A Pedestrian Derivation of Heisenberg's Uncertainty Principle on Stochastic Phase-Space

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Without using the common methodologies of quantum mechanics – albeit, methodologies that always involve some demanding mathematical concepts, we herein demonstrate that one can derive in a very natural, logical and trivial manner, Heisenberg's quantum mechanical uncertainty principle on the new phase-space whose name we have herein coined *Stochastic Phase-Space*. This stochastic phase-space – is a mathematical space upon which we previously demonstrated [2] the naturally implied existence of the First Law of Thermodynamics from Liouville's theorem. In addition to Heisenberg's uncertainty principle, we derive an upper limiting uncertainty principle and it is seen that this upper limiting uncertainty principle describes non-ponderable tachyonic particles.

It must have been one evening after midnight when I suddenly remembered my conversation with Einstein and particularly his statement, 'It is the theory which decides what we can observe.' I was immediately convinced that the key to the gate that had been closed for so long must be sought right here. I decided to go on a nocturnal walk through Faelled Park and to think further about the matter . . . Werner Karl Heisenberg (1901-1976). Adapted from [3, p. 6].

1 Introduction

The present paper is the third in a five part series where we make the endeavour to understand the meaning and origins of what drives the unidirectional forward arrow of thermodynamic entropy. In our first instalment [4, hereafter Paper I], we demonstrated that the *Second Law of Thermodynamics* (SLT) can possibly be understood if there exists a new kind of probability measure, p_r, which drives thermodynamic processes and this thermodynamic probability evolves in such a manner that, whenever this thermodynamic probability changes its value when a system moves from one state to the next, it always takes higher values than the value it previously held – i.e. $dp_r \geq 0$, at all physical and material times. In a nutshell, thermodynamic events will at the very least, progressively evolve from a probabilistically less likely state – to a probabilistically more likely state. Such an evolution sequence is what is naturally expected from probability calculus anchored on common binary logic where natural systems are expected to steadily progress into their most likely state.

In the construction of our new ideas, naturally, we expected that this thermodynamic probability p_r, would turn out to be the usual Boltzmann probability, i.e.

$$p_r = Z^{-1} \exp\left(-E_r/k_{\rm B}T\right) ,$$

where p_r is the probability that for a system at temperature T, the microstate with energy E_r, will be occupied and Z is the partition function. As will be demonstrated in the sequel paper [5, hereafter Paper IV], this probability p_r, cannot be the usual Boltzmann probability, but a new kind of probability associated not with the occupation of the given microstate, but its evolution; where by evolution, it is understood to mean – moving or progression from its present state to a new state altogether.

Further on, in the paper [2, hereafter Paper II], we demonstrated that Liouville's theorem [6] can actually be viewed as a subtle statement of the *First Law of Thermodynamics* (FLT). This we did by defining the Liouville density function, $\delta\varrho$, in-terms of some new physical quantity, $\delta\mathcal{S}_{\rm TD}$, that we called the *thermodynamic phase* (or the *thermodynamic action*), i.e. $\delta\varrho = \exp(\delta\mathcal{S}_{\rm TD}/\hbar)$, where $\hbar$ is Planck's normalized constant. Furthermore, in Paper IV, we shall identify $\delta\varrho$ as the appropriate thermodynamic probability of evolution, that is, the thermodynamic probability responsible for the SLT.

In the present paper, we shall demonstrate that when cast as a probability measure, $\delta\varrho$ naturally yields the universally celebrated quantum mechanical uncertainty principle of Heisenberg [1]. In addition to Heisenberg's lower limiting (i.e. $\delta E\,\delta t \geq \hbar/2$ and $\delta p\,\delta x \geq \hbar/2$) uncertainty principle, we derive an upper limiting uncertainty principle – i.e. $\delta E\,\delta t \leq \hbar/2$ and $\delta p\,\delta x \leq \hbar/2$. As initially pointed out in [7], this upper limiting uncertainty principle strongly appears to describe non-ponderable tachyonic particles.

Without a doubt, Heisenberg's quantum mechanical uncertainty principle is certainly one of the most famous aspects of quantum mechanics and this very aspect of the theory is universally regarded as the most distinctive feature of the theory. It is a unique characteristic feature which makes quantum mechanics differ radically from all classical theories of the physical world. For example, the uncertainty principle for position and momentum $\delta p\,\delta x \leq \hbar/2$ states that one cannot simultaneously assign exact values to the position and momentum of a physical system. Rather, these quantities can only be determined with some intrinsic, inherent and characteristic uncertainties that cannot – simultaneously – become arbitrarily small.

In its popular understanding, the Heisenberg uncertainty principle is assumed to be a principle to do with the accuracy in the results of measurements of physical variables such as momentum, position, energy, *etc*. Strictly speaking, this is not true. For example Millette [8] argues that the Heisenberg uncertainty principle arises from the dependency of momentum on wave number ($\boldsymbol{p} = \hbar\boldsymbol{k}$) that exists at the quantum level, and that ultimately the uncertainty principle is purely a relationship between the effective widths of Fourier transform pairs of conjugate variables. Our ideas propagated herein do support these views and as an addition, these quantum mechanical uncertainties associated with physical variables are seen to arise from pure stochastic processes occurring on some new phase-space that we have coined the stochastic phase space.

Now, in closing this introductory section, we shall give a synopsis of the present paper – i.e. this paper is organised as follows: in §2, we derive the uncertainty relations that govern ordinary ponderable matter and thereafter in §3, we derive the uncertainty relations that govern exotic non-ponderable matter. Lastly, in §4, we give a general discussion.

2 Derivation of the uncertainly principle

As stated in the introductory section, we are going to demonstrate in this section (i.e. in §2.2) that one can derive in a very natural and logical manner, the position-momentum and energy-time quantum mechanical Heisenberg uncertainty principle on the newly proposed *Stochastic Phase-Space* (hereafter $\delta\Gamma$-space) upon which we demonstrated [2] the naturally implied existence of the FLT from Liouville's theorem. In addition to Heisenberg's uncertainty principle, we will also derive in §3, upper limiting position-momentum and energy-time uncertainty principles and these upper limiting uncertainty principles describe non-ponderable tachyonic particles.

Before we proceed, we need to explain what it is we mean by *upper limiting uncertainty principle*. If there is an upper limiting uncertainty principle, from the viewpoint of common logic, there also must be a *lower limiting uncertainty principle*. Indeed, the uncertainty principle of Heisenberg is a lower limiting uncertainty principle because it gives the lowest possible value that the product of the energy (δE) *&* time (δt), and momentum (δp) *&* position (δr) uncertainties would ever take. That is to say, the products $\delta E\,\delta t$ and $\delta p\,\delta r$, can take whatever value they can or may take for so long as this value does not exceed the minimum threshold value of $\hbar/2$, hence, in this way, it becomes pristine clear that the Heisenberg uncertainty principle ($\delta E\,\delta t \geq \hbar/2$ and $\delta p\,\delta r \geq \hbar/2$) is indeed a lower limiting uncertainty principle.

Now, if – by the sleight of hand, we are to flip the sign in the Heisenberg lower limiting uncertainty principle so that we now have $\delta E\,\delta t \leq \hbar/2$ and $\delta p\,\delta r \leq \hbar/2$, the resulting uncertainty principle is an upper limiting uncertainty principle since it now gives an upper limit in the value that the products ($\delta E\,\delta t$ and $\delta p\,\delta r$) of the uncertainties can ever take. Whence, we must hasten at this point and say we already have discussed the implications of a upper limiting uncertainty principle in our earlier works (i.e. in [7]) where we argued that if such particle exist to being with, not only will they travel at superluminal speeds – they also will have to be non-ponderable as-well; that is to say, they must be invisible and absolutely permeable. In simpler colloquial terms, such particles must be capable of passing through solid walls with no hindrance at all whatsoever.

2.1 Preliminaries

Now, before we can go on to present our derivation of Heisenberg's uncertainty principle in §2.2, we will need to set-up the stage for that event. First, in order for that, we shall give in §2.1.1, a description of the particle system that we shall consider, and, in §2.1.2, we shall describe the normalization across all spacetime for the thermodynamic probability function $\delta\varrho$ and in §2.1.3, we shall describe the normalization across a given space-and-momentum axis for the thermodynamic probability function, $\delta\varrho$. Lastly, in §2.1.4, we present some useful mathematical equations that we will need in our endeavours to derive the Heisenberg uncertainty principle.

2.1.1 Description of particle system

As initially suggested in Paper II, we envisage the existence of two mutually exclusive spacetimes and these we have termed – the *Classical Canonical Spacetime* (hereafter, CC-Spacetime), and, the *non-Canonical Spacetime* (hereafter, NC-Spacetime). The NC-Spacetime can also be called the *Stochastic Spacetime*. On the deterministic CC-Spacetime, a particle has its usual deterministic classical four position (x, y, z, c_0t) that we are used to know, while on the non-deterministic NC-Spacetime, the non-deterministic jittery quantum randomness and fuzziness associated with the usual deterministic classical canonical position ($\delta x, \delta y, \delta z, c_0\delta t$) are defined on this non-deterministic NC-Spacetime.

For example, considering only the x-axis, a particle will have x as its canonical position and δx as its associated non-canonical position as defined on the NC-Spacetime. It is δx that should give this particle the quantum fuzziness leading to the weird quantum probabilistic nature of physical systems. For the human observer – assuming zero human-induced error in measuring the position of the particle – the effective position $\hat{x}$ of the particle at any given time is $\hat{x} = x \pm \delta x$. So, in general, x^{μ} is the canonical four position of the particle and

δx^{μ} is the associated quantum randomness that leads to the mysterious, strange and bizarre fuzzy quantum probabilistic nature of natural systems.

In our description above, when we say particle, we mean a point-particle – i.e. particles of zero spatial dimension, hence zero volume. Obviously, there will be some trouble in accepting this – as point-particles, are – in physics – no more than idealization of real finite-sized particles that are smeared-out in a finite region of space. That is, a point-particle is in general an appropriate or convenient representation of any object whatever its size, shape, and structure – all these details of size, shape, and structure, *etc*, are irrelevant under the general particle model.

To further complicate this issue of the particle description of matter, we all know pretty well that the existence of point-particles is strictly forbade by Heisenberg's uncertainty principle. With this in mind, of these particles, what we envisage is them having all their charge such as their gravitational mass and electrical charge being concentrated on that very single point with this point being trapped in the finite sized spherical region of radius: $\delta r = \sqrt{(\delta x)^2 + (\delta y)^2 + (\delta z)^2}$, with the the centre of this finite spherical region fixed in space about the canonical position (x, y, z). Because of the fields that the trapped charged point-particle carries – i.e. fields with which this particle interacts with other particles; the fuzzy, random wandering and dotting back-and-forth, up-and-about of this particle inside this finite region should create the impression of a solid billiard-like ball of radius δr with oft cause the bulk of its charge (gravitational, electrical, *etc*) expected to be trapped in this spherical region. Surely, such a particle-system will be localized and it will have the property of ponderability that we experience with electrons, protons, *etc*. Let us call such a particle-system, a *Ponderable Material Particle*.

Now, for a minute, let us assume that the above described point-particle is not trapped. If that were the case, then, what is it that we are going to have for such a particle-system? Clearly, it must be an unbounded point-particle that is free to roam all of the Universe's length, breath and depth – from one end of the Universe, to the other in an instant! Such a particle-system should have its charge (gravitational, electrical, *etc*) spread-out evenly throughout the entire Universe. Not only this, while such a particle-system will have a definite fixed canonical position, the entire particle-system must be invisible as it will not have the property of ponderability (localization). Likewise, let us call such a particle-system, a *non-Ponderable Material Particle*.

Now, as shall soon become clear in our derivation of Heisenberg's uncertainty principle, two classes of particles will emerge and the first is that class whose random quantum fuzziness as described on the NC-Spacetime obeys the usual quantum mechanical uncertainty principle of Heisenberg, i.e. $\delta E\, \delta t \geq \hbar/2$ and $\delta p\, \delta r \geq \hbar/2$; and these particles travel at speeds less than, or equal to the speed of light in *vacuo*. The second class is that of particles whose quantum fuzziness as described on the NC-Spacetime obeys not the usual quantum mechanical uncertainty principle Heisenberg, but obey the converse of Heisenberg's uncertainty principle, namely $\delta E\, \delta t \leq \hbar$ and $\delta p\, \delta r \leq \hbar$ and these particles travel at speeds that are at the very least, greater than the speed of light in *vacuo*.

At this juncture, we feel very strongly that we have prepared our reader to meet the strange new proposal of invisible particles that travel at superluminal speeds, thus – assuming the reader somewhat accepts or at the very least, finds some modicum of sense in what we have had to say above – we shall quietly proceed to the main business of this paper – that of demonstrating the natural existence of Heisenberg's uncertainty principle on the proposed NC-Spacetime where the jittery, fuzzy quantum randomness has here been defined.

2.1.2 Normalization across all space

If $\delta\varrho$ is assumed to be some probability function, then it must be normalizable. Normalization is oft cause one of the most fundamental and most basic properties that a probability function must satisfy. As is the norm: normalization of this function, $\delta\varrho$, across all of the six dimensions of $\delta\Gamma$-space requires that:

$$\frac{1}{\hbar^3} \overbrace{\underbrace{\int\!\!\int\!\!\int}_{\delta p_{\min}}}^{\delta p_{\max}} \overbrace{\underbrace{\int\!\!\int\!\!\int}_{\delta r_{\min}}}^{\delta r_{\max}} (\delta\varrho_+)\, d^3x d^3p = 1, \tag{1}$$

where: $\delta\varrho_+ = \delta\varrho_x^+ \delta\varrho_y^+ \delta\varrho_z^+ \delta\varrho_0^+$. In writing $\delta\varrho$ in (1), we have appended a subscript + and this is not a mistake, it is deliberate. This + appendage has been instituted – for latter purposes – so that a distinction can be made between a thermodynamic system with a positive $\delta\mathcal{S}_{\text{TD}}$ thermodynamic phase (action) and that with a negative $-\delta\mathcal{S}_{\text{TD}}$ thermodynamic phase (action), i.e.: $\delta\varrho_+ = \delta\varrho_+(\delta\mathcal{S}_{\text{TD}})$, while: $\delta\varrho_- = \delta\varrho_-(-\delta\mathcal{S}_{\text{TD}})$. The two functions describe two different kinds of phenomenon, namely $\delta\varrho_+$ describes ponderable matter as we know it, while $\delta\varrho_-$ describes some (exotic) non-ponderable (invisible) tachyonic matter. This shall be made clear as we go, hence the need to make a distinction of $\delta\varrho_+$ and $\delta\varrho_-$.

Now, the normalization in (1) is the probability of finding the particle in the spatial $(\hat{r})$ and momentum $(\hat{p})$ region:

$$\begin{aligned} \delta r_{\min} \leq \quad & \hat{r} \quad \leq \delta r_{\max} \\ \delta p_{\min} \leq \quad & \hat{p} \quad \leq \delta p_{\max}\,, \end{aligned} \tag{2}$$

where $\hat{r}$ and $\hat{p}$ are the actual measured radial coordinate and magnitude of the momentum of the particle as measured from the spatial canonical point of origin of the particle (system) in question.

2.1.3 Normalization across a given axis

Now, given that $\delta\varrho_+ = \exp(\delta\mathcal{S}_{\rm TD}/\hbar)$, where:

$$\delta\mathcal{S}_{\rm TD} = \delta\boldsymbol{p}\cdot\delta\boldsymbol{r} - \delta E\delta t = \delta p_\mu \delta x^\mu\,, \tag{3}$$

it follows that the quantities $\delta\varrho_x^+$, $\delta\varrho_y^+$, $\delta\varrho_z^+$, $\delta\varrho_0^+$ are such that:

$$\begin{aligned} \delta\varrho_x^+ &= \exp\left(\frac{\delta\mathcal{S}_x}{\hbar}\right) = \exp\left(\frac{\delta p_x\delta x}{\hbar}\right) \quad \ldots \quad (a)\\ \delta\varrho_y^+ &= \exp\left(\frac{\delta\mathcal{S}_y}{\hbar}\right) = \exp\left(\frac{\delta p_y\delta y}{\hbar}\right) \quad \ldots \quad (b)\\ \delta\varrho_z^+ &= \exp\left(\frac{\delta\mathcal{S}_z}{\hbar}\right) = \exp\left(\frac{\delta p_z\delta z}{\hbar}\right) \quad \ldots \quad (c)\\ \delta\varrho_0^+ &= \exp\left(-\frac{\delta\mathcal{S}_0}{\hbar}\right) = \exp\left(-\frac{\delta E\delta t}{\hbar}\right) \quad \ldots \quad (d) \end{aligned} \tag{4}$$

where oft cause $\delta\mathcal{S}_x = \delta p_x\delta x$, $\delta\mathcal{S}_y = \delta p_y\delta y$, $\delta\mathcal{S}_z = \delta p_z\delta z$, and, $\delta\mathcal{S}_0 = \delta p_0\delta x_0 = \delta E\delta t$. Clearly, written in this manner, these functions $\delta\varrho_x^+$, $\delta\varrho_y^+$, $\delta\varrho_z^+$, $\delta\varrho_0^+$ are the thermodynamic probability evolution functions describing the particle across the δx-δp_x axis, δy-δp_y axis, δz-δp_z axis, and the δt-δE axis respectively.

The probability of finding the particle along the x-p_x, y-p_y, z-p_z and t-E axis respectively, in the region of its bounds is unity and this is expressed:

$$\frac{1}{\hbar}\int_{\delta p_{\rm min}}^{\delta p_{\rm max}}\int_{\delta r_{\rm min}}^{\delta r_{\rm max}} (\delta\varrho_x^+)\,dx dp_x = 1\,, \tag{5}$$

$$\frac{1}{\hbar}\int_{\delta p_{\rm min}}^{\delta p_{\rm max}}\int_{\delta r_{\rm min}}^{\delta r_{\rm max}} \left(\delta\varrho_y^+\right) dy dp_y = 1\,, \tag{6}$$

$$\frac{1}{\hbar}\int_{\delta p_{\rm min}}^{\delta p_{\rm max}}\int_{\delta r_{\rm min}}^{\delta r_{\rm max}} \left(\delta\varrho_z^+\right) dz dp_z = 1\,, \tag{7}$$

$$\frac{1}{\hbar}\int_{\delta E_{\rm min}}^{\delta E_{\rm max}}\int_{\delta t_{\rm min}}^{\delta t_{\rm max}} \left(\delta\varrho_0^+\right) dt dE = 1\,. \tag{8}$$

Before we can deduce the Heisenberg uncertainty principle from the above equations (5)-(8), we shall lay down some necessary mathematical formulae.

2.1.4 Necessary mathematical equations

In our derivation of Heisenberg's uncertainty principle in §2.2 and §2.3, we are going to encounter the function e^{ax}/x, where x is the variable and a is some constant. Of this function, we will need to know its integral and limit as $x \mapsto 0$. It is not difficult to show that:

$$\int\left(\frac{e^{ax}}{x}\right)dx = \frac{e^{ax}}{ax} + k\,, \tag{9}$$

where k is some integration constant and:

$$\lim_{x\mapsto 0}\left(\frac{e^{ax}}{x}\right) = a\,. \tag{10}$$

Now, we are ready to derive Heisenberg's uncertainty principle (and more).

2.2 Position-momentum uncertainty

In this section, we are now going to derive a lower and upper bound uncertainty principle for momentum and position. Taking (5) and substituting $\delta\varrho_x^+ = \exp(\delta p_x\delta x/\hbar)$, we will have:

$$\frac{1}{\hbar}\int_{\delta p_{\rm min}}^{\delta p_{\rm max}}\int_{\delta r_{\rm min}}^{\delta r_{\rm max}} \exp\left(\frac{\delta p_x\delta x}{\hbar}\right) dx dp_x = 1\,. \tag{11}$$

Now, using the result of (9) to integrate (11) with respect to x, and evaluating the resulting integral, we will have:

$$\int_{\delta p_{\rm min}}^{\delta p_{\rm max}}\left(\frac{e^{\delta p_x\delta r_{\rm max}/\hbar} - e^{\delta p_x\delta r_{\rm min}/\hbar}}{\delta p_x}\right)dp_x = 1\,. \tag{12}$$

Further, we need to integrate (12) with respect to p_x. In doing so, we will encounter again an integral of the form given in (9). The result of this integration is therefore:

$$\hbar\left[\frac{e^{\delta p_x\delta r_{\rm max}/\hbar}}{\delta p_x\delta r_{\rm max}} - \frac{e^{\delta p_x\delta r_{\rm min}/\hbar}}{\delta p_x\delta r_{\rm min}}\right]_{\delta p_{\rm min}}^{\delta p_{\rm max}} = 1\,. \tag{13}$$

Evaluating this, we will have:

$$\overbrace{\frac{\hbar e^{\delta p_{\rm max}\delta r_{\rm max}/\hbar}}{\delta p_{\rm max}\delta r_{\rm max}}}^{\textbf{Term I}} - \overbrace{\frac{\hbar e^{\delta p_{\rm min}\delta r_{\rm max}/\hbar}}{\delta p_{\rm min}\delta r_{\rm max}}}^{\textbf{Term II}} - \underbrace{\frac{\hbar e^{\delta p_{\rm max}\delta r_{\rm min}/\hbar}}{\delta p_{\rm max}\delta r_{\rm min}}}_{\textbf{Term III}} + \underbrace{\frac{\hbar e^{\delta p_{\rm min}\delta r_{\rm min}/\hbar}}{\delta p_{\rm min}\delta r_{\rm min}}}_{\textbf{Term IV}} = 1\,. \tag{14}$$

Furthermore, for ponderable material particles, as discussed in §2.1.1, we want our particle system to be bounded (trapped) between the regions $0 \leq \hat{r} \leq \delta r_{\rm max}$ and $0 \leq \hat{p}_x \leq \delta p_{\rm max}$. This means that we must evaluate (14) in the limits $\delta r_{\rm min} \mapsto 0$ and $\delta p_{\rm min} \mapsto 0$.

Now, making use of the limit given (10), it follows that as:

$$\begin{aligned} \delta r_{\rm min} &\mapsto 0\,,\\ \delta p_{\rm min} &\mapsto 0\,, \end{aligned} \tag{15}$$

for the Terms I, II, III and IV in (14), we will have:

$$\begin{aligned} \text{Term I} &= \frac{\hbar e^{\frac{\delta p_{\rm max}\delta r_{\rm max}}{\hbar}}}{\delta p_{\rm max}\delta r_{\rm max}}\,,\\ \text{Term II} &\mapsto 1\,,\\ \text{Term III} &\mapsto 1\,,\\ \text{Term IV} &\mapsto 1\,, \end{aligned} \tag{16}$$

hence from (16), it follows from this that (14) will reduce to:

$$\hbar e^{\delta p_{\rm max}\delta r_{\rm max}/\hbar}/\delta p_{\rm max}\delta r_{\rm max} - 1 = 1\,,$$

where, after some re-arrangement, we will have:

$$\frac{\frac{1}{2}\hbar}{\delta p_{\max}\delta r_{\max}} = e^{-\delta p_{\max}\delta r_{\max}/\hbar}\,. \tag{17}$$

From a meticulous inspection of (17), it is clear and goes without saying that in order for this equation to hold true $\delta p_{\max}\delta r_{\max} > 0$, hence:

$$\frac{\frac{1}{2}\hbar}{\delta p_{\max}\delta r_{\max}} = e^{-\delta p_{\max}\delta r_{\max}/\hbar} < 1\,, \tag{18}$$

thus, we will have:

$$\delta p_{\max}\delta r_{\max} > \frac{1}{2}\hbar\,. \tag{19}$$

With the subscript "max" removed from $p_{\max}$ and $r_{\max}$, this (19) is without any doubt whatsoever the famous 1927 position-momentum quantum mechanical uncertainty principle of Heisenberg. One can work this out for the other three cases – i.e. for the $(\delta y, \delta p_y)$ dimension as given in (6) and the $(\delta z, \delta p_z)$ dimension as given in (7) and they would arrive at the same result.

It is important to note that the exact Heisenberg upper uncertainty principle involves a *greater than or equal to sign*, that is "≥", yet in (19), the equal sign "=" is missing. This issue shall be addressed in Paper IV where it shall be seen that this case represents only those particles that travel at the speed of light. Next, we consider the energy-time uncertainty relation.

2.3 Time-energy

Now, in §2.3.1 and §2.3.2, we are going to derive a lower and an upper bound uncertainty principle for energy and time and as we do this, we must have at the back of our mind that stable ponderable particles ought to have no upper bound in their temporal fluctuations. Yes, they can only have a lower bound in their temporal fluctuations and this lower bound must coincide with the moment of their creation. On the contrary, unstable ponderable particles ought to have a finite upper bound in their temporal fluctuation.

2.3.1 Lower bound energy-time uncertainty

We are now going to derive the energy-time uncertainty principle. The derivation is similar to the one given in §2.2 above. To that end, from (4*d*) and (8), we know that:

$$\frac{1}{\hbar}\int_{\delta E_{\min}}^{\delta E_{\max}}\int_{\delta t_{\min}}^{\delta t_{\max}} \exp\left(-\frac{\delta E\delta t}{\hbar}\right) dt dE = 1\,. \tag{20}$$

Now, using (9) to evaluate (20), we obtain the following:

$$\overbrace{\frac{\hbar e^{-\delta E_{\max}\delta t_{\max}/\hbar}}{\delta E_{\max}\delta t_{\max}}}^{\textbf{Term I}} - \overbrace{\frac{\hbar e^{-\delta E_{\min}\delta t_{\max}/\hbar}}{\delta E_{\min}\delta t_{\max}}}^{\textbf{Term II}} - \underbrace{\frac{\hbar e^{-\delta E_{\max}\delta t_{\min}/\hbar}}{\delta E_{\max}\delta t_{\min}}}_{\textbf{Term III}} + \underbrace{\frac{\hbar e^{-\delta E_{\min}\delta t_{\min}/\hbar}}{\delta E_{\min}\delta t_{\min}}}_{\textbf{Term IV}} = 1\,. \tag{21}$$

In the limit as:

$$\begin{aligned} \delta t_{\min} &\mapsto 0\,, \\ \delta E_{\min} &\mapsto 0\,, \end{aligned} \tag{22}$$

for Terms I, II, III and IV in (21), according to (10), we will have:

$$\begin{aligned} \text{Term I} &= \frac{\hbar e^{\frac{\delta E_{\max}\delta t_{\max}}{\hbar}}}{\delta E_{\max}\delta t_{\max}}\,, \\ \text{Term II} &\mapsto 1\,, \\ \text{Term III} &\mapsto 1\,, \\ \text{Term IV} &\mapsto 1\,, \end{aligned} \tag{23}$$

hence, it follows from this – that (21) will reduce to:

$$\hbar e^{\delta E_{\max}\delta t_{\max}/\hbar}/\delta E_{\max}\delta t_{\max} - 1 = 1\,,$$

where, after some algebraic re-arrangement, we can rewrite this equation as:

$$\frac{\frac{1}{2}\hbar}{\delta E_{\max}\delta t_{\max}} = e^{-\delta E_{\max}\delta t_{\max}/\hbar}\,. \tag{24}$$

Similarly, from an inspection of (24), one will clearly obtain that for this equation holds true $\delta E_{\max}\delta t_{\max} > 0$, hence:

$$\frac{\frac{1}{2}\hbar}{\delta E_{\max}\delta t_{\max}} = e^{-\delta E_{\max}\delta t_{\max}/\hbar} < 1\,, \tag{25}$$

thus:

$$\delta E_{\max}\delta t_{\max} > \frac{1}{2}\hbar\,. \tag{26}$$

Once again, this is the famous 1927 energy-time quantum mechanical uncertainty principle of Heisenberg. Just as in (19), the reason for having the *greater than sign* and not the *greater than or equal to sign* are the same as those given in the case of (19). This uncertainty relation (i.e. (26)) describes a ponderable (spatially bound) material particle that is unstable and has a lifetime τ that is such that $\tau < \delta t_{\max}$.

2.3.2 Upper bound energy-time uncertainty

Now, for the same reason given in §2.3.1, we are going to proceed further and consider the case of a ponderable material particle system that has no upper bound in its temporal fluctuations – i.e. a stable ponderable material particle system that can live forever (e.g. like an electron or a proton). Such a particle will have δt_{max} and δE_{max} being such that:

$$\begin{aligned} \delta t_{\text{max}} &\mapsto \infty\,, \\ \delta E_{\text{max}} &\mapsto \infty\,. \end{aligned} \tag{27}$$

According to (10) under the given conditions (i.e. (27)), for the Terms I, II, III and IV in (21), we will have:

$$\begin{aligned} \text{Term I} &\mapsto 0\,, \\ \text{Term II} &\mapsto 0\,, \\ \text{Term III} &\mapsto 0\,, \\ \text{Term IV} &= \frac{\hbar e^{-\dfrac{\delta E_{\text{min}}\delta t_{\text{min}}}{\hbar}}}{\delta E_{\text{min}}\delta t_{\text{min}}}\,, \end{aligned} \tag{28}$$

hence, it follows from this that (21) will reduce to:

$$\hbar e^{-\delta E_{\text{min}}\delta t_{\text{min}}/\hbar}/\delta E_{\text{min}}\delta t_{\text{min}} = 1\,,$$

where, after some basic algebraic re-arrangement, we can rewrite this equation as:

$$\frac{\hbar}{\delta E_{\text{min}}\delta t_{\text{min}}} = e^{\delta E_{\text{min}}\delta t_{\text{min}}/\hbar}\,. \tag{29}$$

As before, it is not difficult to see that for (29) to hold true, this requires that $\delta E_{\text{min}}\delta t_{\text{min}} > 0$, hence, and from this, it clearly follows that:

$$\frac{\hbar}{\delta E_{\text{min}}\delta t_{\text{min}}} = e^{\delta E_{\text{min}}\delta t_{\text{min}}/\hbar} < 1\,, \tag{30}$$

thus:

$$\delta E_{\text{min}}\delta t_{\text{min}} < \hbar\,. \tag{31}$$

Insofar as its interpretation is concerned, by no stitch of the imagination is this (31) related to the famous 1927 energy-time quantum mechanical uncertainty principle of Heisenberg and this is so because of the *less-than-sign* "<" appearing in it. What this equation is "telling" us is that the energy and time fluctuations are not bound above, but below. When it comes to the lifetime of the particle in question, this translates to the reality that the particle can live forever – i.e. $\tau = \infty$. Therefore, this uncertainty relation describes stable ponderable particle systems – i.e. ordinary electrons and protons, which by-and-large strongly appear to be stable particle systems.

3 Non-ponderable matter

From a symmetry and *bona fide* mathematical standpoint, if we have the physics of particles described by the thermodynamic phase $+\delta\mathcal{S}_{\text{TD}}$, there surely is nothing wrong, but everything natural and logical for one to consider the physics of particle systems described by the opposite thermodynamic phase – i.e. $-\delta\mathcal{S}_{\text{TD}}$. Such necessary and beautiful symmetry considerations is what lead the great English theoretical physicist – Paul Adrian Maurice Dirac (1902-1984) to foretell the existence of antimatter [9–11]. We here consider the said particle systems whose thermodynamic phase is $-\delta\mathcal{S}_{\text{TD}}$.

Before even going into investigating the said particle systems, natural questions will begin to flood the mind, questions such as: *Will such particles violate the FLT?* The answer is: *No, they will not*. To see this, one simply substitutes $-\delta\mathcal{S}_{\text{TD}}$ into the equations of Paper II, where-from they certainly will come to the inescapable conclusion that these particles will indeed obey the FLT. Further – a question such as: *Will these particle systems violate the SLT?* may also visit the curious and searching mind. An answer to this will be provided in Paper IV.

Furthermore – in the extreme and zenith of one's state of wonderment, they might excogitate: *Will such particles be visible and ponderable?* By visible it is understood to mean: will these particle systems emit or reflect electromagnetic radiation that we are able to sense? And by ponderable, we mean will such particle systems be able to clump-up and form touchable materials like rocks, *etc*? This is the question we are going to answer. To preempt our findings, such particle systems will be invisible and non-ponderable.

To commence our expedition, we shall start by writing down the functions $\delta\varrho_x^-$, $\delta\varrho_y^-$, $\delta\varrho_z^-$, $\delta\varrho_0^-$ and these are such that:

$$\begin{aligned} \delta\varrho_x^- &= \exp\left(-\frac{\delta\mathcal{S}_x}{\hbar}\right) = \exp\left(-\frac{\delta p_x\delta x}{\hbar}\right) \;\ldots\; (a) \\ \delta\varrho_y^- &= \exp\left(-\frac{\delta\mathcal{S}_y}{\hbar}\right) = \exp\left(-\frac{\delta p_y\delta y}{\hbar}\right) \;\ldots\; (b) \\ \delta\varrho_z^- &= \exp\left(-\frac{\delta\mathcal{S}_z}{\hbar}\right) = \exp\left(-\frac{\delta p_z\delta z}{\hbar}\right) \;\ldots\; (c) \\ \delta\varrho_0^- &= \exp\left(\frac{\delta\mathcal{S}_0}{\hbar}\right) = \exp\left(\frac{\delta E\delta t}{\hbar}\right) \;\ldots\; (d) \end{aligned} \tag{32}$$

Now, just as in the case of ponderable matter in the previous section, in order for us to derive the implied uncertainty relations from (32), we are going to consider (in §3.1, §3.2 and §3.3, respectively) the normalization of $\delta\varrho_x^-$ and $\delta\varrho_0^-$.

3.1 Lower bound position-momentum uncertainty

As before, normalization of $\delta\varrho_x^-$ requires that:

$$\frac{1}{\hbar}\int_{\delta p_{\text{min}}}^{\delta p_{\text{max}}}\int_{\delta r_{\text{min}}}^{\delta r_{\text{max}}}\exp\left(-\frac{\delta p_x\delta x}{\hbar}\right)dx\,dp_x = 1\,. \tag{33}$$

Just as we have already done with (11) and (20); integrating and evaluating (33), we obtain:

$$\overbrace{\frac{\hbar e^{-\delta p_{\max}\delta r_{\max}/\hbar}}{\delta p_{\max}\delta r_{\max}}}^{\textbf{Term I}} - \overbrace{\frac{\hbar e^{-\delta p_{\min}\delta r_{\max}/\hbar}}{\delta p_{\min}\delta r_{\max}}}^{\textbf{Term II}} - \underbrace{\frac{\hbar e^{-\delta p_{\max}\delta r_{\min}/\hbar}}{\delta p_{\max}\delta r_{\min}}}_{\textbf{Term III}} + \underbrace{\frac{\hbar e^{-\delta p_{\min}\delta r_{\min}/\hbar}}{\delta p_{\min}\delta r_{\min}}}_{\textbf{Term IV}} = 1. \quad (34)$$

Likewise, with (34) in place, one may try to bound the particle in space and momentum, in much the same way as it has been done in §2.2 by instituting the asymptotic conditions $\delta r_{\min} \mapsto 0$ and $\delta p_{\min} \mapsto 0$. So doing, they surely would obtain the unpleasant result:

$$\hbar e^{-\delta p_{\max}\delta r_{\max}/\hbar}/\delta p_{\max}\delta r_{\max} = 0\,.$$

This result is surely unpleasant because it means that we must have $\delta p_{\max}\delta r_{\max} = \infty$. Overall, this means that this particle system has no upper bounds in quantum of action $\delta p_{\max}\delta r_{\max}$; this surely is uncomfortable as the quantum of action must be bound either above or below. Given this uncomfortable result $\delta p_{\max}\delta r_{\max} = \infty$, a much better way to approach this particle system is to start off by setting no upper bounds in space and momentum and in the end obtain finite lower bounds in the quantum of action $\delta E\,\delta t$, that is to say, start off by setting:

$$\begin{aligned} \delta r_{\max} &\mapsto \infty\,, \\ \delta p_{\max} &\mapsto \infty\,. \end{aligned} \quad (35)$$

Instituting the above (35) limits into (34), for the Terms: (I), (II), (III) and (IV), one obtains:

$$\begin{aligned} \text{Term I} &\mapsto 0\,, \\ \text{Term II} &\mapsto 0\,, \\ \text{Term III} &\mapsto 0\,, \\ \text{Term IV} &= \frac{\hbar e^{-\frac{\delta p_{\min}\delta r_{\min}}{\hbar}}}{\delta p_{\min}\delta r_{\min}}\,, \end{aligned} \quad (36)$$

hence:

$$\frac{\hbar e^{-\frac{\delta p_{\min}\delta r_{\min}}{\hbar}}}{\delta p_{\min}\delta r_{\min}} = 1\,. \quad (37)$$

In much the same fashion as in the preceding sections, re-arranging this (37), we will have:

$$\frac{\hbar}{\delta p_{\min}\delta r_{\min}} = e^{\frac{\delta p_{\min}\delta r_{\min}}{\hbar}} > 1\,, \quad (38)$$

hence:

$$\delta p_{\min}\delta r_{\min} < \hbar\,. \quad (39)$$

This means the fuzziness in the momentum and spatial location of the particle about its canonical centre is bounded above and not below.

3.2 Lower bound energy-time uncertainty

Further, for the energy-time uncertainty relation, normalization of $\delta\varrho_0^-$ requires that:

$$\frac{1}{\hbar}\int_{\delta E_{\min}}^{\delta E_{\max}}\int_{\delta t_{\min}}^{\delta t_{\max}} \exp\left(\frac{\delta E\delta t}{\hbar}\right) dt dE = 1\,. \quad (40)$$

As before, integrating and evaluating this (40), we obtain:

$$\overbrace{\frac{\hbar e^{-\delta E_{\max}\delta t_{\max}/\hbar}}{\delta E_{\max}\delta t_{\max}}}^{\textbf{Term I}} - \overbrace{\frac{\hbar e^{-\delta E_{\min}\delta t_{\max}/\hbar}}{\delta E_{\min}\delta t_{\max}}}^{\textbf{Term II}} - \underbrace{\frac{\hbar e^{-\delta E_{\max}\delta t_{\min}/\hbar}}{\delta E_{\max}\delta t_{\min}}}_{\textbf{Term III}} + \underbrace{\frac{\hbar e^{-\delta E_{\min}\delta t_{\min}/\hbar}}{\delta E_{\min}\delta t_{\min}}}_{\textbf{Term IV}} = 1\,. \quad (41)$$

In the limit as:

$$\begin{aligned} \delta t_{\max} &\mapsto \infty\,, \\ \delta E_{\max} &\mapsto \infty\,, \end{aligned} \quad (42)$$

for the Terms I, II, III and IV in (41), we will have:

$$\begin{aligned} \text{Term I} &\mapsto 0\,, \\ \text{Term II} &\mapsto 0\,, \\ \text{Term III} &\mapsto 0\,, \\ \text{Term IV} &= \frac{\hbar e^{-\frac{\delta E_{\min}\delta t_{\min}}{\hbar}}}{\delta E_{\min}\delta t_{\min}}\,, \end{aligned} \quad (43)$$

hence, it follows from this that (41) will reduce to:

$$\hbar e^{-\delta E_{\min}\delta t_{\min}/\hbar}/\delta E_{\min}\delta t_{\min} = 1\,,$$

where, after some re-arrangement, we can rewrite:

$$\frac{\hbar}{\delta E_{\min}\delta t_{\min}} = e^{\delta E_{\min}\delta t_{\min}/\hbar}\,. \quad (44)$$

As before, from a meticulous inspection of (44), it is abundantly clear that $\delta E_{\min}\delta t_{\min} > 0$, hence:

$$\frac{\hbar}{\delta E_{\min}\delta t_{\min}} = e^{\delta E_{\min}\delta t_{\min}/\hbar} < 1\,, \quad (45)$$

thus:

$$\delta E_{\min}\delta t_{\min} < \hbar\,. \quad (46)$$

Just as with (39), (46) means that the fuzziness in the energy and temporal fluctuations of the particle are bounded above and not below.

3.3 Upper bound energy-time uncertainty

Lastly, we now consider the case of a non-ponderable material particle system that has no upper bound in its temporal fluctuation – i.e. a stable non-ponderable material particle system that can live forever. Such a particle will have $\delta t_{\max}$ and $\delta E_{\max}$ such that:

$$\begin{aligned} \delta t_{\max} &\mapsto \infty\,, \\ \delta E_{\max} &\mapsto \infty\,. \end{aligned} \tag{47}$$

Under the given conditions (i.e. (47)), for the Terms I, II, III and IV in (41), we will have:

$$\begin{aligned} \text{Term I} &\mapsto 0\,, \\ \text{Term II} &\mapsto 0\,, \\ \text{Term III} &\mapsto 0\,, \\ \text{Term IV} &= \frac{\hbar e^{-\frac{\delta E_{\min}\delta t_{\min}}{\hbar}}}{\delta E_{\min}\delta t_{\min}}\,, \end{aligned} \tag{48}$$

hence, it follows from this that (21), will reduce to:

$$\hbar e^{-\delta E_{\min}\delta t_{\min}/\hbar}/\delta E_{\min}\delta t_{\min} = 1\,,$$

where, after some re-arrangement, we can rewrite:

$$\frac{\hbar}{\delta E_{\min}\delta t_{\min}} = e^{\delta E_{\min}\delta t_{\min}/\hbar}. \tag{49}$$

Likewise, for it to hold true always, (49) requires that $\delta E_{\min}$ $\delta t_{\min} < 0$, hence:

$$\frac{\hbar}{\delta E_{\min}\delta t_{\min}} = e^{\delta E_{\min}\delta t_{\min}/\hbar} < 1\,, \tag{50}$$

thus:

$$\delta E_{\min}\delta t_{\min} < \hbar\,. \tag{51}$$

Again, we here have an upper bounded uncertainty relation.

4 General discussion

Since the inception of Heisenberg's uncertainty principle in 1927, several attempts see e.g. [8,12–15, and references therein] have been made to derive this mysterious mathematical relationship from much more fundamental soils of physics than those on which Heisenberg [1] derived this relation. In his original paper, Heisenberg began by deriving the uncertainty relation for position and momentum on the basis of a supposed experiment in which an electron is observed using a γ-ray microscope and second, by consideration of the theory of the Compton effect, he proceeded to argue that the precision of the determination of position and momentum are connected by the uncertainty relation.

In 1929, using the usual definition of expectation values (inner product) of Hermitian Hilbert-space operators (observables) and the mathematical property of the *Cauchy–Bunyakovsky–Schwarz inequality*, Robertson [12] proceeded in a rigorous manner, to demonstrate a more general and fundamental origin of the quantum mechanical uncertainty principle. The present attempt is just but one such derivation – albeit – on the soils of a new kind of phase space – the Stochastic Phase Space.

However, unlike all previous attempts on the derivation of the uncertainty principle, what makes the present attempt different is that we have not only derived the lower limit uncertainty principle, but an upper bound uncertainty principle that seems to describe invisible non-ponderable particles that travel at superlumical speeds. This unique prediction seem to suggest not only the existence of darkmatter, but darkenergy as well. Dark matter is already required by physicists in order to explain the flat rotation curves of spiral galaxies, while dark energy is required to explain the supposed accelerated expansion of the Universe. This subject of invisible non-ponderable particles, dark matter and dark energy would require a separate and lengthy paper in order to cover it in a just manner.

Another important point to note about the present derivation is that the enigmatic jittery quantum randomness leading to the uncertainty principle is here an intrinsic and inherent property of all quantum mechanical systems, it (i.e. the jittery quantum randomness) is not induced by the act of measurement as is the case of Heisenberg's uncertainty principle and its latter versions or attempts at a derivation of this relation. Yes, human measure will introduce statistical errors that are statistically predictable. The stochastic quantum randomness is not predictable at all – not even by the most rigours known (or unknown, or yet to be unknown) statistical methods.

In closing, allow us to say that we have always held central to our philosophy of Physics the strong and seemingly unshakeable belief system similar to that of Albert Einstein – namely, that the fundamental laws of Nature are exact, and as such, one day it will be shown that this is the case. That is to say, in the character of Einstein's philosophy, we have held fast to his influential and deep philosophy that indeed *God does not play dice with the World*, and that *The moon exists whether or not one is looking at it or not*. Contrary to this, we must admit and say that as we continue to peer deeper into the fabric and labyrinth of physical and natural reality as it lies bare for us to marvel at, this dream or belief system now stands shattered into minuscule pieces – for it now seems clearer to us that the enigmatic jittery quantum randomness must be real.

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