the correct mass term. We have also derived the self-energy expressions for bosons including photons, leptons including neutrinos, and quarks.

It is important to note that

1. The expressions derived are for stationary (time independent) defects.
2. The case of time-dependent screw and edge dislocations moving with velocity v is covered in §16.1.2 and §16.2.2 of [11] respectively. The calculations involve integrals of the form

$$\int_y \frac{1}{\alpha y} \arctan\left(\frac{x-vt}{\alpha y}\right) \mathrm{d}y = -\frac{i}{2}\left[\mathrm{Li}_2\left(-i\,\frac{x-vt}{\alpha y}\right) - \mathrm{Li}_2\left(i\,\frac{x-vt}{\alpha y}\right)\right] \tag{45}$$

   where

$$\alpha = \sqrt{1-\frac{v^2}{c^2}} \tag{46}$$

   and where $\mathrm{Li}_n(x)$ is the polylogarithm function which arises in Feynman diagram integrals. For $n = 2$ and $n = 3$, we have the dilogarithm and the trilogarithm special cases respectively. This is a further indication that the interaction of strain energies are the physical source of quantum interaction phenomena described by Feynman diagrams as discussed in section 4.

The results obtained are found to provide a physical explanation of QED phenomena in terms of the interaction resulting from the overlap of defect strain energies in the spacetime continuum in *STCED*.

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