Length Stretching and Time Dilation in the Field of a Rotating Body

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As was found in the first paper of this series of papers, the rotation of space produces a significant curvature (Progr. Phys., 2022, v.18, 31–49). In the second paper, we showed that light rays and mass-bearing particles are deflected near a rotating body due to the curvature of space caused by its rotation (ibid., 50–55). In this article we show that, since the rotation of the Earth around its axis curves the Earth's space making it "stretched" along the geographical longitudes, the measured length of a standard rod is greater when the rod is installed in the longitudinal direction. Due to the same reason, there is a time loss on board an airplane flying to the East (the direction in which the Earth's space rotates), and also a time gain when flying in the opposite direction, to the West. Both of the above effects are maximum at the equator (where the curvature of the Earth's space caused by its rotation is maximum and, therefore, space is maximally "stretched") and descrease towards the North and South Poles.

This paper is dedicated to the memory of Joseph C. Hafele, the outstanding American experimental physicist known due to his famous around-the-world-clocks experiment.

This is the third paper in the series of our papers on the effects of the space curvature caused by the rotation of space.

Recall that in the first paper [1], besides many other scientific results, it was found that the rotation of space produces a significant curvature due to its space-time non-holonomity (non-orthogonality of the time lines to the three-dimensional spatial section). In the second paper [2] that followed the first one, it was shown that light rays and mass-bearing particles are deflected near a rotating body due to the space curvature caused by the rotation of its space.

In particular, according to the formulae we have obtained, the curvature of the Earth's space, caused by its rotation, decreases from the equator, where it is maximum, to the geographical poles, and its effect depends on the direction of the measurement path with respect to the direction in which the Earth rotates.

This small paper is based of the previous two. We will calculate here the effects of length stretching and time loss/gain, which are due to the curvature of the Earth's space, caused by its rotation.

As always, we use the mathematical apparatus of chronometric invariants, which are physically observable quantities in the General Theory of Relativity. This mathematical apparatus was created in 1944 by our esteemed teacher A. L. Zelmanov (1913–1987) and published in his presentations [3–5], among which [5] is most complete. For a deeper study of this subject, read either our first article in this series [1] or the respective chapters in our monographs [6, 7].

Chronometrically invariant quantities are projections of four-dimensional (general covariant) quantities onto the line of time and the three-dimensional spatial section, which are linked to the physical space of a real observer, and are invariant everywhere along the spatial section (his observed space). They are calculated using operators of projection, which take the structure of space into account. Since a real space can be curved, inhomogeneous, anisotropic, deforming, rotating, be filled with distributed matter etc., the lines of real time can have different density of time coordinates, and the three-dimensional coordinate grids can have different density of real three-dimensional coordinates. Therefore, chronometrically invariant quantities are truly physically observables registered by the observer.

In particular, the physically observable chr.inv.-projection of the four-dimensional interval dx^α onto the time line of an observer is the interval of physically observable time

$$d\tau = \sqrt{g_{00}}\, dt - \frac{1}{c^2}\, v_i\, dx^i,$$

and the physically observable chr.inv.-projections of dx^α onto his spatial section are the regular three-dimensional coordinate intervals dx^i. Here v_i is the linear velocity of the three-dimensional rotation of space, which arises due to the non-holonomity of the space-time (non-orthogonality of the time lines to the three-dimensional spatial section). It is determined as

$$v_i = -\frac{c\, g_{0i}}{\sqrt{g_{00}}}\,, \qquad v^i = -c\, g^{0i}\sqrt{g_{00}}\,,$$

where g_{00} is expressed through the gravitational potential w as usually, i.e., $\mathrm{w} = c^2\,(1-\sqrt{g_{00}})$.

The fundamental metric tensor $g_{\alpha\beta}$, projected onto the three-dimensional spatial section of an observer, gives the chr.inv.-metric tensor h_{ik} of his space

$$h_{ik} = -g_{ik} + \frac{1}{c^2}\, v_i\, v_k\,, \quad h^{ik} = -g^{ik}, \quad h^i_k = -g^i_k = \delta^i_k\,,$$

which has all properties of $g_{\alpha\beta}$ in the three-dimensional spatial section. Using the chr.inv.-metric tensor, we can lift and lower indices in chr.inv.-quantities, and also get their squares. Thus, the square of the three-dimensional physically observable interval on the spatial section is calculated as

$$d\sigma^2 = h_{ik}\,dx^i dx^k = \left(-g_{ik} + \frac{1}{c^2}\,v_i v_k\right) dx^i dx^k .$$

In our further calculations, we will use the same space metric that we used in two previous papers. This is the metric of a space, where the three-dimensional space rotates due to the non-holonomity of the space-time, but there is no field of gravitation. More precisely, we neglect the influence of the Earth's gravitation, since in our further examples we do not change the altitude above the Earth's surface, so the influence of the gravitational potential remains constant.

Assuming that the space rotates along the equatorial axis φ, i.e., along the geographical longitudes, with the linear velocity $v_3 = \omega\, r^2 \sin^2\theta$ (here $\omega = const$ is the angular velocity of this rotation), we obtain g_{03} from the definition of v_i,

$$v_3 = \omega\, r^2 \sin^2\theta = -\frac{c\, g_{03}}{\sqrt{g_{00}}}\,,$$

and then we obtain the metric of such a space

$$ds^2 = c^2 dt^2 - 2\,\omega r^2 \sin^2\theta\, dt\, d\varphi - dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right).$$

The non-zero components of the fundamental metric tensor $g_{\alpha\beta}$ of this metric are obvious from the above

$$g_{00} = 1\,, \qquad g_{03} = -\frac{\omega\, r^2 \sin^2\theta}{c}\,,$$

$$g_{11} = -1\,, \quad g_{22} = -r^2, \quad g_{33} = -r^2 \sin^2\theta\,,$$

and the non-zero components of the chr.inv.-metric tensor h_{ik}, calculated from the above, are equal to

$$h_{11} = 1\,, \quad h_{22} = r^2, \quad h_{33} = r^2 \sin^2\theta \left(1 + \frac{\omega^2 r^2 \sin^2\theta}{c^2}\right).$$

Now we have everything that is required for our further calculations.

So forth, we will calculate two effects due to the curvature of the Earth's space caused by its rotation.

First, we will calculate the effect of length stretching of a rod depending on its direction (in the equatorial, latitudinal and radial directions), as well as on the geographical latitude of the measurement site. According to the formulae for the Ricci curvature tensor and the scalar curvature, which we have obtained in the first paper [1, p. 45], the Earth's space is curved due to its rotation in the equatorial (longitudinal) direction, and its curvature decreases with the latitude from the equator, where the Earth's space is maximally "stretched", to the geographical poles of the Earth. Therefore, the measured length of a standard rod is expected to be greater when the rod is installed in the direction along the geographical longitudes, and this effect of length stretching is maximum at the equator and decreases with the geographical longitudes towards the North and South Poles.

Second, we will calculate the difference in time on board an aircraft flying Westward and Eastward. It is expected that the rotation of the Earth's space causes a time loss when flying Eastward, the direction in which the Earth's space rotates, and a time gain when flying in the opposite direction, to the West. We also expect that the mentioned effects of time loss and time gain are greater when the airplane travels along the equator (where the curvature of the Earth's space caused by its rotation is maximum and, therefore, space is maximally "stretched") and decrease from the equator towards the North and South Poles.

1. Consider a standard rigid rod of an elementary length dl_0, which is installed in a laboratory located somewhere on the surface of the Earth. Assume that the rod is installed in stages in three different positions: in the equatorial direction φ (along the geographical longitudes), in the polar direction θ (along the geographical latitudes), and in the radial direction r read from the centre of the Earth.

Using the formula for the square of the three-dimensional physically observable interval $d\sigma^2 = h_{ik}\,dx^i dx^k$ and the components of the physically observable chr.inv.-metric tensor h_{ik} we have obtained for an Earth-like rotating space (see above), we calculate the rod's length measured in each of the three indicated positions. It is respectively equal to

$$dl_r = \sqrt{h_{11}\,dr^2} = dr = dl_0\,,$$

$$dl_\theta = \sqrt{h_{22}\,d\theta^2} = r\,d\theta = dl_0\,,$$

$$dl_\varphi = \sqrt{h_{33}\,d\varphi^2} = \sqrt{1 + \frac{\omega^2 r^2 \sin^2\theta}{c^2}}\; r \sin\theta\, d\varphi = \sqrt{1 + \frac{\omega^2 r^2 \sin^2\theta}{c^2}}\; dl_0\,,$$

where $dr = dl_0$ is the length of an elementary segment along the radial r-axis, $r\,d\theta = dl_0$ is the length of an elementary arc along the latitudinal θ-axis (where θ is the polar angle read from the North Pole), and $r \sin\theta\, d\varphi = dl_0$ is the length of an elementary arc along the equatorial φ-axis.

As you can see from the above formulae, the rod retains its original physically observable length dl_0, when installed in the positions along the radial direction ($dl_r = dl_0$) and along the geographical latitudes ($dl_\theta = dl_0$).

However, when the rod is installed in the position along the geographical longitudes, i.e., along the equatorial direction in which the Earth's space rotates, its physically observable length dl_φ becomes greater by a small amount δl depending on the factor specific of the curvature of space caused by its rotation [1, p. 45], i.e.,

$$dl_\varphi = \sqrt{1 + \frac{\omega^2 r^2 \sin^2\theta}{c^2}}\, dl_0 \simeq \left(1 + \frac{\omega^2 r^2 \sin^2\theta}{2c^2}\right) dl_0 \,,$$

$$\delta l \simeq \frac{\omega^2 r^2 \sin^2\theta}{2c^2}\, dl_0 \,.$$

Let us calculate the numerical value of this length stretching δl. The angular velocity of the Earth's rotation is equal to $\omega = 1$ rev/day $= 1.16 \times 10^{-5}$ rev/sec. The Earth's radius is equal to $r = 6.4 \times 10^8$ cm. Then the length stretching of a rod installed at the equator of the Earth in the direction along the longitudinal axis φ is equal to

$$\delta l \simeq 3.1 \times 10^{-14}\, dl_0$$

of the original length dl_0 of the rod. At the latitude of the Greenwich Observatory (51° North Lat., $\theta = 90° - 51° = 39°$) the length stretching of a rod installed along the longitudinal axis φ is less than at the equator and is equal to

$$\delta l \simeq 1.2 \times 10^{-14}\, dl_0 \,,$$

and this effect of length stretching vanishes at the geographical poles of the Earth, since there $\sin\theta = 0$ and, hence,

$$\delta l = 0 \,, \qquad dl_\varphi = dl_0 \,.$$

So, we clearly see that the curvature of the Earth's space along the equatorial (longitudinal) axis, caused by the rotation of the Earth, and, as a result, the "stretching" of physical coordinates along the geographical longitudes, lead to the stretching of the physically observable length of a rod, installed in the position along the geographical longitudes.

The mentioned effect of length stretching is maximum at the equator, where the curvature of the Earth's space and the longitudinal stretching of physical coordinates caused by the Earth's rotation is maximum, and decreases towards the geographical poles, where the length stretching vanishes.

2. Consider an atomic clock installed on board an airplane flying, in stages, Westward and Eastward around the Earth. In this case, according to the definition of physically observable time, and taking the characteristics of an Earth-like rotating space into account (see above), the flight time τ registered on board the airplane is equal to

$$\tau = \left(1 - \frac{1}{c^2}\, v_3\, u^3\right) t = \left(1 - \frac{\omega r^2 \sin^2\theta}{c^2}\, u^3\right) t \,,$$

where t is the reference (coordinate) time counted using a reference clock installed at the point of departure (which is the same as at the point of arrival in an around-the-world flight), and u^3 is the linear coordinate velocity of the airplane, which is measured along the third, equatorial (longitudinal) axis φ as the difference in the geographical longitudes traveled by the airplane per second.

If the airplane stays at the airport, its coordinate velocity is equal to zero $u^3 = 0$ and, therefore, the second term in the above formula vanishes. In this case, the clock installed on board the airplane count the same time as the reference clock at the airport ($\tau = t$).

Since the Earth rotates from West to East, an airplane, when flying Eastward, travels in the same direction in which the Earth's space rotates (the airplane's velocity is co-directed with the rotation velocity of the Earth's space). As a result, the clock installed on board the airplane should register a time loss, the amount of which is calculated as

$$\delta\tau_{\text{East}} = -\frac{\omega r^2 \sin^2\theta}{c^2}\, u^3 t \,.$$

When an airplane flies Westward, its velocity is directed opposite the rotation velocity of the Earth's space. Accordingly, in this case, the clock on board the airplane should register a time gain, the amount of which is

$$\delta\tau_{\text{West}} = +\frac{\omega r^2 \sin^2\theta}{c^2}\, u^3 t \,.$$

Assume that the airplane flies along the equator around the Earth at a constant cruising speed of 800 km/hour, which means that $u^3 = +5.5 \times 10^{-6}$ rev/sec when flying Eastward and $u^3 = -5.5 \times 10^{-6}$ rev/sec when flying Westward. Thus, the airplane returns to its point of departure in a time interval $t = 1.8 \times 10^5$ sec. The angular velocity of the Earth's rotation is equal to $\omega = 1$ rev/day $= 1.16 \times 10^{-5}$ rev/sec and the Earth's radius is equal to $r = 6.4 \times 10^8$ cm. Thus, we obtain that the clock on board this airplane should register a time loss when flying Eastward and a time gain when flying Westward, which are respectively equal to

$$\delta\tau_{\text{East}} = -5.3 \text{ nanosec}, \quad \delta\tau_{\text{West}} = +5.3 \text{ nanosec}.$$

That is, the rotation of the Earth's space results in a 5.3 nanosecond loss in time on board an Eastward-flying airplane travelled around the world along the equator, i.e., in the direction in which the Earth's space rotates, and a 5.3 nanosecond gain of time when travelled around the world in the opposite direction, to the West.

The above effect of time loss and time gain caused by the rotation of the Earth's space decreases with the geographical latitude due to the sine of the polar angle, which is a multiplier in the above formulae. For example, when flying Eastward and Westward around the Earth along the Greenwich parallel (51° North Lat., $\theta = 39°$), the effect of time loss and time gain is respectively equal to

$$\delta\tau_{\text{East}} = -2.1 \text{ nanosec}, \quad \delta\tau_{\text{West}} = +2.1 \text{ nanosec}.$$

This effect obviously vanishes at the geographical poles of the Earth, since there $\sin\theta = 0$.

Yes, the expected loss/gain in the flight time is only 5.3 nanoseconds at the equator, and it decreases to the geographical poles. Compare, in the Hafele-Keating around-the-world-clocks experiment [8–10], the common effect of the relativistic addition of the Earth's rotation velocity to the airplane's velocity and also the decrease of the gravitational potential of the Earth with the flight altitude resulted a time loss of -59 ± 10 nanoseconds Eastward and a time gain of $+273 \pm 7$ nanoseconds Westward. The UK's National Measurement Laboratory commonly with the BBC repeated the Hafele-Keating experiment on its 25th anniversary in 2005, on board a London-Washington-London flight and with a better precision of ± 2 nanoseconds [11]. But even such a high measurement precision does not allow us to reliably register the expected 5.3 nanosecond loss/gain in the flight time (and this effect decreases to 2 nanoseconds at the middle latitudes).

Fortunately, the loss/gain in the flight time, caused by the rotation of the Earth's space, is a "cumulative effect": it depends linearly on the flight time (see the formula above). That is, when an airplane will "wind circles" around the Earth, the effect of time loss/gain on its board, caused by the rotation of the Earth's space, will increase with each revolution. And, after three-four-five revolutions around the Earth, the expected effect caused by the rotation of the Earth's space will be many times (or even dozens of times) higher than the measurement precision.

This is the real way to register the effect of time loss/gain, caused by the rotation of the Earth's space. "Winding circles" around the Earth is easier not using an airplane, but on board a spacecraft orbiting the Earth because it travels around the Earth two dozen times a day anyway and without the need of aviation kerosene. Thus, having an atomic clock installed on board an orbital spacecraft, the effect of time loss/gain, caused by the rotation of the Earth's space, can be accumulated to a surely measurable numerical value in just a few days, without doing anything for this.

The aforementioned effects of length stretching and time loss/gain, occurring due to the curvature of the Earth's space caused by its rotation, are new fundamental effects of the General Theory of Relativity. They are in addition to the effect of deflection of light rays and mass-bearing particles in the field of a rotating body, which we theoretically discovered earlier, and the well-known Einstein effect of deflection of light rays in the field of a gravitating body.

We dedicate this paper to the memory of Joseph C. Hafele (1933–2014), the outstanding American experimental physicist who initiated and later performed (together with Richard E. Keating) the famous around-the-world-clocks experiment, now known as the Hafele-Keating experiment [8–10].

We had an extensive correspondence with Joseph Hafele in the 2010s, in which we discussed various problems. Unfortunately, his sudden death had interrupted our acquaintance. He was a truly gentleman, good Catholic and a honest scientist who never compromised [12].

Surely he would be happy, if he read this article and saw it published.

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