GR = QM: Revealing the Common Origin for Gravitation and Quantum Mechanics via a Feedback Signal Approach to Fundamental Particle Behavior

Franklin Potter

8642 Marvale Drive, Huntington Beach, CA 92646 USA. E-mail: frank11hb@yahoo.com

By allowing the fundamental particles of the Standard Model to communicate via "feedback signals" within a vacuum lattice of mathematical nodes at the Planck scale, one learns that this approach toward understanding fundamental physics reveals the surprising common origin of quantum mechanics and of general relativity. This "feedback signal" approach is shown to be equivalent to the path integral approach but also the underlying reason for its success.

1 Introduction

The GR = QM in the title refers to a recent suggestion [1] that perhaps the long-standing theoretical conflict between general relativity and quantum mechanics is not insurmountable. In fact, the conjecture has been that they may actually be closely related, or at least they could have the same fundamental origin.

Herein I establish the common fundamental origin for gravitation and quantum mechanics. A non-traditional approach to fundamental particle behavior is required, one that agrees with the successful effective Standard Model (SM) of leptons and quarks [2] but treats these particles as harmonic oscillators emitting and receiving scalar waves at their Compton frequencies [3]. A fundamental particle, such as an electron, communicates with the surrounding discrete vacuum lattice of mathematical nodes via these scalar "feedback signals". Therefore, a particle itself actively determines its subsequent behavior even in the absence of the SM local gauge fields.

The surprising result is that the common origin of quantum mechanics and of general relativity arises directly by simply analyzing particle behavior in sufficient geometrical detail.

2 A brief particle physics review

In this section I offer a brief review of some of the physics consequences if one considers both the internal symmetry space for defining the particle states of the SM and our (3+1)-D spacetime to be discrete spaces. Such possibilities may be necessary in order to justify (1) treating the internal symmetry space and spacetime as C² unitary space lattices of mathematical nodes and (2) proposing the leptons, hadrons, and electroweak (EW) bosons to be 3-D particles behaving as harmonic oscillators. If one chooses to accept these concepts outright, one can skip forward to Section 3 for the details of the feedback signal approach.

Recall that the SM describes the known local gauge interactions, color and electroweak, via its $SU(3)_C \times SU(2)_L \times U(1)_Y$ lagrangian, so I will ignore these gauge interactions in the discussion ahead. The leptons, the hadrons formed from quarks and gluons, and the EW interaction bosons W^{\pm} , Z^0 , and γ , are the fundamental particles defined [2] in the internal symmetry space whose behavior in spacetime will be explained in terms of the feedback signal approach. That is, I am treating these three categories of fundamental particles as 3-D objects and not as point particles. The justification is provided below.

The proposed feedback signal approach can only be selfconsistent if each fundamental fermion, i.e., lepton or quark, "gathers in" the immediate surrounding lattice nodes in its own unique way. That is, I assume that (3+1)-D spacetime is a discrete lattice of mathematical nodes, and a particle's collection of lattice nodes, perhaps at the Planck scale, must have a different discrete rotational symmetry for each different fundamental fermion family. These assumptions are in contrast to the same SU(2) point particle continuous symmetries for each family in the traditional interpretation of the SM.

Specifically, one finds that only discrete symmetry binary subgroups of the unit quaternion group Q, which is equivalent to SU(2), suffice, with each binary subgroup of Q having two EW isospin $\pm \frac{1}{2}$ states in each fermion family. Therefore, being binary subgroups of Q, and of SU(2) x U(1), all the mathematical machinery of the SM remains valid. Moreover, the important left-handed fermion state preference for the weak interaction is dictated by the mathematical properties of the quaternion multiplications for the weak interaction.

I have identified 3 discrete symmetry binary subgroups of Q that define the 3 physical lepton families [4–6]. They are these specific 3 binary subgroups acting in the R^3 subspace of C^2 : the [332] binary subgroup for the electron family; the [432] binary subgroup for the muon family; and the [532] binary subgroup for the tau family. They are known also as the binary tetrahedral group 2T, the binary octahedral group 2O, and the binary icosahedral group 2I, respectively, and correspond to special discrete binary rotations of 3-D objects called regular polyhedrons in the 3-D real space R^3 . No more lepton families are predicted because there are no more binary subgroups of Q that require a 3-D space.

The fact that Nature agrees with the 3 lepton families representing these 3 binary subgroups of Q is verified by the

first principles derivation [6,7] of the neutrino PMNS mixing angles from their three quaternion generators by collectively mimicking the SU(2) generators, i.e., the three Pauli generators. The empirical values of the lepton mixing angles now agree within 1σ to each of these theoretical absolute values: $\theta_{12} = 34.281^\circ$, $\theta_{23} = 42.859^\circ$, $\theta_{13} = 8.578^\circ$. Conceptually, this EW flavor state mixing to produce the mass states occurs because a valid renormalizable conformal field theory requires a continuous symmetry such as in the lagrangian of the SM. This lepton family mixing therefore guarantees that the 3 discrete symmetry binary subgroups defining the lepton families collectively behave as the SU(2) of the SM.

I have identified also 4 related discrete symmetry binary subgroups [4, 5, 8] that define four 4-D quark families in \mathbb{R}^4 : [333], [433], [343], and [533], corresponding to the only regular polytopes in \mathbb{R}^4 . The mathematical and physical consequences of these discrete symmetry groups for 4 quark families are discussed in Appendix A. The 4-D quarks and 4-D gluons combine according to QCD to form the 3-D hadrons, the baryons and mesons, or one can use intersection theory to establish the same results.

Note that the 3-D lepton states in \mathbb{R}^3 and the 4-D quark states in \mathbb{R}^4 both fit into the proposed 2-D unitary space \mathbb{C}^2 . Our (3+1)-D spacetime for discussing the particle bahavior also fits into \mathbb{C}^2 . I am assuming that the two spaces, the internal symmetry space for particle definition and spacetime for the physics behavior join together seamlessly. Therefore, this $\mathbb{C}^2 = \mathbb{R}^4$ space is proposed to be the one I need to consider to be discrete and composed of mathematical nodes. The nodes are equally spaced on average at the Planck scale when no fundamental particles are in existence.

Each fermion family with its own unique discrete symmetry binary subgroup has two Q or SU(2) orthogonal $\pm \frac{1}{2}$ states, but they will be mass-energy degenerate unless they form the two new physical orthogonal states of different energies as dictated by QM. Therefore, each lepton and each quark family has two weak isospin flavor states that have different mass values with a characteristic oscillation occurring between the two original mathematical states at the Compton frequency and Compton wavelength

$$\omega_C = \frac{mc^2}{\hbar}, \quad \lambda_C = \frac{h}{mc}.$$
 (1)

For the electron, its Compton values are $\omega_C \approx 7.8 \times 10^{20}$ Hz and $\lambda_C \approx 2.4 \times 10^{-12}$ meters. Therefore, the Compton wavelength of each fundamental particle will be many orders of magnitude larger than the Planck distance of about 10^{-35} meters. Consequently, the proposed vacuum lattice structure of nodes appears to be a continuous space for the fundamental particles.

Although the effective SM lagrangian has the continuous symmetry local gauge group $SU(3)_C \ge SU(2)_L \ge U(1)_Y$, add-ons called horizontal discrete symmetry groups are now be-

coming acceptable alternatives for defining the lepton family states, particularly with the advent of neutrino mixing and non-zero neutrino mass states [2]. However, the discrete symmetry binary subgroups of the unit quaternion group Q that I have proposed for the leptons and quarks retain the successful predictions of the SM without the need to introduce any additional horizontal discrete symmetries to its lagrangian.

That is, all the successes of the SM have been retained by my specific discrete symmetry approach for the fermions while the geometrical sources of some of its physical properties have been elucidated. I cannot overemphasize this retention of the SM mathematical and physical properties, with perhaps the SM being a useful approximation even down to the Planck scale.

The above brief review of my discrete symmetry approach to the SM has been included in order to introduce some of the mathematical connections that propose some unconfirmed physics possibilities and also to justify using a discrete spacetime of mathematical nodes as both the origin of the fundamental fermions of the SM and as an active participant in their physical behavior. I will show how this approach leads directly to the special theory of relativity (STR), path integrals, quantum mechanics (QM), and the general theory of relativity (GTR), as explained in the discussion ahead.

3 The feedback signal approach

Spacetime itself at the Planck scale of about 10^{-35} meters could be a discrete space described by a uniform lattice of mathematical nodes. Therefore, I assume that our physical (3+1)-D spacetime agrees with a uniform lattice in the unitary space C² (or equivalently R⁴) at or near the Planck scale and that each fundamental lepton family forms its particle states by "gathering in" lattice nodes to form its own unique discrete symmetry 3-D objects. This "gathering in" process distorts the lattice locally with the amount of lattice distortion extending outward in a decreasing manner with increasing distance, i.e., as inverse distance.

If I assume that the undistorted, uniformly spaced lattice has no net energy density, then the positive mass-energy of a fundamental particle is related to the amount of lattice distortion in some yet-to-be-determined way. I expect this massenergy to be balanced by an equal negative energy value that retains the overall net zero energy total even for the distorted lattice. Perhaps the increased "stretch distance" between the nodes outside the particle definition volume provides negative energy that is the balancing factor for an assumed zero total energy for the Universe.

Recall that Clifford algebra and Bott periodicity [9] dictate a conjugate $R^4 = C^2$ space. In this conjugate space for anti-particles, the same mathematical properties of the uniformly spaced lattice would apply, again producing a positive mass-energy for the anti-particle states.

Each fundamental particle oscillating at its characteris-

tic frequency, its Compton frequency ω_C , is proposed to be emitting scalar waves, call them "feedback signals", into the surrounding vacuum lattice to eventually reach everywhere. The particle source could be undergoing "breathing mode" oscillations and emitting spherical waves isotropically into its environment. One must not identify these oscillations with electromagnetic waves because they are just propagating lattice distortions that allow lattice nodes to communicate with their nearest neighbors.

According to the special theory of relativity (STR), there exists a limiting speed for mass-energy transfer. I will take this maximum speed to be c, the speed of light in a vacuum, although there could be a higher speed limit if some day a photon is determined to possess a very tiny mass value.

Let a particle oscillate at its Compton frequency

$$\omega_C = \frac{mc^2}{\hbar},\tag{2}$$

with *m* the particle's mass value, *c* the speed of light in a vacuum, and \hbar being Planck's constant divided by 2π .

The feedback signals obey the standard scalar wave equation, a hyperbolic partial differential equation in three spatial variables x, y, z, and one time variable t. Its scalar function u(x,y,z,t) obeys

$$\nabla^2 u - \frac{\partial^2 u}{c^2 \partial t^2} = 0. \tag{3}$$

Solutions of this equation for spherical symmetry have no angular dependence, so the feedback signal amplitude u(r,t) depends only upon the radial distance according to

$$\left(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{\partial^2}{c^2\partial t^2}\right)u(r,t) = 0.$$
 (4)

The solutions for a single frequency ω have the form

$$u(r,t) = \frac{A}{r} e^{i(\omega t \pm kr)}$$
(5)

where the wavenumber $k = \omega/c$ and the peak intensity $I(r) = |A|^2/r^2$, i.e., the inverse square dependence.

This feedback signal approach requires the fundamental particle to behave as a microscopic 'antenna' moving within and communicating with the lattice and with other particles via its feedback signals. For example, the electron oscillating at $\omega_C = 7.77 \times 10^{20}$ Hz disturbs the surrounding lattice at the same frequency ω_C , and this oscillatory disturbance propagates radially outward in all directions at speed *c*. By treating the particle as an antenna, the particle not only emits its feedback signals but also can absorb its own feedback signals returning from scatterings in the lattice environment.

I can describe the electron's oscillation in more detail. Although I have its oscillations only at the Compton frequency ω_C , such ideal behavior cannot be maintained once signals return from the environment, even when the electron is at rest. There will exist a small spread in frequency values about its Compton frequency according to Fourier analysis. Therefore, a Q value can be assigned to represent the small spread in frequency values, just as for any other harmonic oscillator. The signal emissions have a small spread in frequencies also, but for simplicity I will ignore this property unless needed for clarification purposes. Therefore, I will continue to use a single characteristic Compton frequency ω_C even though we understand that the oscillator does not have an infinite Q value.

The lattice nodes act as a *transponder* to the feedback signals, absorbing and immediately emitting them equally in all directions for all frequencies, all amplitudes, and with no phase shift. That is, each small volume element in the lattice must absorb some of the incident feedback signals and then emit immediately the feedback signals at the original frequency into all directions isotropically. One can think of a single lattice node or of a specific collection of lattice nodes acting together as a transponder, but considering the same type of transponder everywhere for simplicity.

If one wishes to introduce a non-zero phase shift at each transponder, then a simple modification could be to have the phase shift value be the same for all the transponders and be independent of the feedback signal frequency. Either constraint can be eliminated for a more complicated vacuum lattice. I have chosen the simplest assumption of no phase shift and equal response for all frequencies and amplitudes.

I had initially allowed the feedback signals to have an arbitrary velocity v_0 . However, I learned that if one lets the speed of the feedback signals $v_0 = c$, the speed of light in a vacuum, then this simple feedback signal approach permits the direct derivation of the phenomena and equations of special relativity, general relativity, and quantum mechanics, with all of them agreeing with the present theories. The biggest surprise occurred when I learned that general relativity and quantum mechanics would then have the same fundamental origin.

In the sections ahead I will use many parts of my original 1982 attempt toward establishing this feedback signal approach as a viable approach but with some added updates here and there to provide a 21st century perspective. The identification of the gravitational interaction is one recent addition.

4 Single particle behavior at uniform velocity

Let a lone fundamental particle, such as a single electron in the Universe, be a 3-D physical harmonic oscillator oscillating at its Compton frequency ω_C with its antenna-like behavior emitting its feedback signal oscillations into the surrounding discrete lattice of uniformly spaced mathematical nodes, perhaps separated by the Planck distance of about 10^{-35} meters. As far as the electron is concerned, with its Compton wavelength of about 2.4 x 10^{-12} meters, the lattice appears to be continuous. Likewise for all other particles composed of leptons and of quarks, i.e., the hadrons, as well as the interaction bosons of the SM.



Fig. 1: Feedback signals are emitted by an electron at its previous successive equal-phase positions A_i . P_1 and P_2 are two of numerous transponders in the surrounding 3-D space on equal-phase ellipsoids for the signals from electron position A_1 only. This uniform velocity electron has moved forward at 0.866*c* to B where the returning feedback signals define its present location.

Either way, having a discrete space or a continuous space, the oscillations of the particle will appear as feedback signals traveling in the surrounding space \mathbb{R}^3 (the subspace of \mathbb{R}^4 and \mathbb{C}^2) and progress through the space at the constant velocity *c* with decreasing amplitude as the radial distance from the particle increases. Why require a decreasing amplitude? Because we must consider the concept of energy conservation associated with these outgoing and incoming feedback signals.

For simplicity only, I ignore at first the "permanent" space distortion of the lattice caused by the formation and presence of each fundamental particle. Therefore, the feedback signals propagate through a lattice in which the average lattice node spacings remain the same separation distance everywhere. Later on I will remove this restriction in order to discuss gravitational effects between two fundamental particles.

Both a coordinate space and a momentum space description of this feedback signal approach is considered. Single particle behavior in coordinate space is shown in Fig. 1. If the electron had been at rest, then all the positions A_i and position B would coincide and the ellipses would be circles centered at B to exhibit the spherical symmetry. However, this electron has been moving at a uniform velocity in the +x direction and is now at location B receiving feedback signals from the transponders P_i everywhere in space. The surrounding ellipsoids are equal-phase locations for the outgoing feedback signals emitted by the electron at previous equallyspaced successive electron positions A_i for i = 1,2,3,4,5.

In this lab frame as the electron moves by, the diagram shows three feedback signal rays, from A_1 to P_1 to B, from A_1 to P_2 to B, and from A_1 to P_3 to B, of equal total length that have feedback signals arriving at B exactly in-phase with the particle oscillation when the particle arrives at B. These rays are a few examples of the feedback signals that have been emitted isotropically into 4π solid angle by the particle when at A₁.

Only a specific subset of all the equal-phase ellipsoids are shown in Fig. 1. Note also that each larger ellipsoid represents a lesser signal amplitude at the transponders along the ellipsoid, being a further distance away from the source, and that all feedback signals returning from the same 3-D ellipsoid have identical amplitudes and phases because their total path distances are equal. Because the transponders in space are everywhere, all emitted signals will eventually reach one of them. I will later explain how all the multiple scattering paths from the A_i to B are related to the path integral concept considered by R.P. Feynman in his approach to quantum mechanics and classical mechanics [10].

If the particle has just come into existence, then the signals will have not reached very far into the surrounding space. In almost all practical cases the particle has existed for a time long enough so that the signals will have permeated to tremendous distances and an approximate steady-state condition will have been established, with the outgoing and incoming signal amplitude totals approximately matching at the particle's new location B.

Recall that I have chosen no phase delay for the transponders. Incoming feedback signals to the transponder from any direction are immediately emitted into all directions. Their spherically symmetrical emission pattern, shown at each P_i , assumes that all space locations, and therefore all transponders, are identical, behave identically, and will "scatter" feedback signals. This ideal transponder behavior is the simplest possible for determining the subsequent behavior of the particle.

5 Frequency shifted feedback signals

The feedback signals sent forward and backward along the electron's velocity (momentum) vector in the x-direction experience frequency shifts. Signals sent in the forward direction with frequency ω_C return from those transponders at a higher frequency $\omega_C + \Delta \omega$ because the moving particle encounters the equally-spaced equal-phase maximum signal amplitudes at shorter time intervals than when the particle is at rest. That is, these returning signals at frequency $\omega_C + \Delta \omega$ are blue-shifted according to the relativistic Doppler expression

$$\omega' = \omega_C + \Delta \omega = \sqrt{\frac{1 + v/c}{1 - v/c}} \,\omega_C. \tag{6}$$

And those feedback signals returning from transponders in the backward direction are red-shifted to the lower frequency by taking the opposite sign of the electron's velocity *v*.

One important consequence of this feedback signal approach is that a steady-state equilibrium can be maintained for the electron moving at a constant velocity. There is symmetric behavior in the two coordinate directions perpendicu-

lar to the velocity direction but a constant asymmetric reach in the x-direction of motion. For example, in Fig. 1 consider the outermost ellipsoid scattering the feedback signals emitted from position A_1 . The backward sampling distance for a particular ellipsoid is shorter than the forward sampling distance in the environment.

In the steady-state condition for a single electron in the universe, the returning signals from all directions should not change the electron's constant velocity because there is no amplitude change in any of the returning signals, and their phases from all directions agree at the new electron position B. If there were no frequency shifts in the x-component of the feedback signal frequencies, then one might calculate the contributions by either of two methods: (1) adding up the returning signals from the rear and from the front by considering cones of equal solid angles on opposite sides of B and using elliptic functions of the second kind, or (2) adding up the returning signals along a line through B at any angle θ with respect to the velocity vector direction. Using the second method, one would add contributions along a line at angle θ to achieve

$$-\sqrt{\frac{1+v\cos\theta_f/c}{1-v\cos\theta_f/c}}\nu + \sqrt{\frac{1-v\cos\theta_b/c}{1+v\cos\theta_b/c}}\nu = 0, \quad (7)$$

where the first term represents signals returning from the forward direction at angle θ_f and the second term returning signals from the back at angle θ_b . Because one can constrain $0 \le |\theta| \le \pi/2$ for the forward direction, then along the same line $\theta_b = -(\pi/2 + \theta_f)$ and the sum is always zero because the cosines have opposite signs in diagonally opposite quadrants.

However, that method does not apply for this situation. Why not? Because we must account for the frequency shifts by integrating over the surface area of each ellipsoid separately for the feedback signals returning from the forward direction and those returning from the backward direction in order to determine the net effect. In Fig. 1, one recognizes that the plane passing through points P_3 and B perpendicular to the x-axis separates the two surface parts for each ellipsoidal surface integral, thereby separating the backward returning feedback signals from the forward returning ones.

In terms of the semi-major axis *b* and the semi-minor axis *a*, the ellipsoid's eccentricity

$$\epsilon = \sqrt{(b^2 - a^2)/b^2}.$$
(8)

The solid angle of the ellipsoidal cap on the right of B subtended from A_1 is

$$\Omega_{cap} = 2\pi \left(1 - \cos \theta\right) \tag{9}$$

where θ is the angle between the ray from A₁ to P₃ and the x-axis. The solid angle subtended by the left side is

$$\Omega_{left} = 4\pi - \Omega_{cap} = 2\pi (1 + \cos \theta). \tag{10}$$

Substituting the pertinent geometrical values, one obtains

$$\Omega_{cap} = 2\pi \left(1 - \frac{2\epsilon^3}{\sqrt{1 + 4\epsilon^6}} \right). \tag{11}$$

These geometrical factors are multiplied by the frequencies returning from each point on the ellipsoidal surfaces. Along the x-axis one obtains:

$$\Omega_{cap}\,\omega' = 2\pi \left(1 - \frac{2\epsilon^3}{\sqrt{1+4\epsilon^6}}\right) \sqrt{\frac{1+v/c}{1-v/c}}\,\omega_C,\qquad(12)$$

and

$$\Omega_{left}\,\omega' = 2\pi \left(1 + \frac{2\epsilon^3}{\sqrt{1+4\epsilon^6}}\right) \sqrt{\frac{1-v/c}{1+v/c}}\,\omega_C.$$
 (13)

Substituting $\epsilon = \beta = v/c$, assuming $v \ll c$, and expanding the expressions in a Taylor series, their difference becomes

Diff
$$\approx -4\pi \,\omega_{\rm C} \,\beta \,(\beta^2 - 1) \approx 4\pi \,\omega_{\rm C} \,\beta,$$
 (14)

i.e., proportional to the velocity v as expected, verifying that the uniform velocity will be maintained along the x-axis.

If one desires to check the result for relativistic velocities, the complete integration over the cap and the surface area remainder would be necessary. The frequency shifts can be large enough to put the returning feedback signals outside the high Q absorption curves. However, the integration verifies that the uniform velocity is maintained.

6 Inertia and Mach's principles

The idea of inertia considered in the early 1600s by Galileo and others proposed that a body maintains its state of uniform motion unless acted upon by an outside net force.

In the previous section, my feedback signal approach reveals the origin for this Law of Inertia. That is, the vacuum lattice itself plays an active and important role in maintaining the state of a particle's uniform motion. The feedback signals scatter from the transponders to arrive back in-phase to determine the particle's new location.

Information about the environment is brought back to determine the continuous behavior of the particle. Long-lived particles can establish a steady-state communication with the environment, but short-lived particles learn only transient information about their immediate environment. Fast particles near the speed of the feedback signals sample only an extremely small distance perpendicular to the trajectory direction.

The distant parts of the Universe play their role in determining the particle motion locally because feedback signals from way out there are added to the closer contributions to determine its new location. Mach's principle connecting local behavior to the influences from far reaches of the Universe therefore fits well in this feedback signal approach. The origin of the inertia concept is established.



Fig. 2: The momentum-space x-component amplitude contribution at B of the returning feedback signals for the electron at rest [solid rectangle] versus the contributions of the feedback signals [dashed rectangle] in the x-direction for the electron at a uniform velocity.

7 The momentum-space description

What does the particle do with its own returning signals? And with other particle's signals, which may be at the same frequency or at other frequencies? The response to any feedback signals by the particle depends upon whether the feedback signals lie within the response range of frequencies for its inherent harmonic oscillator, meaning that feedback signals are absorbed if they lie within the absorption response curve defined by its Q value. That is, a particle is not a transponder and will be frequency selective. And, in contrast to the transponders, which maintain their initial properties forever, the future behavior of the particle can be affected.

The x-component momentum-space behavior of the electron's feedback signals is shown in Fig. 2. The gray rectangle represents the equal x-momentum contributions from all 4π solid angle for the electron at rest in the lab frame, being symmetrical about $k_x = 0$. Left to right, from $-k_x$ to 0 to $+k_x$, one has the momentum-space total amplitude contributions from the x-components of the returning feedback signals. The dashed rectangle represents the same electron moving at a constant velocity v, so this dashed rectangle is the original rectangle displaced by the x-momentum of the particle. Out-of-phase returning feedback signals will change the distribution.

8 Time asymmetry

In addition to continuous Lie symmetries, discrete symmetries are important in particle physics. Experiments in the 1950s and 1960s established both parity P and charge-parity CP violation for the weak interaction. Theoretically, one expects CPT invariance, which includes the time reversal operation T, and to this date all evidence points toward CPT conservation [2]. CP violation occurs for the weak interaction, so then T violation must occur for the weak interaction, so then T violation must occur for the weak interaction also in order to maintain CPT invariance. The mathematical source [6] of the weak interaction CP violation is simply the mathematics of products of unit quaternions in the group Q, the leptons, quarks, and weak bosons all being represented by quaternions.

This feedback signal approach to particle behavior possesses a fundamental time asymmetry, the expected T violation. Consider a free particle with its Compton frequency ω_C in uniform motion in the lab frame. To the moving particle, as we demonstrated earlier, its returning feedback signals from the forward direction are blue-shifted to a higher frequency and those returning from the backward direction are red-shifted to a lower frequency.

Now introduce time reversal via the operator T, i.e., have the electron move backwards at the same uniform velocity as if running a video backwards. The particle will be emitting bluish feedback signals in the new backward direction and their returning signals from the transponders would be redshifted back to the original Compton frequency ω_C . The new forward emitted reddish signals will return as blue-shifted back to the original ω_C also. Therefore, the environment appears symmetrical in the forward and backward directions, so the particle should not be moving. There is a conflict with the hypothesis of time reversal symmetry. Therefore, time reversal symmetry is violated. Time reversal cannot occur in Nature.

Hence, a definite time direction is an inherent feature of the feedback signal approach. The moving particle "knows" its forward direction in the time coordinate. All particles would possess this time asymmetry property. For the antiparticles, which exist in the mathematically conjugate space to our normal space, they would also have one time direction only, forward for them but perhaps in the backward direction mathematically for us.

Consequently, time travel backwards in time would be impossible in our Universe of particles unless, perhaps, one changes all the material particles to their anti-particles that are conjectured to have the opposite time direction in the conjugate space. And time travel forward in time faster than normal would be impossible also because there would exist a conflict with the particle behavior we have established via the feedback signal approach.

9 Origin of Special Relativity

Does this feedback model of particle behavior, as developed so far, lead to the special theory of relativity (STR)? If one examines the successive series of ellipsoids shown in Fig. 1, these ellipsoids belong to a set of curves with eccentricity $\epsilon = \beta = v/c$, the ratio of the electron's velocity divided by the speed of light. Therefore, as $\beta = v/c \rightarrow 1$, then also $\epsilon \rightarrow 1$.

In order to derive the expected STR equations, two assumptions about the feedback signals must be accepted:

- 1. the speed of the feedback signals in all reference frames is the same constant *c*, and
- 2. the perpendicular distances are invariant.

In the laboratory frame the feedback signals from each A_i to an ellipsoidal shell and back to the electron now at B will arrive in-phase at B, the definition of the new location of the free electron. A specific path within an ellipsoidal shell



Fig. 3: Constant phase ellipsoid parameters for deriving special relativity relations in a vacuum with an eccentricity $\epsilon = \beta = v/c$.

is shown in Fig. 3. The feedback signal goes from A_i at one focus of the ellipsoid to B at the other focus in the same time that the electron goes from A_i to B via a straight trajectory through the origin O.

I can now do the standard derivation, with the feedback signals instead of with light rays. Let the lab frame be the primed frame. The perpendicular distance from O to *a*, the semi-minor axis distance, and back is

$$2\Delta y = 2ct,\tag{15}$$

and, using the geometrical properties of the ellipsoid,

$$2\Delta y' = 2s \left[\frac{c^2}{v^2} - 1\right]^{1/2},$$
 (16)

with s = vt'. Because the perpendicular distances in the two reference frames are equal, $\Delta y' = \Delta y$, the time intervals are related by

$$t' = \frac{t}{\sqrt{1 - v^2/c^2}}$$
(17)

and the distance intervals along the velocity vector in the xdiection are related by

$$\ell' = \ell \, \sqrt{1 - v^2/c^2}.$$
 (18)

These relations are the fundamental equations of STR for the coordinate and time measurements. In Subsection 9.2 the relativistic energy and momentum expressions are derived. But first some geometrical properties of ellipsoids must be introduced.

9.1 Ellipsoidal geometry

In terms of the semi-major axis length *b* and the semi-minor axis length *a*, the ellipsoid's eccentricity is given by Eq. 8. If the perpendicular semi-minor axis length *a* is held fixed in both perpendicular directions to the x-direction as $\beta = \epsilon \rightarrow 1$, the semi-major axis value

$$b = \frac{a}{\sqrt{1 - \epsilon^2}} \to \infty.$$
(19)

At the same time the surface area of the ellipsoid as a prolate spheroid becomes

$$S.A. = 2\pi a^2 + 2\pi \frac{ab \sin^{-1} \epsilon}{\epsilon} \sim 2\pi a^2 + 2\pi ab \to \infty, \quad (20)$$

while the ellipsoid volume increases as

Volume =
$$\frac{4}{3}\pi ba^2 \rightarrow \infty$$
. (21)

With the ellipsoids stretching out along the x-axis, the velocity direction, as a consequence of $\beta = \epsilon \rightarrow 1$, the number of in-phase ellipsoids that can "scatter" feedback signals from the A_i to B is rapidly decreasing. Or so it seems that way! As a check, consider the feedback signal that goes rearward from A_i to -b and then is scattered forward to B. If the electron's velocity $v \sim c$, then immediately after the feedback signal's emission directed toward -b comes the return feedback signal to arrive at B simultaneously and in-phase with the electron. Consequently, only a very small distance into the environment behind and sideward will be sampled to determine the electron's behavior.

The minimum sampling distance in the direction perpendicular to the x-axis might seem to be the semi-minor axis distance

$$a = \frac{ct'}{2} \sqrt{1 - \beta^2} \to 0.$$
 (22)

However, the particle's Compton wavelength, or actually half the Compton wavelength, is the minimum sampling distance when $v \sim c$.

9.2 Energy and momentum

Using Fig. 3 again, one can determine several other important consequences in STR via the feedback signal approach. Relativistic energy and momentum can be related to the volume of the ellipsoid. If this statement is true, then the electron at rest has its mass-energy $E = mc^2$ determined by its "spherical volume" density when $\epsilon = 0$. Note that this fundamental particle volume will maintain a discrete rotational symmetry corresponding to the binary subgroup properties of each fundamental particle. So the "spherical volume" is an idealized spherical approximation in which the particle exists.

The ellipsoid volume when $\beta \ll 1$ is expressed as

$$V = \frac{4}{3}\pi ba^2 = \frac{4}{3}\pi \frac{a^3}{\sqrt{1-\epsilon^2}} \simeq \frac{4}{3}\pi a^3 \left(1 + \frac{1}{2}\beta^2 + \ldots\right) \quad (23)$$

or, when multiplied by c^2 , is

$$Vc^{2} = \left(\frac{4}{3}\pi a^{3}\right)c^{2} + \frac{1}{2}\left(\frac{4}{3}\pi a^{3}\right)v^{2} + \dots , \qquad (24)$$

which can be compared favorably to the familiar STR expansion of $m = m_0/\sqrt{1 - v^2/c^2}$ as

$$mc^2 = m_0 c^2 + \frac{1}{2} m_0 v^2 + \dots$$
 (25)

in which the second term on the right in Eqs. 24 & 25 expresses the increase of the mass-energy due to the particle's velocity, also known as the kinetic energy, and defines p = mv.

The simplest conclusion is that mass-energy is directly associated with the distorted volume of the space lattice occupied by the electron and depends upon the mass density

$$\rho(m_0) = \frac{6}{\pi} \frac{m_0^4 c^3}{h^3},\tag{26}$$

which reminds us that each type of fundamental particle distorts the lattice space in its own way to pack in its unique amount of mass-energy.

But there is more to behold! The vacuum, i.e., the lattice of mathematical nodes, must contribute the energy per unit volume which can be assimilated into the moving particle to increase its total energy according to STR. Until now I have assumed that the uniformly-spaced lattice does not have energy per unit volume, which is probably correct, but now we learn that the *distorted* lattice created by the particle at rest (and when in motion) is the energy source. At this point in my earlier research in the 1970s and 1980s I realized that each fundamental particle in Nature should have a different symmetry in order to agree with my discovery of the mass-energy relation to the volume enclosed.

In 1984, by accident, I found the significant clue to the lepton family symmetries that indicated that they could be representing specific discrete symmetry binary subgroups of SU(2), i.e., the unit quaternion group Q. That is, the 3 lepton families could be representing the specific 3-D discrete symmetry binary subgroups of Q named [3,3,2], [4,3,2], and [5,3,2], and also exhibit properties and behavior that suggests that the SM is a good theory all the way down to the Planck scale with its possible discrete lattice of mathematical nodes.

10 Origin of Feynman path integrals

Physicist R.P. Feynman is credited with providing a relativistic path integral approach to quantum mechanics (QM) in the 1940s and applying this method to better understand the foundations of physics. Today, practically all areas of physics continue to use path integrals to investigate the behavior of Nature at all levels [11].

The fundamental idea behind the path integral calculation is that a particle, such as an electron, "sniffs out" all possible paths between its initial location A and its final location B. Each possible path contributes its QM amplitude and phase angle to the path integral. Most paths contribute very little to this limit of the sum because their path lengths from A to B are so long that not only are their QM amplitude values reduced significantly but also their phase values differ enough to cancel each other. Two path examples are shown in Fig. 4 that will have significantly different contributions to the amplitudes at B.



Fig. 4: Two vastly different paths from A to B: (1) Path A,1,2,3,4,B, and (2) Path A, a, B. Feedback signals travel both paths. Or, in the path integral approach, the electron "sniffs out" both paths.

The actual classical path taken will be among the paths that collectively make the biggest contribution to the path integral, because this classical path will be the one for which the nearby paths have almost the exact same contribution to the path integral. Note that this path integral approach is based upon the mathematical principle of least time, which dictates that the actual classical path will be the one for which many nearby paths have the least time difference for going from A to B. Fundamentally, the method agrees with the least action principle.

The path integral approach is a proven method that works for all of physics, quantum and classical, meaning that the path integral results agree with all the known fundamental laws of Nature. Therefore, if the feedback signal approach is the source of the path integral method, then one can explain why path integrals successfully describe all of physics! Or vice-versa!

Feedback signals are emitted by a fundamental particle into all directions and undergo multiple transponder scatterings between the initial position A of the electron and its next position B, such as the simple 5-component path in Fig. 4. All the possible paths taken by these feedback signals going from A to B can be considered collectively identical to the "sniffing" out all possible paths from A to B in the path integral approach. Each feedback signal path is then a contributor to the path integral with its specific amplitude and phase angle.

Therefore, the underlying mathematical reason why the path integral approach works so impeccably well is that fundamental particles are using feedback signals to sample their environment in order to determine their subsequent behavior. Thus, one could use path integrals as the preferred mathematical method to describe all the results of the particle feedback signal behavior.

There exist many mathematical ways to represent the path integral method. One interesting visual way [12] to represent this limit of the sum over all paths is to use equal length arrows for each path and point them in the correct phase direction in an Argand diagram shown in Fig. 5. That is, each path



Fig. 5: Argand diagram of the phases for the different paths. Only 13 different paths are shown here, but the general idea of finding their total contribution to the amplitude is represented by the length of the arrow from A to B.

to the current position will have a different phase, therefore a different angle with respect to the horizontal real axis and the vertical imaginary axis in this complex 2-D space.

Nearby paths will have almost the same phase angle, will point in nearly the same direction, and will add a significant distance to the total vector sum in the diagram. Those arrows with opposite directions may cancel out completely. Each phase arrow is produced by a different path from the start to the current position B. The path integral amplitude is the length of the long straight arrow from beginning to end, A to B in the diagram, and the probability to be at the current position is the absolute value of its square.

In summary, each arrow also represents a feedback signal path and its phase contribution at location B, the current position of the electron. Again, one must add up all the feedback signal amplitudes arriving at B to find their total amplitude, which will depend upon the distance traveled and the phase at arrival at B. The electron position will be at the new maximum amplitude value. Therefore, we have conceptual and mathematical agreement with the path integral.

11 Origin of quantum mechanics

The rules of quantum mechanics (QM) can be derived from the path integral approach. But the path integral approach has its origin in the feedback signal approach as described above. At this point I could simply consider using path integrals to derive the 3 rules of QM. But deriving QM by the feedback signal approach provides a better "feeling" for how any particle behaves in the single slit and double slit experiments. There is no surprise because the feedback signal approach has been shown to be equivalent to the path integral approach.



Fig. 6: While passing through the single slit the particle will experience diffraction spreading in the y direction because the feedback signals returning from the wall will produce phase shifts in the shaded regions (approximate idealized representation).

Here are those 3 rules of QM from which all its consequences can be derived [13]. But first I must recall the definition of an event in relativistic QM. A QM event is defined as a set of initial and final conditions, e.g., an electron leaves the source, arrives at the detector, and nothing else happens. The first principles of QM [i.e., the 3 rules] are:

- 1. Each event in an ideal experiment is described by a complex number ψ that is called the probability amplitude, the event probability P being the square of the absolute value $|\psi|^2$.
- 2. When an event can occur in several alternative ways, the total probability amplitude Ψ for the event is the sum of the probability amplitudes for each way considered separately. There is an interference term $2\psi_1\psi_2$:

$$\begin{split} \Psi &= \psi_1 \; + \; \psi_2 \\ P &= \mid \psi_1 \; + \; \psi_2 \mid^2 \, . \end{split}$$

3. If an experiment is performed that is capable of determining whether one or the other alternative is actually taken, the probability of the event is the sum of the probabilities for each alternative. The interference is lost:

$$P = P_1 + P_2.$$

Note that one does not need to actually do the measurement for this sum of probabilities to apply. Simply having the capability to do the measurement is enough to eliminate the interference terms.

11.1 Diffraction

Consider a fundamental particle moving along the x-axis approaching a narrow vertical slit extending upward along the z-axis in a solid material wall that extends to infinite distances perpendicular to the x-axis. The slit is symmetrical about the x-axis in both perpendicular directions. The particle approaches the slit from the left, goes through the slit, and recedes away from the slit to the right. One can put a "screen" of particle detectors behind the slit to measure the particle's arrival pattern.

As the particle approaches the slit the returning feedback signals define its new positions as before. Those signals re-



Fig. 7: After passing through the double slits the electron will experience diffraction and interference spreading in the y direction because the feedback signals returning from the wall will produce phase shifts in the shaded regions (approximate idealized representation).

turning directly from the open slit portion of the wall introduce no phase shift, and the returning signals from the volume of "empty" space on either side of the wall, front and back, also do not introduce a phase shift.

We now need to determine the phase shift effects of the wall on the behavior of the particle. Its matter content introduces phase shifts δ^a on the approach and δ^r on the recession, with the same phase shift values for each of the infinite series of feedback signal ellipsoids. The resulting amplitude values at the particle's position will depend also upon the total round trip distance.

Our concern is what happens in k-space on the back side of the slit both along the x-direction of the electron's travel and what happens in both perpendicular y and z directions. The angular distribution of the feedback signals from each ellipsoid will produce changes in the y-amplitude $A(k_y)$ according to the actual distribution of matter around the slit. The new k_y amplitudes are shown in Fig. 6 for inside and immediately behind the slit. Within the free particle rectangular box are shaded regions for possible examples of the phaseshifted signals returning from the particles in the wall around the slit.

With left-right symmetry in the slit region itself in the y direction, there exist symmetrical amplitude decreases as shown in Fig. 6 but no net acceleration. Instead, the change in the distribution of the amplitude in k_y space leads to a symmetrical spreading of the particle according to Fourier analysis. If the wall effectively stretches to infinity, then the major contribution comes from the slit region around $k_y = 0$. One has a broadened diffraction pattern produced which has the amplitude

$$U'(y) = U(y) + 2\Delta k A'(k_0) \exp \left[i(\omega(k_{0y})t - k_{0y}y) \right] \\ \times \frac{\sin \Delta k_y (y - v_{0y}t)}{\Delta k_u (y - v_{0u}t)}.$$
(27)

The term U(y) is the standard distribution in coordinate space for a free particle. The important result is the increased spread in the y-direction to produce the expected diffraction pattern, as represented by the 2nd term.

11.2 Interference

This feedback signal approach also reproduces the double slit interference pattern for the feedback signals because of the k_y momentum distribution shown in Fig. 7. In coordinate space the behavior of the feedback signals at each slit is wave-like but now one cannot determine in principle whether the particle goes through either slit because the feedback signals pass through both slits simultaneously. The amplitudes are added to produce interference before calculating the total probability.

Only when the experimental setup is such that one could determine the slit used by the particle do we get the addition of the probabilities. The mathematics tells us that whether one "looks" or not is irrelevant, but as long as one "could look", then the interference terms are absent in the probability expression.

I have explained how the particle's feedback signal behavior at a slit exactly dictates the behavior of a particle as described by QM, both for diffraction and interference. Hence, the 3 rules outlined at the beginning of this section for the first principles of QM follow directly from the diffraction and interference of the feedback signals, thereby revealing the origin of QM.

12 Origin of gravitation

Now consider the behavior of two different particles with different mass-energy values. The case of two identical particles exchanging feedback signals is discussed in Appendix B, where the connection between particle spin and quantum statistics agrees with Fermi-Dirac and Bose-Einstein behavior.

The analysis developed here first outlines the feedback signal source of the gravitational interaction. Then I discuss its agreement with the standard geometrical curvature approach to the general theory of relativity (GTR).

As an example, let's bring a muon into the environment of our electron with both particles at rest initially. I ignore their electromagnetic charge interaction, which is understood to be a local interaction described by the Standard Model, requiring the exchange of virtual photons.

Therefore, the muon has its Compton frequency about 207 times higher and a wavelength about 207 times shorter than for the electron. Thus, in Fig. 8, I cannot do justice to both particles at the same time by drawing their feedback signal ellipsoids to relative scale. Consequently, I only show different wavelength signals emitted by each, but they are not to scale.

Both particles emit their characteristic frequency feedback signals into the vacuum lattice. Each high Q particle has a nearly zero ability to absorb the signals from the other particle. Therefore, the biggest contribution to the amplitude and phase changes of the returning feedback signals comes from the lattice distortion surrounding each particle.



Fig. 8: As two "assumed neutral" particles approach each other, the transponders in the vacuum lattice handle both sets of feedback signals simultaneously. The signals returning from these transponders are a different phase than for the free particle. The instantaneous effects in k-space are shown in momentum space with the new k values at the dashed lines (*approximate idealized representation exagerated*).

Meanwhile, the transponders in space continue to behave as before, except that their separations have changed because they no longer have identical average spacings between the nodes. Whereas the node spacings are expected to be closer where the particle is defined by its discrete symmetry, their spacings are further apart outside this immediate region. As conjectured earlier, perhaps this node spacing difference in the two regions keeps the lattice total energy value at zero. One now has a lattice with non-uniform node spacings everywhere compared to the original uniform lattice that has no fundamental particles.

Transponders around the electron will continue to scatter the muon's higher frequency feedback signals isotropically into all directions. The lattice distortion will cause these feedback signals to return to the muon out-of-phase with returning feedback signals from other directions, thereby reducing the total amplitude from the forward direction toward the electron, as shown in the Fig. 8 momentum space diagram.

Therefore, the original spherical symmetry of the returning feedback signals around the muon is gone and the muon must either move toward or away from the electron. One can appreciate that the out-of-phase returning signals reduce the total feedback signal amplitude from the electron's direction, which means that the muon will begin to move toward the electron. Why? As shown in Fig. 8, the center-of-momentum for the muon's feedback signal distribution has moved toward the electron. So there is an attraction toward the other particle.

What does the less massive electron do? The same, but in the opposite direction toward the muon of greater mass M. The feedback signals going to the muon region are returned to the electron out-of-phase. Again, the out-of-phase returning feedback signals reduce the total amplitude arriving from the muon's direction, resulting in electron movement toward the



Fig. 9: Whenever a "chunk" of k-space is absent (the gray area) near $k-\Delta k_x$, there will be an acceleration in the +x direction. Usually the feedback signals returning from the forward direction are out-of-phase, the source being the transponders around other particles in the environment ahead.



Fig. 10: As a neutral particle of mass m approaches from the left toward another neutral particle of mass M, the transponders in the vacuum lattice handle both sets of feedback signals simultaneously. Shown are paths from positions r_1 and r_5 subtending the same angle θ to the distorted space around M as "seen" from the approaching particle. The feedback signals returning from these two rings of transponders around M return with a different phase than for the free particle without the presence of M. (approximate representation exagerated).

muon. That is, the electron's center-of-momentum distribution has moved toward the muon. There is a mutual attraction between the two particles.

The acceleration of each particle occurs when there is a change in phase of the feedback signals arriving from any direction. For example, suppose the particle "senses" that a "chunk" of k-space is absent near k_0 - Δk , as shown in Fig. 9. This situation occurs when returning feedback signals from the forward direction are out-of-phase with the oscillation phase of the particle itself. The center of the momentum rectangle will move from k to k' corresponding to a faster moving electron with k' > k, meaning that the particle has moved ahead of the expected uniform velocity location in the corresponding coordinate diagram.

The acceleration is caused by feedback signal amplitude changes as a result of phase changes in the feedback signals as the particle approaches a mass M, an effect directly related to the distortions in the lattice geometry around M. This distortion produces the spacetime curvature associated with GTR gravitation, as explained in the next Section.

In Fig. 10 are shown our two "neutral" particles of masses m and M, with m approaching the distortion volume around M. One sees immediately for the same angle θ subtended by the feedback signal ray toward M as m approaches M, there will be a shorter distance of roundtrip travel for the feedback

signals as they approach one another. And the feedback signals from m will sample regions of greater and greater lattice distortions upon moving closer to the center of M.

In Fig. 11 is an approximation to the result of both effects on the momentum-space amplitude distribution for the two positions shown in Fig. 10, i.e., r_1 and r_5 . As more and more amplitude is missing, the change in momentum will increase, i.e., the acceleration toward M will increase upon nearing M as the momentum value increases toward + k_x . This type of behavior is expected for the gravitational interaction, because the lattice distortion amount depends upon the mass-energy of M.

The feedback signals are scalar waves given by Eq. 4 in the form $(A/r) \exp[i(\omega(k)t - kx)]$ that are emitted, scattered, and returned, so we can go from the momentum space to coordinate space behavior using the Fourier Transform to obtain the total amplitude at the new, accelerated position for the wave packet

$$U(x) = \int_{k_0 - \Delta k_x}^{k_0 + \Delta k_x} A(k_0) \exp \left[i(\omega(k)t - k_x x)\right] dk_x.$$
 (28)

And if we assume

$$\omega(k) = \omega(k_0) + (k - k_0) \left(\frac{d\omega}{dk}\right), \tag{29}$$

then the composite feedback signal at the electron, i.e., the total amplitude at its new accelerated position is

$$U(x) = 2\Delta k_x A(k_0) \exp\left[i\left(\omega(k_0)t - k_0x\right)\right]$$

$$\frac{\sin \Delta k_x(x - v_0t)}{\Delta k_x(x - v_0t)}.$$
(30)

This modulated monochromatic wave does not spread in time, an important property of this feedback signal approach for the behavior of particles.

As $v \rightarrow c$, the ellipsoids become more prolate, the Δk_x increases with each equal time interval, and the wave packet of the electron adjusts smoothly. In the limit, the sideward sampling of the environment does not extend beyond the Compton wavelength λ_c and the feedback signals are sampling less of the surrounding space, thus reducing any further acceleration. This behavior agrees with the special theory of relativity (STR).

By considering the acceleration in more detail, one would discover that the smaller range in wave numbers in momentum space spreads the particle wave packet in the x-direction. When a new constant velocity is achieved, the particle wave packet reverts to its normal size. In the perpendicular y- and z-directions in which $v_y = v_z = 0$ as before, a symmetrical hole appears in k_y-space and k_z-space during the acceleration but returns to normal when the acceleration is done. Hence, some temporary lateral spreading of the wave packet occurs



Fig. 11: As the particle approaches M, the feedback signals returning from the two transponder rings have a different phase than for the free particle. The possible reduction of the amplitudes in k-space are shown for positions r_1 and r_5 in Fig. 10, with contributions to the k-space distribution removed above the dashed lines for a range of angles. (approximate representation exagerated).

also in these directions perpendicular to the accelerated motion along the x-direction.

One could consider further properties of the electron in terms of its de Broglie wavelength = h/p for non-relativistic momentum values in order to discuss the wave packet behavior for the electron. However, the feedback signal approach is all that's needed to understand the electron's behavior in response to another particle that also distorts the lattice.

I have described particle motion in terms of its dependence upon the integral of all the feedback signals returning from the environment back to the source-receiver location. Equal weighting for all k values has been used. In the idealized acceleration example, a rectangularized "chunk" of kspace was missing. Actually, one should consider that some of the feedback signals are returning from all directions with a different phase with respect to the k-space signals returning in a uniformly spaced euclidean lattice. The phase differences would produce a "hole" in k_x -space that can have positive and negative values. All the possibilities could be examined via computer simulations.

13 Gravitation from the Radius Excess

A lattice distortion occurs not only at the particle's origin but also throughout the surrounding space and spacetime. No longer does the lattice have uniformly spaced nodes. As we move further and further away from the origin of each particle, this lattice distortion becomes less and less.

The physics consequences can be understood by first separating the analysis into two parts: the 3-D space part, and then the time part for the (3+1)-D spacetime of our physical world. The two parts are put together to assemble the spacetime of Einstein's GTR.

13.1 The 3-D space part

In the uniformly spaced 3-D sublattice part of C^2 with nodes but with no particles yet, consider an imaginary thin spherical shell with a radius R >> d, the lattice node spacing. Then euclidean geometry dictates a radius value from its surface area A

$$R = \sqrt{\frac{A}{4\pi}}.$$
 (31)

Now in this 3-D sublattice consider the electron to have been in existence at the origin so that its characteristic distortion exists everywhere with the amount of distortion decreasing inversely with distance. At the one Compton wavelength distance from the center of the particle, that is, greater than about 10^{-12} meters from the electron's distortion center, one is far enough away to consider an imaginary spherical surface surrounding the electron as a good approximation at all further radial distances.

We measure the distance in a discrete space by counting the nodes along a radial path. Therefore, the measured radius r_{meas} from the electron's center to any outside distance will be greater than for the undistorted lattice because nodes will have been pulled inward. In fact, the radial difference between the distorted lattice and their euclidean lattice is called the *radius excess* expressed by [14]

Radius excess =
$$r_{meas} - \sqrt{\frac{A}{4\pi}}$$
. (32)

Note that in the limit when the enclosed mass-energy inside R is reduced to zero, then the radius excess will reduce to the previous zero value. Therefore, let the radius excess be directly proportional to the enclosed mass-energy amount m, in this case the mass of the electron. Then do a dimensional analysis to predict

Radius excess =
$$r_{meas} - \sqrt{\frac{A}{4\pi}} = \frac{G}{3c^2}m.$$
 (33)

The factor 1/3 comes from the geometry of a 3-D sphere and is the numerical factor for the second term in the Taylor expansion of the sine function.

This radius excess is the important quantity which, according to Einstein's GTR, is indeed proportional to the mass of the particle enclosed by the imaginary sphere at radius R. That is, for a fixed R value, the distance measured by counting the nodes will be greater for the more massive particle enclosed. Note that the radius excess defined here is a measure of the 3-D geometrical curvature produced by the massenergy m, and that this radius excess expression actually defines the average curvature just above the chosen surface area.

The quantity $G/3c^2 \sim 2.5 \times 10^{-28}$ meters/kilogram, a very small number. Therefore, in order to get a "feeling" for the radius excess magnitude, insert the pertinent values to learn that the radius excess for the electron is extremely small:

Electron : radius excess =
$$2.3 \times 10^{-60}$$
 meters! (34)

Also, for Earth: 1.5 millimeters; for the Sun: 0.5 kilometers.

13.2 The time part of (3+1)-D spacetime

Now for the time coordinate contribution. The principle of equivalence states that one cannot distinguish between a gravitational field and an accelerated reference frame for a locally uniform gravitational field. Applying this equivalence principle, Einstein found that time varies from place to place.

The time coordinate will be modified near the mass *m*. Let *v* be the relative velocity between a source and a receiver, with the received frequency ω' being related to the emitted frequency ω by Eq. 6 for STR. For $v^2/c^2 << 1$, the approximation is

$$\omega' = \omega \left(1 + v/c\right). \tag{35}$$

If the receiver is accelerating, then the receiver will have an additional velocity gt, where g is the acceleration value and t is the time interval it takes light to travel the distance Hfrom source to receiver.

Using the equivalence principle, the *g* is now the gravitational acceleration and *H* becomes the radial height difference in the gravitational field. For the clock at the radial height h_2 above the clock at height h_1 , with $H = h_2 - h_1$,

$$\omega_2 - \omega_1 = \frac{gH}{c^2},\tag{36}$$

so that the excess rate is

$$\omega_1 \frac{gH}{c^2}.$$
 (37)

From STR, there is the correction factor of the opposite sign for the speed in case of the moving clocks

$$\omega_2 = \omega_1 \sqrt{1 - v^2/c^2},$$
 (38)

which for low speeds $v \ll c$, becomes

$$\omega_2 = \omega_1 (1 - v^2 / 2c^2), \tag{39}$$

predicting the defect in the rate of the moving clock to be

$$-\omega_1 v^2 / 2c^2$$
. (40)

Combining the two effects produces

$$\Delta\omega = \omega_1 \left(\frac{gH}{c^2} - \frac{v^2}{2c^2}\right). \tag{41}$$

This frequency shift of the moving clock means that if one measures a time interval dt on a fixed clock, the moving clock registers the time interval

$$dt\left[1 + \left(\frac{gH}{c^2} - \frac{v^2}{2c^2}\right)\right].$$
(42)

Therefore, the total time excess over the whole trajectory is the integral

$$\frac{1}{c^2} \int \left(\frac{gH}{c^2} - \frac{v^2}{2c^2}\right) dt,\tag{43}$$

which is to be a maximum, thereby obeying the principle of least action. I.e., the particles always take the longest proper time. Note that this law does not rely upon any of the coordinates.

One can see this result better in the alternative formulation by multiplying Eq. 43 by $-mc^2$, where *m* is the mass of the particle, so that the integral is over the kinetic energy minus the gravitational potential energy which, by the principle of least action, must be a minimum.

13.3 The two main laws of GTR

Therefore, the two main laws of GTR have been established by starting from the idea that each fundamental particle distorts the lattice into its own discrete symmetry. The distortion continues to all distances, and phase changes in the returning feedback signals are produced by the distorted lattice.

Equivalently, the distorted lattice around each particle is the source of the radius excess proportional to the enclosed mass producing the distortion, and this radius excess leads to the two main laws of gravitation.

These laws are:

- 1. The curvature expressed in terms of the excess radius is proportional to the mass inside a sphere, by Eq. 33.
- 2. Objects move so that their proper time between two end conditions is a maximum.

The first law, Einstein's field equation, reveals exactly how the geometry of spacetime changes in the presence of matter. The second law, Einstein's equation of motion, reveals how objects move when there are only gravitational forces. So the entire spacetime is distorted in the presence of matter.

Can we understand the factor of about 10^{40} for the relative strength of the electric force to the gravitational force between the two electrically charged particles, two electrons, for example. There is a significant physical and conceptual difference between the two forces. The electric force relies upon the local gauge interaction of the SM by the exchange of virtual photons, whereas the gravitational force as determined by the feedback signal approach does not have the exchange of a virtual particle for a local gauge interaction. The gravitational acceleration results from particle responses to their returning feedback signals from the environment. Whether the factor of about 10^{40} can be derived by exploiting this difference is expected but has not been achieved at present.

14 Review of steps taken

Here are the sequence of steps taken to establish that QM and GTR have a common origin determined by the feedback signal approach, based upon the fact that QM, the SM, STR, and GTR are all successful theories that agree with Nature:

1. The lepton and quark particle states respect the electroweak symmetry $SU(2) \times U(1)$ of the SM, but the actual two orthogonal fundamental particle states per fermion family are dictated by the discrete symmetry binary subgroups of the unit quaternion group Q, or equivalently, SU(2).

- 2. The two physical orthogonal EW flavor states in each lepton and quark family are formed by the linear superposition of the two mathematical states, and they oscillate at the Compton frequency ω_C as 3-D entities in R³. Hadrons combine their 4-D quarks and gluons to make 3-D particles also, obeying QCD.
- 3. One assumes that (3+1)-D spacetime corresponds to a 2-D complex lattice $C^2 = R^4$ filled with uniformly spaced mathematical nodes acting as ideal transponders.
- 4. The fundamental fermion "gathers in" the mathematical nodes to form its correct discrete symmetry binary subgroup with its lattice distortion extending outward into the lattice.
- 5. The "breathing mode" flavor state oscillations of the particle emit scalar waves into the lattice. I have called these "feedback signals".
- 6. The transponders in the lattice "scatter" these feedback signals into all directions isotropically with no phase shift and with the same response for all frequencies and amplitudes.
- 7. STR, the principle of inertia, Mach's principle, the path integral approach, QM, and the one direction of time, are all derived by analyzing the details of the feedback signal behavior.
- 8. The lattice distortion around each fundamental particle is the source of phase changes in the returning feedback signals at the original particle, resulting in an acceleration toward the other particle.
- 9. Gravitational curvature is shown to agree with the lattice distortion associated with each particle, so the acceleration produced by the feedback signal approach is the gravitational acceleration of GTR.
- 10. Therefore, QM and GTR have the common origin as established by the behavior of particles in response to the feedback signals.

15 Summary

This feedback signal approach toward understanding particle behavior successfully explains the origin of QM, the path integral method that allows one to calculate quantum mechanical and classical physics behavior, and gravitational acceleration. The approach involves fundamental particles behaving as "antennas" emitting and absorbing scalar waves at their Compton frequencies, scalar waves that I have called feedback signals. These feedback signals are scattered isotropically by a discrete lattice of nodes representing spacetime. Gravitation has been shown to be the consequence of the lattice distortion around particles by changing the amplitude and phase of the feedback signals that are returning from regions surrounding mass-energy concentrations, in agreement with the radius excess derivation of GTR.

Therefore, I have revealed the common origin for gravitation and quantum mechanics.

The remaining question is whether fundamental particles, such as the electron, do indeed emit and receive these feedback signals as described in this approach. If so, then not only must fundamental particles be using these feedback signals but also all composite entities such as a proton and very massive objects must rely upon them for determining their physical behavior.

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Appendix A: Quark states

I have proposed [4, 5, 8] that the 4 discrete symmetry binary subgroups that define four 4-D quark families in \mathbb{R}^4 are: [333], [433], [343], and [533], corresponding to the only regular polytopes in \mathbb{R}^4 . The predicted quark mixing angles produce values that generally agree with their empirical values in the standard 3x3 CKM submatrix of its 4x4 quark mixing matrix CKM4. This quark family mixing therefore guarantees that the 4 discrete symmetry binary subgroups defining the quark families collectively behave as the SU(2) of the SM.

Having 4 quark families creates two different conflicts: (1) no 4th quark family has been discovered yet, and (2) there needs to be triangle anomaly cancellation, usually assumed to mean 3 lepton families paired against 3 quark families but with no verification of which lepton family pairs with which quark family. With regard to the first conflict, the mass values of the 4th family quarks could be quite large, so that either they cannot be produced at the LHC [15] or they decay too quickly. The triangle anomaly gets resolved directly because the collective lepton family mimicking SU(2) exactly cancels the collective quark family mimicking SU(2), one-to-one.

The influence of the 4th quark family may yet appear in rare decays of the other quarks and might resolve several extant problems, including being the source of the baryon asymmetry of the Universe (BAU) by providing a needed factor of at least a 10^{13} increase [16] in the Jarlskog constant and by also explaining the muon g-2 discrepancy.

Therefore, the 4-D quark states are clearly distinguished from the 3-D lepton states, the leptons not being capable of having a color charge, which is now a 4-D property. The origin of the three color charge states comes directly from 4-D rotations, which require two simultaneous rotations in orthogonal planes, and there are only three different pairs of orthogonal planes in \mathbb{R}^4 . The three different color charges, r,g,b, defined by simultaneous rotations in the three pairs of orthogonal planes, can be shown equivalent to the three color charges of SU(3)-color. Even more important, having quark states and gluon states defined in \mathbb{R}^4 means they cannot exist in R³, so quark confinement becomes geometrically explained also.

Finally, the 4-D quark and gluon states must combine according to quantum chromodynamics (QCD) to make the mesons and baryons, i.e., the 3-D hadrons. Intersection theory in mathematics can handle this geometrical concept of intersecting 4-D objects to make 3-D objects.

However, QCD theory predicts [17, 18] a self-contained world for the quarks and gluons, with only color changes allowed and no possibility of quark decay. So why does Nature need the leptons? The mathematical answer follows from Kuratowski's theorem [19] in graph theory: all graphs will reduce to the K_5 or $K_{3,3}$ graphs, the only graphs that retain their integrity. Fortunately, at least for quarks, the [333] discrete symmetry binary subgroup of the up/down quark family represents the K_5 graph, so all other quarks will decay eventually to this first family. The stability of the electron may also be a consequence because [332] is related to [333].

Also recall that only 4N-dimensional normal spaces have a conjugate space of the same dimension according to Clifford algebra and Bott periodicity [9]. So, there will be the simultaneous existence of the 4-D anti-particle real internal symmetry space as required by the SM. The next larger space with a conjugate space, \mathbb{R}^8 , is equivalent to a 10-D spacetime. For discrete spaces, icosians related to the binary icosaheral group [532] provide a direct connection [20] from our discrete \mathbb{R}^4 to the discrete space \mathbb{R}^8 , which obeys the discrete symmetry operations of Weyl \mathbb{E}_8 .

The particles exist in our discrete SO(3,1) spacetime, so the icosians produce a second discrete symmetry Weyl E_8 for spacetime. Combining discrete spacetime with the discretee internal symmetry group therefore makes the discrete product group Weyl E_8 x Weyl E_8 , equivalent to the discrete symmetry group I call "discrete" SO(9,1). Hence, there exists a *unique* connection from the SM gauge group to "discrete" SO(9,1) in a 10-D spacetime.

Appendix B: Identical particles and quantum statistics

Consider two identical particles. What behavior will the feedback signal approach predict?

Two neutral identical particles are to be considered, so that we can ignore any local gauge interactions of the SM, both particles beginning at rest with respect to each other. In the general case, feedback signals emitted at the same Compton frequency ω_1 by each particle are absorbed, phase shifted, and emitted by the other identical particle back into the surrounding space.

Their existence in each other's environment means that the identical particles can become phase-locked, either with in-phase or with out-of-phase normal modes, as is the case for two identical-frequency quantum harmonic oscillators communicating to each other, with their final locked-in phase relationship becoming 0 or π . The two possible normal mode frequencies for any two harmonic oscillators communicating via an exchange of energy represented by Γ are

$$\Omega = \frac{1}{2}(\omega_1 + \omega_2) \pm \Gamma, \qquad (44)$$

but the two identical high Q fundamental particles will have $\omega_2 = \omega_1$, so

$$\Omega = \omega_1 \pm \Gamma. \tag{45}$$

Which physical property of a particle actually determines the difference between the two phase-locked states? Because the single free particle does not have phase-shifted returning feedback signals, the phase shifts introduced by the other identical particle can be a function of differences only:

phase shift =
$$f(\omega_i - \omega_j, A_i - A_j, P_i - P_j)$$
, (46)

where ω is the Compton frequency, A is the signal amplitude, and the P could be some other factor such as the intrinsic spin.

As we know, the physical factor P called particle intrinsic spin S is the key. Different particle angular momentum spin states need to be considered, such as a scalar S = 0, a spinor S = 1/2, and a vector S = 1, in order to determine the general result.

Consider the scalar particles first, the ones with intrinsic spin S = 0. At first the feedback signals returning from the direction of the other identical scalar particle might not be in-phase, so the two particles are accelerated toward each another because the returning feedback signals from the vacuum transponders in the direction opposite the other particle are inphase. Eventually, the scalar particles can become locked inphase with each other's oscillations and can occupy the same point in space. So these two S = 0 identical particles behave as bosons obeying Bose-Einstein statistics.

Now consider a system of two spin S = 1/2 electrons. QM requires [21] that their overall asymmetric wavefunction be the product of position eigenvalues and the total spin quantum numbers. There are three triplet spinor states having S = 1 symmetric with respect to the exchange of the electrons, with the spatial part being asymmetric so that the probability of the two electrons being at the same point in space is zero. But for the singlet S = 0 spinor state, the spin part is asymmetric and the spatial part is symmetric, thereby enhancing the probability to be at the same point in space, i.e., there is an attraction to one another.

Applying geometry by rotating the two S = 1/2 identical particles together in the triplet S = 1 state by 360°, one determines that the feedback signals will return with a phase that produces an increased amplitude pushing each particle away from the direction of the other identical particle. Therefore, a repulsion occurs to produce an increased separation. Called Pauli repulsion, this response is the source of Fermi-Dirac statistics.

In the total S = 0 case for two spin S = 1/2 particles, i.e., with spins opposite, the feedback signal amplitudes at each particle decrease by adding in the returning signals from the direction of the other identical particle. There is attraction, so this total S = 0 spin state is allowed for two electrons at the same point in space. That is, the spatial wavefunction is even but the spin wavefunction for this total S = 0 state is

anti-symmetric.

Finally, when both particles each have S = 1, the total spin states are S = 2 and S = 0. The geometrical factors will produce a result identical to the total S = 0 Bose-Einstein behavior for two scalar particles, i.e., there is a feedback signal amplitude decrease that results in an attraction.