Gamow Theory for Transmission and Decay of Unbound Diprotons

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Transmission and decay of unbound diprotons have been investigated in accordance with the Gamow theory for the quantum tunneling and radioactive decays. It is shown that a diproton, once formed, will be quickly decayed with two typical decay modes: (1) the proton decay, which causes the diproton to be separated into two separate protons and (2) the β^+ decay, which causes the diproton to be changed and fused into a deuteron after emitting a positron and a neutrino. For both of the decay modes, the transmission probabilities rapidly increase with the energy of the emitted particle. The β^+ decay from a diproton is much rarer ($< 10^{-4}$ times less) in general than the proton decay. The lifetimes for both of the two decay modes slowly decrease with the energy of the emitted particle and are extremely short to about 10^{-21} s. In addition, we have also modeled the diproton decay of a typical proton-rich radioactive heavy nucleus such as ^{15}Ne and obtained result of lifetimes consistent with measurements

1 Introduction

Helium-2 or ^{2}He is an isotope of helium. Its nucleus consists of only two protons and is usually called a diproton. It is extremely unstable and believed to be in an unbound state with a negative binding energy due to the spins of the two protons to be anti-aligned according to the Pauli Exclusion Principle [1, 2]. A diproton can be formed in two ways: (1) by combination of two separate protons or (2) by decay from radioactive heavy nuclei. Two separate protons, when they collide with enough energy to tunnel through the Coulomb barrier between them, form a diproton, $^1\mathrm{H}+^1\mathrm{H}+\mathrm{Energy} \longrightarrow {}^2\mathrm{He}$. On the other hand, some proton-rich (or neutron-rare) heavy nuclei have been experimentally found to emit diprotons. For instances, the radioactive nuclei ^{15}Ne and ^{11}O can decay, respectively, to ^{13}O and ^{9}C after emitting a diproton [3,4]. This type of event for a diproton to be emitted from a radioactive nucleus is usually called the diproton decay.

A diproton, once formed via either one of the two ways as described above, will quickly decay through either one of the two different modes [5]. It most likely undergoes a proton decay to change immediately back to two separated protons, $^2\mathrm{He} \longrightarrow {}^1\mathrm{H} + {}^1\mathrm{H}$, with a probability greater than 99.99%. In this case, both of the emitted particle and the leftover nucleus are protons. The formed diproton can also very rarely undergo a positron (or β^+) decay and get fused to form a deuteron, $^2\mathrm{He} \longrightarrow {}^2\mathrm{H} + e^+ + \nu_e$, with a probability less than 0.01%. In this case, one of the two protons in the formed diproton decays to a neutron after emitting a positron and a neutrino. Meanwhile, the neutron immediately fuses with the other proton to form a deuteron and release nuclear energy. It can be seen that the β^+ decay of diprotons is much rarer (about ten thousand or more times rarer) than the proton decay of diprotons. The lifetime of a diproton is extremely short and believed to be much much less than 10^{-9} s. Up to now, scientists have only provided these upper bound values for both of the rareness of β^+ decay and the lifetime of diprotons. The actual rareness of the β^+ decay and the lifetime of diprotons are still uncertain.

The Sun is a giant natural fusion reactor with an emission power of 3.85×10^{26} W from the nuclear fusion of its core's 1.2×10^{56} protons at a rate of about 3.6×10^{38} protons per second to produce helium nuclei or α-particles [6]. A diproton is an intermediate in the first step of the proton-proton chain nuclear reaction that is occurring in the cores of stars including our Sun. Therefore, the instability of diprotons critically affects the rate of nuclear fusion reactions in the core of the Sun. From classical physics, no proton should be able to overcome the 820 keV Coulomb barrier between protons to form a diproton and then get fused in the Sun's core, where the temperature is about 1.5 keV. According to Gamow's theory or model for the quantum tunneling probability [7], however, one part per million of the core's protons can penetrate or tunnel through the Coulomb barrier to form diprotons. Considering the high ion-collision frequency (over about 20 terahertz), one can find approximately 10^{63} sufficient collisions for diprotons to be formed in one second in the core of the Sun. Even though as mentioned above less than 0.01% of diprotons are fused to deuterons via the β^+ decay, the fusion reaction rate in the core of the Sun is still around 10^{21} times higher in magnitude than the actually observed fusion reaction (or power emission) rate. This extremely high fusion rate would lead the Sun to have an intensive explosion, if there does not exist any other fusion inhibitors.

Recently, the author proposed that the plasma waves, globally destabilized in the core of the Sun, can significantly reduce the nuclear fusion reaction rate to the observed power emission rate or luminosity and thus effectively prevent the Sun from an instantaneous explosion [8]. Through significantly reducing the electric permittivity of the core plasma, plasma waves can extremely raise the Coulomb barrier and shift the Gamow peak to a higher energy of particles to extremely inhibit the fusion reaction. It has been shown that, if the frequency of plasma waves that are globally generated in the core plasma of turbulences is about 1.28 times the plasma frequency, the Sun can have the actual fusion rate or shine on at the currently observed luminosity. This implies that, in addition to the quantum tunneling effect and rareness of β^+ decay, plasma waves are also playing the essential role in solar nuclear fusion and power emission.

In this paper, we study the transmission and lifetime for the proton and β^+ decays of unbound diprotons according to the Gamow theory for the quantum tunneling. We obtain that the transmission probability and lifetime of unbound diprotons depend on the energy of the emitted or decayed particles. When the energy of emitted protons is about 800 keV or higher, more than 99.99% of diprotons will decay into separate protons. When the energy of emitted positrons is about 10 eV or lower, less than 0.01% of diprotons will decay and fuse to deuterons. The lifetimes of a diproton via both of the two decay modes decrease with the energy of emitted particles and are about 10^{-21} s or shorter. The speeds of a proton with hundreds of keV and an electrons with several eV are typically valued at about 10^6 m/s.

2 Gamow theory for transmission and decay of diprotons

In 1928, George Gamow proposed a theory for α-decay of radioactive heavy nuclei [7]. Since the α particle, i.e. the helium nucleus, is a positively charged particle (with charge $Z_1 e$, where $Z_1 = 2$ for the α particle), it will be electrically repelled by and further escape from the leftover nucleus (with charge $Z_2 e$). Here Z_1 and Z_2 are the atomic numbers of the nuclear elements or the proton number in the nucleus of the emitted particle and the leftover nucleus, $\epsilon_0 = 8.85 \times 10^{-12}$ $C^2/(J\ m)$ is the permittivity of free space, and $e = 1.6 \times 10^{-19}$ C is the charge of the proton. Gamow's theory approximately modeled the potential energy by a finite potential square well to represent the attractive nuclear force and joined with a Coulomb repulsive potential tail [9],

$$V(r) = \begin{cases} -V_0 & \text{for } 0 < r < r_1 \\ \frac{1}{4\pi\epsilon_0}\frac{Z_1 Z_2 e^2}{r} & \text{for } r_1 < r < \infty \end{cases} . \tag{1}$$

Fig. 1 sketches the potential energy $V(r)$ given by (1) as a function of radial distance r in all the classical and quantum regions. The width of the potential square well is noted by r_1, which is determined by the radius of the nucleus or by the sum of the radii of both the emitted particle and the leftover nucleus. The depth of the potential square well is noted by V_0, which is much greater than the maximum height of the Coulomb barrier, U_c. The outer turning point (i.e. r_2) can be determined, in terms of the energy E of the emitted α particle to be equal to the potential energy at r_2, by

$$r_2 = \frac{4\pi\epsilon_0 E}{Z_1 Z_2 e^2} . \tag{2}$$

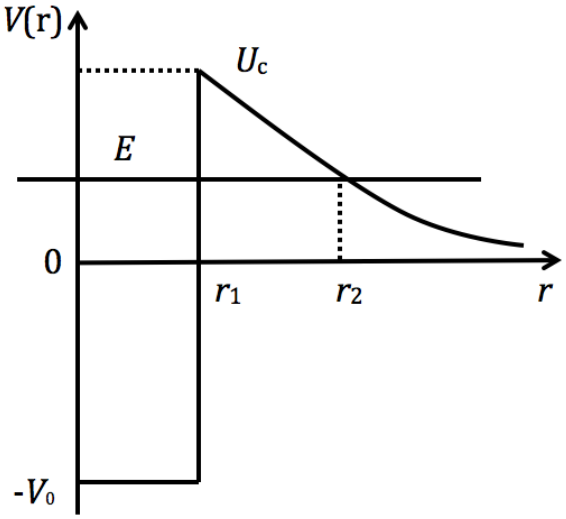

Fig. 1: Gamow's modeling of the potential energy for an electrically charged particle to decay or be emitted from a radioactive nucleus. It consists of the potential energy square well for the attractive nuclear force and the Coulomb potential energy tail for the repulsive electric force between the emitted particle and the leftover nucleus of the decay.

In the central potential $V(r)$, the radial Schrödinger equation is,

$$\frac{d^2u(r)}{dr^2} = \frac{2\mu}{\hbar^2}[V(r) - E]u(r) + \frac{l(l+1)}{r^2}u(r) , \tag{3}$$

where $u(r)$ is the radial wave function, μ is the reduced mass, $\mu = m_1 m_2/(m_1 + m_2)$ with m_1 the mass of the emitted particle and m_2 the mass of the leftover nucleus. The integer l is the quantum number for the magnitude of angular momentum and $\hbar$ is defined by $\hbar = h/2\pi$ with $h = 6.62 \times 10^{-34}$ J s, the Planck constant. A two-body system with a central force or potential can be treated as a system of one body with the reduced mass.

Applying the WKB approximation and considering the case of $l = 0$, one can approximately solve the radial Schrödinger equation and find the radial wave functions to be

$$u(r) = \frac{C}{\sqrt{|p(r)|}} \exp\left[\pm\frac{1}{\hbar}\int |p(r)| dr\right] , \tag{4}$$

where $p(r)$ is defined by

$$p(r) = \sqrt{E - V(r)} . \tag{5}$$

Here it should be pointed out that the general solution of the radial Schrödinger equation should be the combination of these two.

Then, from the solved wave function, the transmission (or tunneling) probability is obtained as

$$T = e^{-2\gamma} , \tag{6}$$

where γ is determined by

$$\begin{aligned}\gamma &= \frac{1}{\hbar}\int_{r_1}^{r_2} dr\,\sqrt{E - V(r)} \\ &= \frac{\sqrt{2\mu E}}{\hbar}\left[r_2\left(\frac{\pi}{2} - \arcsin\sqrt{\frac{r_1}{r_2}}\right) - \sqrt{r_1(r_2 - r_1)}\right].\end{aligned} \tag{7}$$

And the lifetime of the parent nucleus is given by

$$\tau = \frac{2r_1}{v}e^{2\gamma} \tag{8}$$

where $v = \sqrt{2E/m_1}$ is the speed of the emitted (or α) particle. It should be noted that, although being proposed for explaining the α decay of radioactive nuclei, the Gamow model is applicable in general for the decay or emission of any type of charged particles from a radioactive nucleus such as the proton decay from a diproton, β^+ decay from a diproton, and emission of a diproton from a radioactive heavy nucleus (e.g. diproton decays of ^{15}Ne and ^{11}O), and so on.

For the proton decay mode of a diproton, the emitted particle is a proton and the leftover nucleus is also a proton. In this case, we have $Z_1 = Z_2 = 1$, $m_1 = m_2 = m_p$, and $\mu = m_p/2$, where $m_p = 1.67 \times 10^{-27}$ kg is the proton mass. The width of the potential square well or the radius of the diproton can be chosen as $r_1 = 1.75 \times 10^{-15}$ m. With the values of these parameters and (6)–(8), we can plot, in Fig. 2, the transmission probability for the proton decay of the diproton (solid line) and the lifetime of the diproton via the proton decay mode (dashed line) as a function of the energy of the proton. It is seen that the transmission probability increases with the energy. Most diprotons undergo this decay mode when the energy of the emitted particle is greater than about some hundred keV. In other words, diprotons rarely decay into protons with energy much below about the Coulomb barrier such as one hundred keV or less. The lifetime of unbound diprotons via this decay mode is very short and slowly decreases with the energy of the emitted particle. When the energy of the emitted particle is greater than about some hundred keV, the lifetime of diprotons is as short as about 10^{-21} s.

For the β^+ decay mode of a diproton, the emitted particle is a positron and the leftover nucleus is a deuteron. In this case, we have $Z_1 = Z_2 = 1$, $m_1 = m_e$, $m_2 = 2m_p$, $\mu = m_e$, where $m_e = 9.1 \times 10^{-31}$ kg is the electron mass. The width of the potential square well or the radius of the diproton can be chosen again as $r_1 = 1.75 \times 10^{-15}$ m. With the values of these parameters and (6)–(8), we can plot, in Fig. 3, the transmission probability for the β^+ decay of a diproton (solid line) and the lifetime of diproton via this decay mode (dashed line) as a function of the energy of the positron. It is seen that the transmission probability increases with the energy. Diprotons rarely undergo this decay mode when the energy of the positron is less than about some hundred eV. The reason for the β^+ decay of the diproton to be extremely rare is because the energy of the emitted positron is far below the 820 keV Coulomb barrier. For the transmission probability to be about 10^{-21}, the energy of the emitted positron must be less than an eV, which may not be reasonable. Therefore, the result obtained here supports the existence of other physics effects such as plasma oscillations or waves that the author recently proposed to significantly inhibit the nuclear fusion reaction in the core of the Sun [8]. The lifetime of unbound diprotons via this β^+ decay mode is also very short and slowly decreases with the energy of the emitted positron. When the energy of the emitted positron is as high as about some hundred eV, the lifetime of diprotons is also as short as about 10^{-21} s.

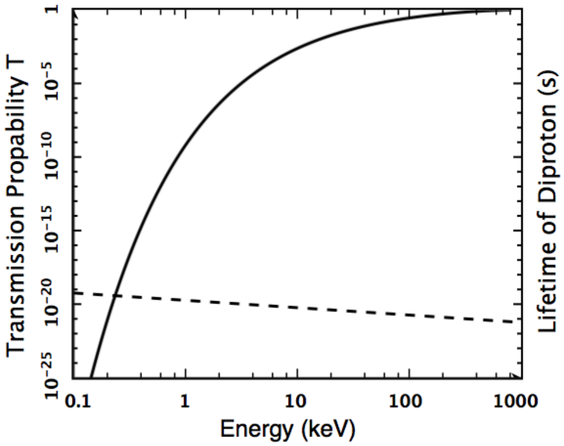

Fig. 2: Proton decay and lifetime of an unbound diproton. The solid line plots the transmission probability of a proton from the unbound diproton in the potential energy well to tunnel through the Coulomb barrier as a function of the energy of the proton. The dashed line plots the lifetime of the diproton.

For the diproton decay of radioactive heavy nuclei such as ^{15}Ne, the emitted particle is a diproton and the leftover nucleus is ^{13}O. In this case, we have $Z_1 = 2$, $Z_2 = 8$, $m_1 = 2m_p$, $m_2 = 13m_p$, $\mu = 1.73\,m_p$, Here we have considered approximately both proton and neutron having about the same mass. The width of the potential square well or the radius of ^{15}Ne nucleus can be chosen as $r_1 = 4 \times 10^{-15}$ m. With the values of these parameters and (6–8), we can plot, in Fig. 4, the transmission probability for the diproton decay from a radioactive nucleus ^{15}Ne (solid line) and the lifetime of the nucleus ^{15}Ne via this diproton decay mode (dashed line) as a function of the energy of the diproton. It is seen that the transmission probability increases with the energy. Most ^{15}Ne nuclei undergo the diproton decay when the energy of the emitted particle is greater than about some MeV. The lifetime of the radioactive nucleus ^{15}Ne via the diproton decay mode is very short and slowly decreases with the energy of the emitted diproton. When the energy of the emitted diproton is as high as about some MeV, the lifetime of the radioactive nucleus ^{15}Ne is

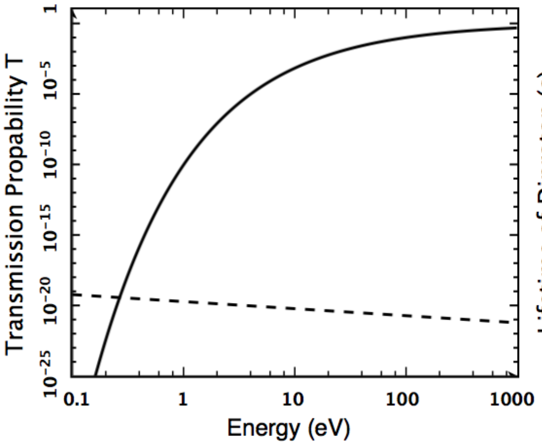

Fig. 3: Positron (or β^+) decay and lifetime of an unbound diproton. The solid line plots the transmission probability of a positron from the unbound diproton in the potential energy well to tunnel through the Coulomb barrier as a function of the energy of the positron. The dashed line plots the lifetime of the diproton.

as short as about 10^{-21} s, consistent with measurements [10]. The diproton decay was also detected from other nuclei such as ^{18}Ne nucleus [11, 12].

3 Discussions and Conclusions

If diprotons are bound, stars would burn about a billion billion times brighter in luminosity or faster in nuclear reaction, resulting in a universe to fail the life support [13, 14]. This diproton disaster can be overcome by plasma oscillations or waves, which have been shown recently to be able to be extremely efficient in inhibiting the nuclear reaction [8], to have the observed luminosity without need to adjust the stars' central temperature, density, and initial number of deuterons. In future study, we will study in more detail the transmission probability of bound diprotons for the fusion reaction.

As a consequence of this study, we have investigated the transmission and decay of unbound diprotons according to the Gamow theory. An unbound diproton is extremely unstable and quickly decays through two types of decay modes with lifetime to be extremely short down to about 10^{-21} s and transmission probability to be significantly energy dependent. A diproton mostly undergoes a proton decay to be two separate protons with a transmission probability higher than 99.99%, and rarely undergoes a β^+ decay to form a deuteron with a transmission probability lower than 0.01%. In the reasonable energy range, the β^+ decay of diproton is not rare enough for the Sun to have the observed reaction rate, which supports the author's recently proposed other inhibition effect such as plasma oscillation in solar nuclear fusion. The result obtained for the diproton decay from a radioactive nucleus can also be consistent with measurements.

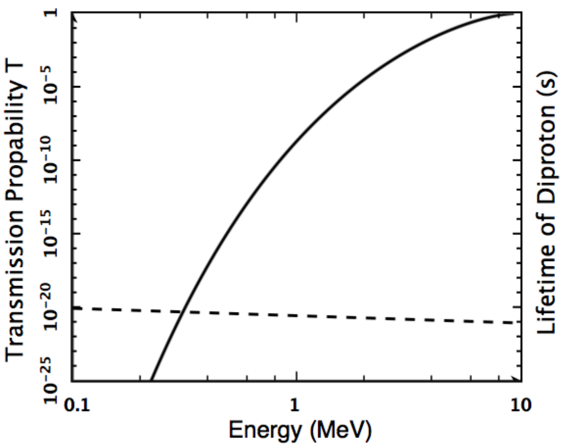

Fig. 4: Diproton decay and lifetime of ^{15}Ne nucleus. The solid line plots the transmission probability of a diproton from the radioactive ^{15}Ne nucleus in the potential energy well to tunnel through the Coulomb barrier as a function of the energy of the diproton. The dashed line plots the lifetime of ^{15}Ne nucleus.

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