Gödel Time Travel With Warp Drive Propulsion

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In the first part of this work,we recall the basic principles of the Alcubierre warp drive space-time within the extrinsic curvature formalism. In the created singular region, we consider a hollow object that carries a charged current all around its external shape which interacts with an electromagnetic potential. As a result, this comoving object placed inside the region will follow a Finslerian geodesic. This allows to re-define a new lapse function that contains the potential-charge interacting term which can be chosen arbitrarily large, in order to lower the energy density required for sustaining the space-time distortion. Ultimately, this new lapse function is adjusted so as to keep the warp drive energy tensor positive thus always satisfying the famous energy conditions. In the second part, we apply this result to the Gödel curves following our previous publication whereby it was shown that Gödel's metric is a physical model not bound to any astrophysical representation. In this perspective, we suggest a possible mode of time travel.

Notations

Space-time Greek indices α, β run from 0, 1, 2, 3.
Spatial Latin indices a, b run from 1, 2, 3.
Space-time signature is: +2 (Part I) and −2 (Part II).

PART I

1 The warp drive metric

1.1 The (3 + 1) formalism or ADM technique

Arnowitt, Deser and Misner (ADM) suggested a technique which leads to decompose the space-time into a family of spacelike hypersurfaces and parametrized by the value of an arbitrarily chosen time coordinate x^0 [1]. This *foliation* displays a proper time element dt between two nearby hypersurfaces labeled $x^0 = const$, $x^0 + dx^0 = const$ and the proper time element $cd\tau$ must be proportional to dx^0, thus we write:

$$cd\tau = N\,(x^\alpha, x^0)\,dx^0, \tag{1.1}$$

where, according to the ADM terminology, N is called the *lapse function*.

Let us now evaluate the 3-vector whose spatial coordinates x^a are lying in the hypersurface $x^0 = const$ and which is normal to it, on the second hypersurface $x^0 + dx^0 = const$, where these coordinates now become $N^a dx^0$. The vector N^a is called the *shift vector*. The 4-metric tensor covariant components are

$$(g_{\alpha\beta})_{\text{ADM}} = \begin{pmatrix} -N^2 - N_a N_b\, g^{ab} & N_b \\ N_a & g_{ab} \end{pmatrix}. \tag{1.2}$$

The line element corresponding to the hypersurfaces separation is therefore written as

$$\begin{aligned}(ds^2)_{\text{ADM}} &= \\ &= -N^2\,(dx^0)^2 + g_{ab}\,(N^a dx^0 + dx^a)(N^b dx^0 + dx^b) = \\ &= (-N^2 + N_a N^a)(dx^0)^2 + 2N_b\,dx^0 dx^b + g_{ab}\,dx^a dx^b,\end{aligned} \tag{1.3}$$

where g_{ab} is the 3-metric of the hypersurfaces. The contravariant components of the ADM metric tensor are

$$(g^{\alpha\beta})_{\text{ADM}} = \begin{pmatrix} -\dfrac{1}{N^2} & \dfrac{N^b}{N^2} \\ \dfrac{N^a}{N^2} & g^{ab} - \dfrac{N^a N^b}{N^2} \end{pmatrix}. \tag{1.4}$$

As a result, the hypersurfaces have a unit time-like normal with contravariant components:

$$u^\alpha = N^{-1}\,(1, -N^a)\,. \tag{1.5}$$

If the universe is approximated to a Minkowski space within an orthonormal coordinates frame of reference and where the fundamental 3-tensor satisfies $g^{ab} = \delta^{ab}$, the metric (1.3) becomes

$$ds^2 = -(N^2 - N_a N^a)\,c^2 dt^2 + 2N^a dx\,cdt + dx^a dx^b \tag{1.6}$$

or, in another notation,

$$ds^2 = -N^2 dt^2 + (dx + N^a\,cdt)^2 + dy^2 + dz^2. \tag{1.6bis}$$

The Einstein action can be written in terms of the metric tensor $(g_{\alpha\beta})_{\text{ADM}}$ as [2]

$$S_{\text{ADM}} = \int cdt \int N\left({}^{(3)}R - K^a_b\,K^b_a + K^2\right)\sqrt{{}^{(3)}g}\,dx^3 + \\ + \text{boundary terms},$$

where $K^a_a K^b_b = K^2$, and ${}^{(3)}R$ is the 3-Ricci scalar and stands for the *intrinsic curvature* of the hypersurface

$$x^0 = const, \qquad \sqrt{{}^{(3)}g} = \sqrt{\det\|g_{ab}\|} \;\leftrightarrow\; \sqrt{-{}^{(4)}g} = N\,\sqrt{{}^{(3)}g}$$

so that

$$K_{ab} = (2N)^{-1}(-N_{a;b} - N_{a;b} + \partial_0\,g_{ab}) \tag{1.7}$$

represents the *extrinsic curvature*, and as such describes the manner in which the hypersurface $x^0 = const$ is embedded in the surrounding space-time. The rate of change of the 3-metric tensor g_{ab} with respect to the time label can be decomposed into "normal" and "tangential" contributions:

- The normal change is proportional to the extrinsic curvature $2K_{ab}/N$ of the hypersurface;
- The tangential change is given by the Lie derivative of g_{ab} along the shift vector N^a, namely:

$$\mathrm{L}_N\, g_{ab} = 2N_{(a;b)}\,. \tag{1.8}$$

With the choice of $N^a = 0$, we have a particular coordinate frame called *normal coordinates* according to (1.5) which is called an *Eulerian gauge*. Inspection shows that

$$K_{ab} = -u_{a;b} \tag{1.9}$$

which is sometimes called the *second fundamental form* of the 3-space. Six of the ten Einstein equations imply for K^a_b to evolve according to

$$\begin{aligned}\frac{\partial K^a_b}{c\,\partial t} + \mathrm{L}_N\, K^a_b = \nabla^a \nabla_b\, N + \\ + N\left[R^a_b + K^a_a\, K^a_b + 4\pi\,(T-C)\;\delta^a_b - 8\pi T^a_b\right],\end{aligned} \tag{1.10}$$

$$C = T_{\alpha\beta}\, u^\alpha u^\beta, \tag{1.11}$$

where C is the matter energy density in the rest frame of normal congruence (time-like vector field) with $T = T^a_a$. Using the Gauss-Codazzi relations [3] one can express the Einstein tensor as a function of both the intrinsic and extrinsic curvatures. It is convenient here to introduce the 3-momentum current density $I_a = -u_c T^c_a$. So the remaining four equations finally form the so-called *constraint equations*

$$H = \frac{1}{2}\left({}^{(3)}R - K^a_b K^b_a + K^2\right) - 8\pi C = 0\,, \tag{1.12}$$

$$H_b = \nabla_a \left(K^a_b - K\,\delta^a_b\right) - 8\pi I_b = 0\,. \tag{1.13}$$

Therefore, another way of writing (1.11) eventually leads to the formula

$$C = \frac{1}{16\pi}\left({}^{(3)}R - K_{ab}K^{ab} + K^2\right). \tag{1.14}$$

1.2 Salient features of Alcubierre's theory

1.2.1 The Alcubierre metric

In 1994, M. Alcubierre showed that an arbitrary large velocity (superluminal) can be achieved by building a so-called *space-time warped region* (*bubble-like region*) progressing along the x-direction which is a time-like trajectory, without violating the law of relativity [4]. Inside the bubble, the proper time element $d\tau$ is equal to the coordinate time dt which is also the proper time of a distant observer, so any object in the bubble does not suffer any time dilation as it moves. Outside and inside the bubble, space-time remains flat. In the classical interpretation, the warp drive requires *contraction* of the front space, and *expansion* behind the same bubble in the chosen direction, quite in analogy to the inflationary phase of the expanding universe.

In terms of the ADM formalism, the Alcubierre metric is defined from a flat space-time, while the lapse function and the shift functions are chosen as

$$\left.\begin{aligned} &N = 1\\ &N^1 = -v_s(t)\, f(r_s, t)\\ &N^2 = N^3 = 0\end{aligned}\right\}. \tag{1.15}$$

Next, we define

$$r_s(t) = \sqrt{(x - x_s(t))^2 + y^2 + z^2} \tag{1.16}$$

as the distance outward from the center of a spaceship placed in the bubble, variable until R_B, which is the *radius of the bubble*. With respect to a distant observer, the apparent velocity of the ship (thus the bubble), is given by:

$$v_s(t) = \frac{dx_s(t)}{dt}\,, \tag{1.17}$$

where $x_s(t)$ is the trajectory of the bubble along the x-direction. Such a region is transported forward with respect to distant observers, along the x-direction, and any spacecraft placed at rest inside, has no local velocity, but always moves along a time-like curve, regardless of $v_s(t)$. We then have the line element of the *Alcubierre metric*

$$(ds^2)_{\mathrm{AL}} = -c^2 dt^2 + [dx - v_s\, f(r_s, t)\, cdt]^2 + dy^2 + dz^2, \tag{1.18}$$

$$d\tau = dt\,, \tag{1.19}$$

Inside the spacecraft, the occupants will never suffer acceleration and so it is not difficult to show that the 4-velocity of a distant observer called *Eulerian observer* [5], has the following components:

$$(u^\alpha)_{\mathrm{E}} = \{c,\, v_s\, cf(r_s, t),\, 0,\, 0\}\,, \tag{1.20}$$

$$(u_\alpha)_{\mathrm{E}} = \{-c,\, 0,\, 0,\, 0\}\,. \tag{1.21}$$

The Eulerian observer is a special type of observer which refers to the Eulerian gauge defined above but with $N^1 \neq 0$, and as such, it follows time-like geodesic orthogonal to euclidean hypersurfaces. This observer starts out just inside the bubble shell at its first equator with zero initial velocity. Once during his stay inside the bubble, this observer travels along a time-like curve: $x = x_s(t)$ with a constant velocity nearing the ship's velocity: $v_s = dx_s/dt$. The Eulerian observer's velocity will always be less than the bubble's velocity unless $r_s = 0$, i.e., when this observer is at the center of the spaceship located inside. After reaching the second region's equator, this observer decelerates and is left at rest while going out at the rear edge of the bubble.

The Eulerian observer's velocity is needed to evaluate the energy density required to create the bubble.(see below) The function $f(r_s,t)$ is so defined as to cause space-time to contract on the forward edge and equally expanding on the trailing edge of the *bubble* as stated above. This is easily verified by using the expansion of the volume elements $\theta = (u^\alpha)_{\mathrm{E};\alpha}$ given by

$$\theta = \frac{v_s\, df}{(dx)_{\mathrm{AL}}}\,. \tag{1.22}$$

1.2.2 The Alcubierre function

The function $f(r_s,t)$ is often referred to as a *top hat function* and Alcubierre originally chose the following form

$$f(r_s,t) = \frac{\tanh\{\sigma(r_s+R_B)\} - \tanh\{\sigma(r_s-R_B)\}}{2\tanh\{\sigma R_B\}}\,, \tag{1.23}$$

where $R_B > 0$ is the *radius of the bubble*, and σ is a *bump parameter* which can be used to "tune" the wall thickness of the bubble. The larger this parameter, the greater the contained energy density, for its shell thickness decreases. Moreover the absolute increase of σ means a faster approach of the condition

$$\lim f(r_s,t) = 1, \ \text{ for } \ r_s \in (-R_B, R_B)\,, \ \text{ otherwise } \ \sigma \to \infty.$$

In the ADM formalism the expansion scalar is shown to be

$$\theta = \partial_1 N^1 = -\mathrm{Trace}\, K_{ab}\,, \tag{1.24}$$

which, with (1.13), becomes

$$\theta = v_s \frac{df}{dr_s}\frac{x_s}{r_s}\,. \tag{1.24bis}$$

Note that the Natàrio warp drive evades the problem of contraction/expansion, by imposing the divergence free constraint to the shift vector $\nabla[v_s^2 f^2(r_s,t)] = 0$ [6].

Obviously, the shape of the function f induces both a volume contraction and expansion ahead and behind of the bubble. Let us now write down the Alcubierre metric in the equivalent form

$$\begin{aligned}(ds^2)_{\mathrm{Al}} = -\left[1 - v_s^2 f^2(r_s,t)\right] c^2dt^2 - \\ - 2v_s f\, cdt\, dx + dx^2 + dy^2 + dz^2,\end{aligned} \tag{1.25}$$

which puts in evidence the covariant components of the metric tensor

$$\left.\begin{aligned}&(g_{00})_{\mathrm{Al}} = -[1 - v_s^2 f^2(r_s,t)]\\ &(g_{01})_{\mathrm{Al}} = (g_{10})_{\mathrm{Al}} = -v_s f(r_s,t)\\ &(g_{11})_{\mathrm{Al}} = (g_{22})_{\mathrm{Al}} = (g_{33})_{\mathrm{Al}} = 1\end{aligned}\right\}. \tag{1.26}$$

1.2.3 Energy conditions

With the components (1.26), the Einstein-Alcubierre tensor is written

$$(G^{\alpha\beta})_{\mathrm{Al}} = (R^{\alpha\beta})_{\mathrm{Al}} - \frac{1}{2}\,(g^{\alpha\beta})_{\mathrm{Al}}\, R\,, \tag{1.27}$$

$$(T^{\alpha\beta})_{\mathrm{Al}} = \frac{c^4}{8\pi}\,(G^{\alpha\beta})_{\mathrm{Al}}\,. \tag{1.28}$$

The *weak energy condition* (WEC) stipulates [7] that we must always have

$$C_{\mathrm{Al}} = (T^{\alpha\beta})_{\mathrm{Al}}\,(u_\alpha)_{\mathrm{E}}\,(u_\beta)_{\mathrm{E}} \geqslant 0 \tag{1.29}$$

From (1.14) we see that there in the Alcubierre space-time ${}^{(3)}R = 0$. Thus we get

$$C_{\mathrm{Al}} = \frac{1}{16\pi}\left(K^2 - K_{ab}K^{ab}\right), \tag{1.30}$$

$$\begin{aligned}C_{\mathrm{Al}} = \frac{1}{16\pi}\Big[(\partial_1 N^1)^2 - (\partial_1 N^1)^2 - \\ - 2\,(\partial_2 N^1)^2 - 2\,(\partial_3 N^1)^2\Big],\end{aligned} \tag{1.31}$$

$$\begin{aligned}(T^{00})_{\mathrm{Al}}\,(u_0)_{\mathrm{E}}\,(u_0)_{\mathrm{E}} = (T^{00})_{\mathrm{Al}} = \\ -\frac{c^4}{32\pi}\,v_s^2\left[\left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2\right] < 0\,.\end{aligned} \tag{1.32}$$

By taking into account the form of (1.23) we find the energy density:

$$(T^{00})_{\mathrm{Al}} = -\frac{c^4}{32\pi}\,(v_s)^2\left(\frac{df}{dr_s}\right)^2\frac{y^2+z^2}{r_s^2}\,. \tag{1.33}$$

This expression is unfortunately negative as measured by the Eulerian observer, and therefore it violates the weak energy conditions.

2 Reducing the energy density

2.1 A new configuration

Inside this bubble a spacecraft is engineered with a surrounding "shell" of thickness, $R_e - R_i$, where R_e is the outer radius, and R_i the inner radius. Now, let us consider a fluid of density ρ carrying a charge μ which fills this shell. By applying an electromagnetic field with a 4-potential A_α inside the shell, the whole spacecraft surrounded by the charge density will follow a specific *Finslerian geodesic* [8] provided the ratio μ/ρ remains constant all along the trajectory

$$ds_{\mathrm{shell}} = ds + \frac{\mu}{\rho}\,A_\alpha\, dx^\alpha, \tag{2.1}$$

where $ds = \sqrt{\eta_{\alpha\beta}\, dx^\alpha dx^\beta}$.

Therefore we may write the metric (neglecting the non-quadratic term)

$$(ds^2)_{\mathrm{shell}} = ds^2 + \left(\frac{\mu}{\rho}\,A_\alpha\, dx^\alpha\right)^2. \tag{2.2}$$

Now, the shell containing the charge μ which is acted upon by the potential A_α, must be included in the formulation of the metric (1.25). This can be achieved in a manner not too dissimilar to the one chosen in [9, 10]. First we have for the time component of the interaction term

$$\frac{\mu}{\rho}\, iA_0\, dx^0 = \frac{\mu}{\rho}\, \Phi\, cdt\,, \tag{2.3}$$

where Φ is the scalar potential. The metric tensor time component in (2.2) becomes

$$g_{00} = -\left(1 + \frac{\mu}{\rho}\,\Phi\right)^2. \tag{2.4}$$

The remaining spatial components $(\mu/\rho)A_a dx^a$ can be neglected if the 3-velocity of the global charges carrier (spacecraft) is low, since in this case the 3-density current is equal to $j_a = \mu \mathrm{v}_a \approx 0$. Hence, the metric (2.2) would reduce to

$$ds^2 = -\left(1 + \frac{\mu}{\rho}\,\Phi\right)^2 + dz^2 + dx^2 + dy^2. \tag{2.5}$$

In the framework of the Alcubierre metric, the spaceship shell is part of the warp drive bubble and as such the interaction term should be a function of r_s, R_B, σ, and the thickness $(R_e - R_i)$ but not the speed v_s.

Therefore we are led to define the lapse function as

$$N = \sqrt{1 + iS^2}\,, \tag{2.6}$$

where

$$S = \frac{1}{2}\left\{1 + \tanh\left[\sigma(r_s + R_e)^2\right]\right\}^{-\frac{a\Phi\mu}{\rho}}. \tag{2.7}$$

The dimensionless factor a delimits the shell thickness

$$\mathrm{a} = (R_e - R_i)^{-1}\int_{R_i}^{R_e} dR\,, \tag{2.7bis}$$

and (2.7) is verified from the center of the spacecraft location to the ext. bubble wall R_e, where $f = 1$.

The Alcubierre metric (1.25) can then be re-written as

$$\begin{aligned} ds^2 = -\left[N^2 - v_s^2 f^2(r_s)\right] c^2 dt^2 - \\ - 2 v_s f(r_s)\, cdt\, dx + dz^2 + dx^2 + dy^2. \end{aligned} \tag{2.8}$$

From the internal radius R_i throughout the spacecraft interior, there is no charge, and we see that the space is Minkowskian so that the spacecraft and its occupants will never suffer any tidal forces nor time dilation as per (1.10bis).

From the metric (2.8), it is now easy to infer the Eulerian observer's velocity components. We have

$$c^2 = -c^2\left(N^2 - v_s^2 f^2\right)\left(\frac{dt}{d\tau}\right)^2 - 2 v_s f c\,\frac{dt}{d\tau}\, u_{\mathrm{E}} + u_{\mathrm{E}}^2\,. \tag{2.9}$$

The Eulerian observer travels along the geodesic where he "sees"

$$\frac{dt}{d\tau} = N^{-1}, \tag{2.10}$$

which yields

$$0 = u_{\mathrm{E}}^2 - 2 v_s f c N^{-1} u_{\mathrm{E}} + v_s^2 f^2 c^2 N^{-2} \tag{2.11}$$

and finally we obtain

$$u_{\mathrm{E}} = v_s f c N^{-1}, \tag{2.12}$$

$$(u^\mu)_{\mathrm{E}} = \left\{cN^{-1},\, v_s f c N^{-1},\, 0,\, 0\right\}, \tag{2.13}$$

$$(u_\mu)_{\mathrm{E}} = \{-cN,\, 0,\, 0,\, 0\}\,. \tag{2.14}$$

2.2 The energy required for the propulsion

If we insert N into (1.30), the formula

$$C_{\mathrm{Al}} = (u_0)_{\mathrm{E}}\, (u_0)_{\mathrm{E}}\, T^{00} \tag{2.15}$$

yields the new energy density requirement

$$T^{00} = -\frac{c^4}{32\pi}\,\frac{v_s^2(y^2 + z^2)}{N^4 r_s^2}\left(\frac{df}{dr_s}\right)^2. \tag{2.16}$$

Now, recalling the form (2.6) for N, we have

$$N^4 = \left(1 + iS^2\right)^2 < 0\,. \tag{2.17}$$

Thus the energy conditions $T^{00} \geqslant 0$ are obviously always satisfied. Therefore we may choose the factor N (thereby S) arbitrarily large so as to substantially reduce the required energy density for the ship frame.

The higher the charge and the potential, the lower the energy requirement.

In the closed volume V of the spacecraft shell one can inject a flow of electrons according to the constant ratios

$$\frac{\mu}{\rho} = \frac{\sum_V e}{\sum_V m}\,. \tag{2.18}$$

We see that the leptonic lightweight would have the capacity to lower the negative energy even further. The splitting shell-inner part of the spacecraft frame, is really the hallmark of the theory here: it implies that the proper time τ of the inner part of the spacecraft is not affected by the term N.

PART II

In "The Time Machine" (1895), the novel by H. G. Wells, an English scientist constructs a machine which allows him to travel back and forth in time. The history of fascinating idea of time travel can be traced back to Kurt Gödel who found a solution of Einstein's field equations that contains closed time-like curves (CTCs) [11]. Those make it theoretically feasible to go on journey into one's own past. In our previous publication [12], we formally demonstrated that Gödel's model was not just a mere (speculative) cosmological model as is was always accepted, but an ordinary metric with own physical properties.

Upon these results we develop here the bases for a possible time travel mode of displacement.

3 Reformulation of Gödel's metric (reminder)

The classical Gödel line element is generically given by the interval

$$ds^2 = a^2 \left(dx_0^2 - dx_1^2 + dx_2^2 \,\frac{e^{2x_1}}{2} - dx_3^2 + 2\,e^{x_1} dx_0\, dx_2 \right) \quad (3.1)$$

or, equivalently,

$$ds^2 = a^2 \left[-dx_1^2 - dx_3^2 - dx_2^2 \,\frac{e^{2x_1}}{2} + (e^{x_1} dx_2 + dx_0)^2 \right], \quad (3.2)$$

where $a > 0$ is a constant.

In our theory, we assumed that a is slightly space-time variable and we set

$$a^2 = e^{2U}. \quad (3.3)$$

As a result, the Gödel metric tensor components are conformal to the real Gödel metric tensor $g_{\mu\nu}$

$$(g_{\mu\nu})' = e^{2U} g_{\mu\nu}\,, \qquad (g^{\mu\nu})' = e^{-2U} g^{\mu\nu}. \quad (3.4)$$

The exact Gödel metric reads now

$$(ds^2)' = e^{2U} \left[dx_0^2 - dx_1^2 + dx_2^2 \,\frac{e^{2x_1}}{2} - \right. \\ \left. - dx_3^2 + 2\,e^{x_1} (dx_0\, dx_2) \right]. \quad (3.5)$$

This implies that this metric is a solution of the field equations describing a peculiar perfect fluid [13–15]

$$G_{\mu\beta} = \varkappa \left[(\rho + P)\, u_\mu u_\beta - P g_{\mu\beta} \right]. \quad (3.6)$$

The model is likened to a fluid in rotation with mass density ρ and pressure P. The positive scalar U is shown to be:

$$U(x^\mu) = \int \frac{dP}{\rho + P}\,. \quad (3.7)$$

From (3.4) and (3.6) one formally infers that the flow lines of matter of the fluid follow conformal geodesics given by

$$s' = \int e^U ds\,. \quad (3.8)$$

The hallmark of the theory is the substitution (3.3): the Gödel space-time is no longer the representation of a cosmological model but it is relegated to the rank of an ordinary metric where its physical properties could allow for a possible replication.

4 Closed time-like curves

With Gödel one defines new coordinates (t, r, ϕ) which in the reformulated version lead to the line element

$$ds^2 = 4\,e^{2U} \Big[dt_G^2 - dr^2 + (\sinh^4 r - \sinh^2 r)\, d\phi^2 + \\ + 2\sqrt{2}\, \sinh^2 r\, d\phi\, dt \Big]. \quad (4.1)$$

This metric exhibits the rotational symmetry of the solution about the chosen Gödel t_G-time axis where $r = 0$ orthogonal to the hyperplane (x, y, z), since we clearly see that the spatial components of the metric tensor and its covariant derivative do not depend on f. For $r \geqslant 0$, we have $0 \leqslant \phi \leqslant 2\pi$. If a curve r_G is defined by $\sinh^4 r = 1$, that is

$$r_G = \ln\left(1 + \sqrt{2}\right), \quad (4.2)$$

the circle $r > \ln(1 + \sqrt{2})$, i.e. $(\sinh^4 r - \sinh^2 r) > 0$ in the "hyperplane" $t_G = 0$, is a *closed time-like curve* (which is not a *geodesic line*!). Here r_G is referred to as the *Gödel radius*.

The circle of radius r_G is a *light-like curve*, where the light cones are tangential to the hyperplane (x, y, z) of zero t_G. Photons trajectories reaching this radius are closing up, therefore r_G constitutes a *chronal horizon* beyond which an observer located at the origin ($r = 0$) cannot detect them. The following quantity corresponds to r_G, it is $(ds^2)' = e^{2U} ds^2 = 0$ with $e^{2U} \neq 0$.

For $r > r_G$ the light cone opens up and tips over until its future part reaches the negative values of t_G. In this an *achronal domain*, any closed curve is a time-like curve. The conformal line $s' = \int e^U ds$, the integral of which is performed over the curve length is always a time-like geodesic provided the following transformation is applied

$$t = t_G + \tanh\left(\frac{r - r_G}{r_G}\right) \sqrt{x^2 + y^2}\,, \quad (4.3)$$

where $r - r_G$ measures the distance from the Gödel radius onward. So long as $r < r_G$, then t coincides with the Gödel time axis t_G. When $r > r_G$, then $t_G = 0$ and the time coordinate t becomes space-like as viewed from within the Gödel space-time. The Gödel space coordinates should then be transformed as follows

$$x\,(\text{resp. } y, z) = x_G - (x_G + x_N) \tanh\left(\frac{r - r_G}{r_G}\right). \quad (4.4)$$

For $r < r_G$, x (resp. y, z) coincides with the Gödel space-time coordinates x_G (resp. y_G, z_G) of the hyperplane (x, y, z). For $r > r_G$, x (resp. y, z) coincides with a new coordinate x_N (resp. y_N, z_N) distinct from x_G (resp. y_G, z_G).

5 Time displacement mode

5.1 Creating a "bubble" along a Gödel curve

As we demonstrated, the conformal factor e^{2U} is not related to the hypothetical cosmological constant Λ.

It is therefore possible to adjust the factor U in order to create a pressureless singularity within the new Gödel space-time. In the following such a singular region is likened to the warp drive "bubble" which is bound to move along a Gödel curve.

The bubble follows the trajectory $x_s(t)$ where the time coordinate t satisfies here (4.3). Therefore for $R \leqslant R_B$, the bubble is assumed to be ruled by the new Alcubierre metric (2.8) expressed with the signature −2

$$ds^2 = \left(N^2 - v_s^2 f^2\right) c^2 dt^2 - 2 v_s f(r_s)\, cdt\, dx - - dz^2 - dx^2 - dy^2. \tag{5.1}$$

This space-time is thus regarded as *globally hyperbolic* and the bubble will never know whether it moves along a CTC. As a result,the bubble is seen by a specific observer (see below) as being transported forward along the x-direction *tangent* to a CTC beyond the Gödel radius r_G. In the absence of charge outside of the bubble ($R > R_B \to \infty$), there is $f = 0$ and we retrieve Gödel's metric (2.1).

5.2 Gödel chronal horizon

At the origin of the coordinate system, the axis of the light cone is orthogonal to the (x, y, z) hyperplane as described above by the metric (2.1). The circle of radius r_G is a *light-like curve*, where the light cones are tangential to the plane of constant (or zero) t and photons trajectories reaching this radius are closing up, therefore r_G constitutes a *chronal horizon*. Such an horizon is a special type of the *Cauchy horizon* beyond which an observer located at the origin ($r = 0$) cannot detect them. With increasing $r > r_G$ the light cones continue to keel over and their opening angles widen until their future parts reach the negative values of t. In this an *achronal domain*, any closed curve is a time-like curve. As a result, the bubble follows a reversed chronological sequence with respect to the coordinate t.

The bubble moves backwards in time and travels into the past of a specific observer resting at $r = 0$ whose proper time satisfies $\tau = t$. After regressing, once $r < r_G$, the bubble can return to the original causal domain at the departing coordinate time t, thus slightly aging with respect to the rest observer depending on its trip own time duration.

Concluding remarks

Without going into details of a sound engineering, we have just briefly sketched the basic principle of the existing theory using electromagnetism and charged current to suit the warp drive propulsion. Our approach heavily relies on a specific configuration describing a spacecraft located inside a warp drive bubble, which certainly deserves further scrutiny. In order to avoid an additional heavy treatment of the warp drive subject we have skipped some of the important aspects of the topic, as for example the causally separation of the bubble center to the outer edge of the bubble wall and beyond.

For further rigorous studies of classical warp drive physics, one can refer to [16–19]. Unlike our concept all of these theories rely on negative energy contributions also referred to as "exotic energy" or "exotic matter" [20]. Such form of energy has never been detected so far, although its theoretical production based on a L. de Broglie's publication [21] has been suggested in [22]. By introducing a "complex" potential, our warp drive concept does not require any form of exotic matter.

As a space-time short-cut Morris, Thorne et al. [23] derived a specific static wormhole comparable to the Einstein-Rosen-bridge. Combining two wormholes with a distorted one the authors could produce a time lag which would act as a time machine. Of particular interest is the recent paper published by Tippett and Tsang [24] where the Alcubierre warp is applied to a CTC. Like in our theory, a bubble of curvature travels along a closed trajectory and is ruled by a Rindler geometry. At any rate Exotic matter is still required.

Natàrio investigated an "optimal time travel" in the Gödel universe for a particle bound to accelerate along a CTC [25]. For this purpose, the well known Rocket Equation trajectories in general relativity are here applied to a CTC. Natàrio however keeps the factor $a = 1$ (and the cosmological constant $\Lambda = -\frac{1}{2}$), which necessarily restricts again this field of research to a finely tuned universe space-time. In contrast to all those attempts and related theories, the model we suggest here is derived from a reformulated Gödel metric that exhibits consistent physical properties which are known to exist. Because of this reformulation, new physical conditions render plausible a system which may accommodate a potential time machine.

The basic engineering we presented in here, pre-suppose a high level of technological accuracy, which is far from being reached by today's knowledge.

Billions of billions of distant galaxies must certainly harbour quite a great number of inhabitable worlds where advanced civilizations have certainly developed capabilities to allow for such interstellar propulsion modes. Indeed, our universe is 13.7 billion years old compared to the 4 billion years of our (marginal) Earth. Given this scale, an evolution difference of just one million years only between us and other extraterrestrial forms of thinking beings, is not unrealistic, and it implicitly means an incredible exponential degree of superior knowledge which is certainly beyond our common understanding.

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