On the Nature of the Spacetime Continuum

Pierre A. Millette
E-mail: pierre.millette@uottawa.ca, Ottawa, Canada

In this paper, we summarize the nature of the Spacetime Continuum (STC) as provided by the Elastodynamics of the Spacetime Continuum (*STCED*). We note that, in addition to providing a physical explanation for inertial mass and for wave-particle duality, *STCED* covers the Physics of the Spacetime Continuum. We show that the dimensionality of the Spacetime Continuum could be deduced mathematically if the value of the Lamé elastic constants $\bar{\kappa}_0$, $\bar{\mu}_0$ and $\bar{\lambda}_0$ of the Spacetime Continuum could be determined experimentally. From Einstein's field equation for an isotropic and homogeneous STC, we derive the value of the Spacetime Continuum bulk modulus $\bar{\kappa}_0$ in terms of elementary constants. Understanding the nature of the Spacetime Continuum as provided by *STCED* provides a better understanding of the general relativistic spacetime.

1 Introduction

In this paper, we summarize the nature of the Spacetime Continuum (*STC*) as provided by the Elastodynamics of the Spacetime Continuum (*STCED*) [1–3]. *STCED* is a natural extension of Einstein's General Theory of Relativity which blends continuum mechanical and general relativistic descriptions of the Spacetime Continuum. The introduction of strains in the Spacetime Continuum as a result of the energy-momentum stress tensor allows us to use, by analogy, results from continuum mechanics, in particular the stress-strain relation, to provide a better understanding of the general relativistic spacetime.

2 Elastodynamics of the Spacetime Continuum

The stress-strain relation for an isotropic and homogeneous Spacetime Continuum is given by [1, 3]

$$2\bar{\mu}_0\,\varepsilon^{\mu\nu} + \bar{\lambda}_0\,g^{\mu\nu}\varepsilon = T^{\mu\nu} \tag{1}$$

where $\bar{\lambda}_0$ and $\bar{\mu}_0$ are the Lamé elastic constants of the Spacetime Continuum: $\bar{\mu}_0$ is the shear modulus (the resistance of the Spacetime Continuum to *distortions*) and $\bar{\lambda}_0$ is expressed in terms of $\bar{\kappa}_0$, the bulk modulus (the resistance of the Spacetime Continuum to *dilatations*):

$$\bar{\lambda}_0 = \bar{\kappa}_0 - \tfrac{1}{2}\,\bar{\mu}_0 \tag{2}$$

in a four-dimensional continuum. $T^{\mu\nu}$ is the general relativistic energy-momentum stress tensor, $\varepsilon^{\mu\nu}$ the Spacetime Continuum strain tensor resulting from the stresses, and

$$\varepsilon = \varepsilon^{\alpha}{}_{\alpha}\,, \tag{3}$$

the trace of the strain tensor obtained by contraction, is the volume dilatation ε defined as the change in volume per original volume [4, see pp. 149–152] and is an invariant of the strain tensor. It should be noted that the structure of (1) is similar to that of the field equations of General Relativity,

$$R^{\mu\nu} - \tfrac{1}{2}\,g^{\mu\nu}R = -\varkappa\,T^{\mu\nu} \tag{4}$$

where $R^{\mu\nu}$ is the Ricci curvature tensor, R is its trace, $\varkappa = 8\pi G/c^4$ and G is the gravitational constant (see [2, Ch. 2] for more details).

In *STCED*, as shown in [1, 3], energy propagates in the spacetime continuum (*STC*) as wave-like *deformations* which can be decomposed into *dilatations* and *distortions*. *Dilatations* involve an invariant change in volume of the Spacetime Continuum which is the source of the associated rest-mass energy density of the deformation. On the other hand, *distortions* correspond to a change of shape (shearing) of the Spacetime Continuum without a change in volume and are thus massless.

Thus deformations propagate in the Spacetime Continuum by longitudinal (*dilatation*) and transverse (*distortion*) wave displacements. This provides a natural explanation for wave-particle duality, with the massless transverse mode corresponding to the wave aspects of the deformations and the massive longitudinal mode corresponding to the particle aspects of the deformations.

The rest-mass energy density of the longitudinal mode is given by [1, see Eq. (32)]

$$\rho c^2 = 4\bar{\kappa}_0\varepsilon \tag{5}$$

where ρ is the rest-mass density, c is the speed of light, $\bar{\kappa}_0$ is the bulk modulus of the *STC* as seen previously, and ε is the volume dilatation given by (3).

3 The physicality of four-dimensional spacetime

Minkowski [5, 7] first introduced the concept of a four-dimensional spacetime and the description of particles in this spacetime as worldlines in 1908. This has given rise to the question whether four-dimensional spacetime is real or a mathematical abstraction. Eddington [7] considered this question in 1921:

> It was shown by Minkowski that all these fictitious spaces and times can be united in a single continuum of four dimensions. The question is often raised whether this four-dimensional space-time is real, or merely a mathematical construction; perhaps it is sufficient to

> reply that it can at any rate not be less real than the fictitious space and time which it supplants.

Petkov [6, 7] provides a cogent summary of Minkowski's paper. Worldlines of particles at rest are vertical straight lines in a *space−ct* diagram, while particles moving at a constant velocity v are oblique lines and accelerated particles are curved lines. This provides a physical explanation for length contraction as a manifestation of the reality of a particle's extended worldline, where the cross-section measured by an observer moving relative to it (i.e. at an oblique line in the *space−ct* diagram), creates the difference in perceived length between a body at rest and one in movement. This is explored in greater detail in [8, 9]. Minkowski's work demonstrates the physicality of four-dimensional spacetime, and that indeed, four-dimensional physics is spacetime geometry.

The relation (2) between κ, and μ and λ can be generalized to N dimensions, and is given by [10, p. 769]

$$\kappa = \frac{2\mu + N\lambda}{N}\,. \tag{6}$$

The dimensionality of the Spacetime Continuum could thus be deduced mathematically if the value of the Lamé elastic constants $\bar{\kappa}_0$, $\bar{\mu}_0$ and $\bar{\lambda}_0$ of the Spacetime Continuum could be determined experimentally.

4 Physics of the Spacetime Continuum

From General Relativity and *STCED*, one can deduce the properties of the Spacetime Continuum, as *STCED* includes the physics of the Spacetime Continuum as an underlay of the theory.

The Spacetime Continuum is modelled as a four-dimensional differentiable manifold [11] endowed with a metric $g_{\mu\nu}$. It is a continuum that can undergo deformations and support the propagation of such deformations. A continuum that is deformed is strained.

An infinitesimal element of the unstrained continuum is characterized by a four-vector x^μ, where $\mu = 0, 1, 2, 3$. The time coordinate is $x^0 \equiv ct$.

A *deformation* of the Spacetime Continuum corresponds to a state of the *STC* in which its infinitesimal elements are displaced from their unstrained positions. Under deformation, the infinitesimal element x^μ is displaced to a new position $x^\mu + u^\mu$, where u^μ is the displacement of the infinitesimal element from its unstrained position x^μ.

The Spacetime Continuum is approximated by a deformable linear elastic medium that obeys Hooke's law. Under those conditions, for a general anisotropic continuum in four dimensions [12, see pp. 50–53],

$$E^{\mu\nu\alpha\beta}\varepsilon_{\alpha\beta} = T^{\mu\nu} \tag{7}$$

where $\varepsilon_{\alpha\beta}$ is the strain tensor, $T^{\mu\nu}$ is the energy-momentum stress tensor, and $E^{\mu\nu\alpha\beta}$ is the elastic moduli tensor.

The Spacetime Continuum is further assumed to be isotropic and homogeneous. This assumption is in agreement with the conservation laws of energy-momentum and angular momentum as expressed by Noether's theorem [13, see pp. 23–30]. For an isotropic medium, the elastic moduli tensor simplifies to [12]:

$$E^{\mu\nu\alpha\beta} = \bar{\lambda}_0(g^{\mu\nu}g^{\alpha\beta}) + \bar{\mu}_0(g^{\mu\alpha}g^{\nu\beta} + g^{\mu\beta}g^{\nu\alpha}) \tag{8}$$

where $\bar{\lambda}_0$ and $\bar{\mu}_0$ are the Lamé elastic constants of the Spacetime Continuum as seen previously in Section 2. Substituting (8) into (7), we obtain the stress-strain relation (1) seen previously in Section 2, for an isotropic and homogeneous Spacetime Continuum. The Spacetime Continuum is thus modelled as an elastic medium (see [3, pp. 16–18,24]).

Blair [14, p. 3–4] writes Einstein's field equation as

$$\mathbf{T} = \frac{c^4}{8\pi G}\,\mathbf{G}\,, \tag{9}$$

where $\mathbf{T}$ is the stress energy tensor, $\mathbf{G}$ is the Einstein curvature tensor and G is the universal gravitational constant. He notes the very large value of the proportionality constant. This leads him to point out that spacetime is an elastic medium that can support waves, but its extremely high stiffness means that extremely small amplitude waves have a very high energy density. He notes that the coupling constant $c^4/8\pi G$ can be considered as a modulus of elasticity (K) for spacetime. In similarity to the acoustic case, where the specific impedance $z = K/v$, he identifies the quantity c^3/G with the characteristic impedance of spacetime [14, p. 45].

Substituting for the Einstein curvature tensor in (9), the equation becomes

$$T^{\mu\nu} = \frac{c^4}{8\pi G}\,G^{\mu\nu} = \frac{c^4}{8\pi G}\left[R^{\mu\nu} - \tfrac{1}{2}\,g^{\mu\nu}R\right]. \tag{10}$$

For *STCED*, as seen in (7), the single modulus of elasticity of (10) is replaced by the elastic moduli tensor $E^{\mu\nu\alpha\beta}$ of rank 4, consisting of 256 components. For an isotropic and homogeneous Spacetime Continuum, the elastic moduli tensor is given by (8) and simplifies to two moduli, the shear modulus $\bar{\mu}_0$ for transverse waves and the bulk modulus $\bar{\kappa}_0$ for longitudinal waves, as seen previously in (1):

$$T^{\mu\nu} = 2\bar{\mu}_0\,\varepsilon^{\mu\nu} + \bar{\lambda}_0\,g^{\mu\nu}\varepsilon\,. \tag{11}$$

As shown in [2, §2.5], (10) and (11) can be combined and separated into a longitudinal relation

$$\frac{c^4}{8\pi G}\,R = 2(\bar{\mu}_0 + 2\bar{\lambda}_0)\,\varepsilon = 4\,\bar{\kappa}_0\,\varepsilon = \rho c^2 \tag{12}$$

where ρ is the rest-mass energy density present in the Spacetime Continuum, and a transverse relation

$$\frac{c^4}{8\pi G}\,R^{\mu\nu} = 2\bar{\mu}_0\,\varepsilon^{\mu\nu} - (\bar{\lambda}_0 + \bar{\mu}_0)\,g^{\mu\nu}\varepsilon \tag{13}$$

which becomes

$$\frac{c^4}{8\pi G} R^{\mu\nu} = 2\bar{\mu}_0 \left(\varepsilon^{\mu\nu} - \frac{1}{2} \frac{\bar{\lambda}_0 + \bar{\mu}_0}{\bar{\mu}_0} g^{\mu\nu} \varepsilon \right) \tag{14}$$

where $(\bar{\lambda}_0 + \bar{\mu}_0)/\bar{\mu}_0$ is a numerical factor.

We can derive the relationship between the Spacetime Continuum bulk modulus $\bar{\kappa}_0$ and known constants from relation (12) as follows:

$$\frac{c^4}{8\pi G} R = 4\,\bar{\kappa}_0\,\varepsilon\,, \tag{15}$$

where the constant $c^4/8\pi G$ has dimensions of [N], R has dimensions of [m^{-2}], $\bar{\kappa}_0$ has dimensions of [N m^{-2}] or [J m^{-3}], and ε is dimensionless. We need to express R as a dimensionless quantity and combine its constant factor with constant $c^4/8\pi G$. Curvature R is expressed in [m^{-2}]. As shown in [15], the smallest Spacetime Continuum Burgers vector b_0 is equal to Planck's length

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}}\,. \tag{16}$$

The curvature of this smallest surface element will be constant, such that we can write the curvature R as

$$R = \frac{\bar{R}}{\ell_P^2} \tag{17}$$

where $\bar{R}$ is the dimensionless curvature number in terms of the smallest surface element ℓ_P^2.

Substituting (17) and (16) into (15), we obtain

$$\frac{c^7}{8\pi\hbar G^2}\,\bar{R} = 4\,\bar{\kappa}_0\,\varepsilon\,, \tag{18}$$

where the units are [N m^{-2}]. The dimensionless curvature $\bar{R}$ and, as seen in Section 2, the dimensionless volume dilatation ε corresponding to the change in volume per original volume $(\Delta V/V)$ [4, see pp. 149–152], result from the applied stresses leading to the deformation of the Spacetime Continuum.

The latter corresponds to the definition of the bulk modulus. The numerical factors can be included in the definition of the dimensionless curvature $\bar{R}$ and the dimensionless volume dilatation ε to obtain

$$\frac{c^7}{\hbar G^2}\,\frac{\bar{R}}{8\pi} = \bar{\kappa}_0\,(4\varepsilon)\,. \tag{19}$$

One option is to equate the terms having dimensions of [N m^{-2}] to obtain the Spacetime Continuum bulk modulus, with the understanding that there may be a numerical factor on the R.H.S. of (20):

$$\bar{\kappa}_0 = \frac{c^7}{\hbar G^2}\,. \tag{20}$$

From one of my previous articles [1, Eq. (150)], we then have

$$\bar{\mu}_0 = 32\bar{\kappa}_0 = 32\,\frac{c^7}{\hbar G^2}\,. \tag{21}$$

Numerically, $\bar{\kappa}_0 = 4.6\times10^{113}$ J/m^3 and $\bar{\mu}_0 = 1.5\times10^{115}$ J/m^3.

With these constants, we are now in a position to calculate the density of the Spacetime Continuum $\bar{\rho}_0$. Using the relation [1]

$$c = \sqrt{\frac{\bar{\mu}_0}{\bar{\rho}_0}}, \tag{22}$$

the density of the spacetime continuum is

$$\bar{\rho}_0 = 1.7\times10^{98}\ \text{kg/m}^3. \tag{23}$$

This value is in the same ballpark as the vacuum energy density calculated by Carroll [16, see p. 173] ($\sim 10^{112}$ ergs/cm^3) from quantum mechanical considerations.

5 Mass in the Spacetime Continuum

We have considered the origin of inertial mass in the Spacetime Continuum in [17], where we showed that integrating (5) over the 3-D space volume,

$$\int_{V_3} \rho c^2\,\mathrm{d}V_3 = 4\bar{\kappa}_0 \int_{V_3} \varepsilon\,\mathrm{d}V_3\,, \tag{24}$$

and using

$$m = \int_{V_3} \rho\,\mathrm{d}V_3 \tag{25}$$

in (24), where m is the rest mass of the deformation, we obtain

$$mc^2 = 4\bar{\kappa}_0 \int_{V_3} \varepsilon\,\mathrm{d}V_3\,. \tag{26}$$

This demonstrates that mass is not independent of the Spacetime Continuum, but rather mass is part of the Spacetime Continuum fabric itself. Hence mass results from the dilatation of the Spacetime Continuum in the longitudinal propagation of energy-momentum in the Spacetime Continuum. Matter does not warp spacetime, but rather, matter *is* warped spacetime (i.e. dilated spacetime). The missing link in General Relativity is the understanding that the trace of the energy-momentum stress tensor is related to the trace of the Spacetime Continuum strain tensor and is proportional to the mass of matter as given by (5) and (26).

6 Discussion and conclusion

In this paper, we have summarized the nature of the Spacetime Continuum (*STC*) as provided by the Elastodynamics of the Spacetime Continuum (*STCED*), which provides a better understanding of general relativistic spacetime. We have shown that the dimensionality of the Spacetime Continuum could be deduced mathematically if the value of the Lamé elastic constants $\bar{\kappa}_0$, $\bar{\mu}_0$ and $\bar{\lambda}_0$ of the Spacetime Continuum could be determined experimentally. From Einstein's field equation for an isotropic and homogeneous STC, we derive the value of the Spacetime Continuum bulk modulus $\bar{\kappa}_0$ in terms of elementary constants.

STCED provides a physical model of the nature of inertial mass, which also includes an explanation for wave-particle duality. Mass is shown to be the invariant change in volume of spacetime in the longitudinal propagation of energy-momentum in the spacetime continuum. Hence mass is not independent of the spacetime continuum, but rather mass is part of the spacetime continuum fabric itself.

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