Symmetry Breaking Model of Volume Pulsating Walking Droplets

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In this article, we propose a generalized model of dynamic of extended pulsating walking droplets. In the first section, we provide a brief overview of the open problems of walking droplets. In the second section, we analyze some critical issues of the general stroboscopic models. In the third section, we elaborate our proposal of a generalized model of pulsating droplets. Finally, we suggest a link between walking droplets dynamic and the acoustic gravity wave induced on the surface of the vibrating bath.

1 Open problems of walking droplets

In the last fifteen years, the classical study of hydrodynamical Faraday waves has attracted great renewed interest since the discovery of walking droplets and the more general discovery of hydrodynamical pilot wave models [1,2]. Notwithstanding that many papers have cleared and rationalized a lot of phenomena with similarities to quantum mechanics (wave/particle duality, discrete orbits, tunnelling effect, statistical properties, *etc* [3, 4, 6, 7]), we propose that something is missing in the general approach to these issues. For example, as far as the authors are aware, there are no papers which explore an hydrodynamic analogue of the Planck law or of the de Broglie hypothesis or an analogue of the Born statistical interpretation of the wavefunction.

In particular, we propose that the role of volume pulsations and bath deformation may be caused by the impact of the droplet and its influence on the dynamic and conservation of the momentum of the global systems should be explored.

From our point of view, the problem of defining the total momentum of the particle-wave coupled system is deserving of closer study, since we believe that not only the momentum of droplet and the vibrating bath must be considered, but also those of the surface acoustic wave produced by the impact and of the vibrating borders of the vessel.

Our proposal is that the symmetry breaking force of the transition from the bouncing to the walking regime could be due to space asymmetries of one between the deformation of droplet, or the bath's deformation or the acoustic wave pattern or to an asymmetric vibration of the border.

In fact, the actual modellization of the transition between the bouncing regime and the walking regime is based on the surface orography of the vibrating bath, but this model does not yet justify the mechanism by which the surface has a broken symmetry and moreover it assumes that the droplet is punctiform. We assume that the surface bath geometry asymmetry is caused by an acoustic gravity wave and not just by a surface gravity wave [20].

Furthermore, at present there is no model that has a frequency dependent broken symmetry mechanism.

Finally, although there are some experimental studies of the droplet volume pulsation, presently we lack a model that tries to implement this experimental fact. In the following section, we analyze some critical aspects of the stroboscopic model which we believe are yet to be explored.

2 Critical aspects of stroboscopic models

The stroboscopic model of Bush-Molaceck and the generalized integral model of Oza [9, 11, 13] has been till now the most successful and most used model to rationalize walking droplets.

The two major hypotheses on which it is based are the following [5]:

1) The bath height oscillations are described by standing monochromatic waves.

2) The bath Faraday wave field is resonant with the bouncing oscillations (the mode is (2,1)).

The efforts to improve and generalize this model are stimulated by the desire to extend it to multiple droplets dynamics and to describe more accurately the spatio-temporal decay of the bouncing induced Faraday waves.

In the following, we will describe some other hypotheses which we consider need to be better justified and maybe generalized.

The general approach to describe droplet-bath dynamics is to separate the horizontal and the vertical dynamics during flight; on the contrary, we believe that if we want to describe more accurately the real spatio-temporal extended impact between the drop and the bath, we have to consider the successive volume oscillations of the droplet and the acoustic waves beneath the surface bath.

In fact, they persist after the impacts and therefore implement a dynamical memory dependent coupling which moreover hides some energy and momentum whose conservation may be deepened.

The first stroboscopic model [9] contained discrete sums of Bessel functions describing the wavefield and used in the trajectory equation averaged over the bouncing period:

$$m\ddot{x}_i + D\dot{x}_i = -mg\,S\,(h_i(x_i,t))\,\nabla h_i(x_i,t) \tag{1}$$

where D is the drag coefficient, h is the bath height and $S = \sin\Phi$ is the impact phase which is dependent on the mean phase of the wave during the drop contact.

In particular, this model assumes that the height of the vibrating bath is given by a linear superposition of n circular waves each one generated by the drop impact described by the following relation:

$$h(\mathbf{x}, t_N) = \sum_{-\infty}^{N-1} \frac{Ae^{-(x-x_n)/\delta}}{|x - x_n|^{-1/2}}\, e^{-(t_N - t_n)/\tau} \cos(k_F |\boldsymbol{x} - \boldsymbol{x}_n| + \Phi)\,. \quad (2)$$

Recently, some authors [12] have proposed generalizations based on the mean wave field, but all the generalizations are based on the hypothesis of instantaneous and punctiform gradient of the surface wave slope and are aimed to rationalize the wavelike statistics of irregular unstable orbits.

Finally, we wish to note that thus far we lack a self-consistent explanation of the origin of the symmetry breaking force and its associated horizontal momentum transfer.

In the following section, we want to discuss a proposal which attempts to overcome these difficulties and to connect this problem to the search of an energy minimization principle which could explain the main features of the walking droplets stable orbits.

3 Generalized stroboscopic model

We propose to generalize the stroboscopic model by introducing a horizontal force which depends on the frequency and the volume pulsation of the droplet. In particular we implement a memory dependent force taking into account the previous volume oscillation.

Given an horizontal plane of the non-vibrating bath represented by x and y, our generalized symmetry breaking force starting from [10] is the following:

$$\vec{F}_{xy} = \int_{t-t_0}^{t} \nabla p\, \dot{V}\, d\tau = m\, \frac{\Delta \vec{v}_{xy}}{\tau_0} \quad (3)$$

where:

- t_0 is the impact time of the droplet with the bath and it is the inverse of the frequency of the volume pulsation;
- V is the volume of the droplet and $\dot{V}$ is the derivative with respect to time;
- ∇p is the gradient of the bath pressure wave.

This force disappears when $t_0 = 0$, while it converges to that one of the stroboscopic model when the frequency of the pulsation is 0.

Our proposal assumes that this force is present only during the impact and that the pressure on the droplet is due to the potential gravitational energy of the deformed bath.

In fact, differently from the Bush-Molacek model, the real geometrical profile of the vibrating path during the impact is no more sinusoidal. The bath absorbs elastic energy from the bouncing droplet during the impact and consequently it is deformed.

The height difference between the sinusoidal profile and the modified profile gives the potential energy to the deformed droplet.

Our hypothesis is that the pressure p and the height difference are given by the following formula derived from the theorem of conservation of the fluid energy:

$$p + \rho_{bath}\, g_{eff}\, \Delta h = \cos t \quad (4)$$

where ρ is the bath density, g_{eff} is the same used in the stroboscopic model [13] (also denoted as g_*) and p is the pressure induced in the bath after the droplet's impact.

This equation can be generalized since the external vibrating force continuously adds energy to the bath:

$$p + \rho_{bath}\, g_{eff}\, \Delta h = \alpha(t) \quad (5)$$

where $\alpha(t)$ is a periodic function dependent on the oscillatory force and on the volume deformation; Δh is the variation of the harmonic oscillation of the height of the bath caused by the impact of the droplet.

The introduction of this force (which is present only during the impact) requires a generalization of the horizontal dynamics of the walking droplet. Moreover we continue to assume the usual vertical periodic dynamic of stroboscopic model.

If during the impact, we apply to Newton equation (3) using the formalism of the finite difference instead of the derivative, the gradient operator to (5), we arrive at the following model (since we assumed that α depended only on the time t):

$$\begin{aligned} m\, \frac{\Delta \vec{v}_{xy}}{\tau_0 + T_F} + \int_{t-\tau_0}^{t+T_F} D\, \frac{\vec{v}_{xy}\, \dot{V}\, d\tau}{\Delta V} &= \\ = -\int_{t-\tau_0}^{t+T_F} \rho\, \nabla(g_{eff}\, \Delta h)\, \dot{V}\, d\tau & \end{aligned} \quad (6)$$

where:

- T_F is the inverse of the Faraday frequency of the vibrating bath;
- the instantaneous acceleration used by the stroboscopic model has been substituted by the finite difference variation of the velocity during the impact time τ_0;
- $V(t)$ is the time dependent volume pulsation of the droplet that can be assumed to be described by the following formula:

  $$V(t) - V_0 = V_0\, \cos(\omega t)\, e^{-\lambda t}$$

  an exponential decay of an harmonic oscillation with ω the frequency of droplet self-mode oscillation and λ the time decay coefficient;

- g_{eff} is the asymmetric effective gravity dependent on the local frequency given by the following relation:

$$g_{eff} = \gamma \, \sin[2\pi f(x)\, t]$$

  where $f(x)$ is the space-dependent local frequency caused by the asymmetric acoustic wave interference process not considering the dissipation;
- the first integral is a temporal average of the drag force over the past volume pulsation of the droplet of the drag force;
- the second integral has been obtained from (4);
- the gradient in the last integral is due to space asymmetry of the effective gravity of the bath, which we hypothesized could be associated to a space dependence of the bath vibrating frequency.

It is interesting to show that it is possible to recover the main aspect of the stroboscopic model in the following way:

- the first term $m \frac{\Delta \vec{v}_{xy}}{\tau_0 + T_F}$ gives the usual discretized acceleration when the impact time τ_0 goes to zero;
- the second term $\int_{t-\tau_0}^{t+T_F} D \frac{\vec{v}_{xy} \dot{V} d\tau}{\Delta V}$ becomes the dissipative term during the flight when the impact time goes to zero;
- the second member is able to reproduce the slope gradient term $\overline{-F(t)\, \nabla h(x_p, t)}$ introduced by Bush *et al* [11] when τ_0 tends to zero and applying the gradient to (4); in the stroboscopic model the effective gravity is assumed to be space-independent differently from our model.

This model, of course, contains a hidden variable that is the space-dependent frequency vibrating of the bath. This variable allows to fit the numerical model in order to be in agreement with the stroboscopic model, but could be deduced by coupling (6) with another law that relates the pressure with the volume pulsation, assuming that $\alpha(t)$ of (5) is proportional to the second time derivative of the droplet volume [16].

On the contrary to the stroboscopic models, we don't make any ad hoc assumption on the geometric pattern of the surface wave since we think that it should be deduced by investigating experimentally the acoustic spectrum of the surface acoustic gravity wave.

Among many ad hoc and arbitrary hypotheses, we think that a simple option could be the sound emission law taken from [16]:

$$\phi = -\frac{\dot{V}(t)}{4\pi r} \tag{7}$$

where r is the position of a point with respect to the initial impact of the droplet and ϕ is the usual velocity potential of the bath that is related to the effective gravity described by the following formula:

$$\nabla \dot{\phi} = a \cdot g_{eff} \tag{8}$$

with a a dimensional constant.

Finally, we assume that the oscillating acoustic pressure perturbation and the acoustic velocity field obey the following equations of motion:

$$\rho \, d_t \vec{v} = -\nabla p \,, \quad \beta \, d_t p = -\nabla \cdot \vec{v} \,. \tag{9}$$

where ρ is the density of the bath, p is the acoustic pressure, $d_t \vec{v}$ is the convective temporal derivative of the moving fluid, β is the inverse of B the bulk modulus of the acoustic pressure wave [15]; this is a self-consistent system of partial differential equations which determines the coupled dynamic of the system.

This choice is motivated by the link between an oscillating volume and the generation of an acoustic spin wave in a fluid as described in [19]. We suggest that it could be interesting for the experimental researcher to study the change of the acoustic spectrum during the transition from the bouncing regime to the walking regime and could be an operative way to verify or, eventually, falsify the general model proposed.

4 Conclusions

We have studied the problem of the origin of the symmetry breaking force that causes the asymmetry of the wave pattern of vibrating bath. We propose a generalized stroboscopic model of an extended and deformable walking droplet.

In particular, our proposal is based on the hypothesis that each bounce generates an acoustic gravity on the surface and its asymmetric reflection causes a space dependent bath vibrating frequency.

Recently a new class of walking droplets, called superwalkers, have been discovered [21]. These new observations show a strong correlation between the volume of the droplet and the duration of the impact with the velocity of the walking droplet. This property may be interpreted as an indirect confirmation of our hypothesized coupling between the volume deformation and the droplet dynamic.

We hope that our approach will stimulate more extensive experimental research on the energy of the global system (droplet and vibrating bath).

In particular we think that all the models lack an explanation of the role of the energy and its non-conservation and minimization on the discrete orbit of the walking droplets; in fact, the dynamics of stroboscopic models of walking droplets is based on empirical models and not on a general variational principle of this peculiar dissipative system.

Our insight is that the energy and the impulse of the horizontal motion of the walking droplets are associated to the volume oscillation and the deformation of the bath which induces an acoustic gravity wave with momentum and energy.

Furthermore, our opinion is that the volume oscillation would induce density waves in the bath whose turbulence could be explained by onset of turbulence as studied by Francois *et al* [17], whose origin could be caused by helicoidal under surface sound waves.

We think that this hidden energy due to volume pulsation could be experimentally investigated studying the relation with the momentum of the under bath acoustic wave; it is fascinating to speculate that the law behind this could be given by an acoustic hydrodynamic de Broglie-like relation inherent to the energy of droplet volume pulsation:

$$m\,\Delta v_{xy} = H\,k \tag{10}$$

where the first member refers to the kinematic momentum of the droplet, and the second member is related to the acoustic wave momentum with H the hydrodynamic analogue of the Planck constant.

Finally, we think that it could be useful to explore experimentally the possibility to induce the transition from the bouncing regime to the walking regime, making oscillating the vessel keeping constant the frequency and the modulus of the shaker vertical acceleration; we expect that there will be a critical phase transition in a preferred direction from the bouncing to the walking regime.

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