Natural Metrology in Physics of Numerical Relations

Hartmut Müller

Rome, Italy
E-mail: hm@interscalar.com

The paper introduces the natural electron metrology that is based on the electron mass, the speed of light in a vacuum, and the Planck constant. Since the units of the electron metrology are natural, their application gives physical meaning to the numerical properties of the readings and allows to identify and predict physical effects caused by numerical relations. In this paper, the electron metrology is applied to real systems of coupled periodic processes, in particular to the solar system and exoplanetary systems. It is shown that the application of the electron metrology allows to define numerical conditions for lasting stability and to identify evolutionary trends.

Introduction

In physics, measurement is the source of data that allows to develop and verify theoretical models of reality. The result of a measurement is the ratio of physical quantities where one of them is the reference quantity called unit of measurement. Obviously, the value of this ratio depends on the chosen unit of measurement. Moreover, any change of the unit of measurement changes also the numerical properties of the value. For example, a 20 cm microwave and a 7.874 . . . inch microwave both have the same wavelength. However, 20 is integer, but 7.874 . . . is not. Thus, an arbitrarily chosen unit of measurement results in random values of the measured ratios. In this case, also the numerical properties of the measured values are random, and their physical interpretation has no sense. This is why in theoretical physics numerical ratios usually remain outside the realm of interest.

The situation changes fundamentally, if we choose natural units of measurement, for instance, a natural frequency of a real periodical process. In this case, all the harmonics have rational values. Thus, the use of natural units gives physical meaning to the numerical properties of the readings. Now the numerical properties of the measured frequencies provide information about whether they are harmonics or not.

Indeed, the history of metrology shows a clear trend to natural units of measurement. For instance, the current SI definition [1] of a second is based on the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom. One second takes 9,192,631,770 periods of this radiation. However, the number of periods is arbitrarily chosen. Therefore, in the current definition, one second is not a natural unit of measurement, although it is based on the frequency of a natural subatomic process. Also the current SI unit meter is not a natural unit as it is based on the current definition of a second.

The current SI definition of the kilogram is based on the fixed numerical value of the Planck constant, expressed in units of meter and second. Therefore, one kilogram is not a natural unit of measurement. Consequently, all secondary units of measurement based on kilogram, meter and second cannot be considered natural. Therefore, the current SI is not a system of natural units.

The concept of natural units was first introduced in 1874, when George Stoney [2], noting that electric charge is quantized, derived units of length, time, and mass, now named Stoney units in his honor. Stoney chose his units so that the Newtonian gravitational constant, the speed of light in a vacuum, and the electron charge would be numerically equal to 1. In 1899, Max Planck proposed a system of units that is based on the quantum of action. Planck underlined the universality of the new system, writing [3]: ... it is possible to set up units for length, mass, time and temperature, which are independent of special bodies or substances, necessarily retaining their meaning for all times and for all civilizations, including extraterrestrial and non-human ones, which can be called natural units of measure. Planck derived units for length, time, mass, and temperature from the Newtonian gravitational constant, the speed of light, the quantum of action, and the Boltzmann constant.

Regrettably, using Newton's gravitational constant G increases not only the uncertainty of the Planck system, but also its dependence on theoretical assumptions. The constancy of G is only postulated, its value is measured in laboratory scale only, and there is no guaranty of its universality in astronomical scales, because the mass of a planet, planetoid or moon cannot be measured without using G.

In [4] we proposed a system of natural units that is based on the electron mass, the speed of light in a vacuum, the Planck constant, and the Boltzmann constant. The only difference to the Planck system is that we use the electron mass instead of G. However, this difference seems to be significant enough to give physical meaning to the numerical properties of the readings.

In [5] we have shown that in electron units, the masses of elementary particles including the proton have numerical values that approximate integer and reciprocal integer powers of Euler's transcendental number $e = 2.71828\ldots$

As we have shown in [6], the orbital and rotational periods of the planets, planetoids and large moons of the solar system have numerical values that approximate integer powers of Euler's number, if expressed in electron units (table 1). This we have shown also for 1430 exoplanets. Furthermore, the gravitational parameters of the Sun and the planets of the solar system, if expressed in electron units, approximate integer powers of Euler's number [7].

The electron mass is actually the key component in the natural metrology that we propose in this paper. The electron mass defines an absolute reference value, and the Planck constant in combination with the speed of light are interdimensional converters that allow to derive absolute spatial and temporal reference values, which are the Compton wavelength of the electron, and its natural frequency. The Boltzmann constant allows to derive the electron black body temperature as additional natural unit.

The electron is not a rare substance since it is ubiquitous in the universe. The uniqueness of the electron stems from its elementarity and exceptional stability, with an estimated lifetime of over 10^{28} years. In fact, stability and high precision are fundamental requirements for units of measurement. The electron mass is given with an accuracy of 10^{-10}, as shown in table 1. Since the speed of light and the Planck constant are fixed, the accuracy of the electron metrology depends only on the accuracy of the electron mass.

In the following we will show that the application of the electron metrology gives physical meaning to the numerical properties of the readings and allows to identify and predict physical effects caused by numerical relations. For reasons of clarity, in this paper we deal with periodical processes.

ELECTRON UNITS	DEFINITION	VALUE
Electron rest energy	$E = m/c^2$	0.51099895000(15) MeV
Angular frequency	$\omega = E/\hbar$	$7.76344 \cdot 10^{20}$ Hz
Oscillation period	$\tau = 1/\omega$	$1.28809 \cdot 10^{-21}$ s
Compton wavelength	$\lambda = c/\omega$	$3.86159 \cdot 10^{-13}$ m

Table 1: Basic units of the electron metrology. The units are calculated from the measured electron rest energy. The speed of light c in a vaccum, and the Planck constant $\hbar$ are fixed. Data from Particle Data Group [12].

Theoretical Approach

The starting point of our approach is frequency as obligatory characteristic of a periodical process. As the result of a measurement is always a *ratio* of physical quantities, one can measure only *ratios* of frequencies. This ratio is always a real number. Being a real value, this ratio can approximate an integer, rational, irrational algebraic or transcendental number. In [8] we have shown that the difference between rational, irrational algebraic and transcendental numbers is not only a mathematical task, but it is also an essential aspect of stability in systems of coupled periodical processes. For instance, integer frequency ratios, in particular fractions of small integers, make possible parametric resonance that can destabilize such a system [9, 10]. This is why asteroids cannot maintain orbits that are unstable because of their resonance with Jupiter [11]. These orbits form the Kirkwood gaps that are areas in the asteroid belt where asteroids are absent.

According to this idea, irrational frequency ratios should not cause destabilizing parametric resonance, because irrational numbers cannot be represented as a ratio of integers. However, algebraic irrational numbers, being real roots of algebraic equations, can be converted to rational numbers by multiplication. For example, $\sqrt{2} = 1.41421\ldots$ cannot become a frequency scaling factor in real systems of coupled periodical processes, because $\sqrt{2} \cdot \sqrt{2} = 2$ creates the conditions for the occurrence of parametric resonance. Thus, only transcendental ratios can prevent parametric resonance, because they cannot be converted to rational or integer numbers by multiplication. Actually, it is transcendental numbers that define the preferred frequency ratios which allow to avoid destabilizing parametric resonance [13]. In this way, transcendental frequency ratios sustain the lasting stability of coupled periodical processes.

Among all transcendental numbers, Euler's number $e = 2.71828\ldots$ is unique, because its real power function e^x coincides with its own derivatives. In the consequence, Euler's number allows avoiding parametric resonance between any coupled periodical processes including their derivatives.

Because of this unique property of Euler's number, we expect that periodical processes in real systems prefer frequency ratios close to Euler's number and its roots. For rational exponents, the natural exponential function is always transcendental [14]. The natural logarithms of those frequency ratios are therefore close to integer or reciprocal integer values, which are attractors of transcendental numbers of the type e^x, as we have shown in [13]. With reference to the evolution of a planetary system and its stability, we may therefore expect that the ratio of any two orbital periods should finally approximate an integer or reciprocal integer power of Euler's number [15].

The electron shares its exceptional stability with the proton with an estimated lifetime of over 10^{29} years [12]. Within our approach, the stability of the proton results from the numerical properties of the proton-to-electron ratio that approximates the 7^{th} power of Euler's number and its square root [7]. In this way, the metric properties of the proton can be derived from the metric properties of the electron theoretically.

The eigenfrequencies and harmonics of the proton and the electron are natural frequencies of any type of matter, also of the accreted matter of a planet. Conventional models of the solar system do not take into account this aspect, which lies at the core of our numeric physical approach to the electron metrology. Given the enormous number of protons and electrons that form a planet, eigenresonance must be avoided in the long term. This affects any periodical process including orbital and rotational motion. This is why the planets in the solar system and in hundreds of exoplanetary systems have orbital periods that approximate integer and rational powers of Euler's number relative to the natural oscillation periods of the proton and the electron, as shown in my paper [6].

In the following, we discuss exemplary applications of the electron metrology to the analysis of orbital and rotational periods in the solar system.

Exemplary Applications

Kepler's laws of planetary motion do not explain why the planets of the solar system have the orbital periods 87.969, 224.701, 365.256, 686.971 days, and 11.862, 29.457, 84.02, 164.8, 247.94 years, because there are infinitely many pairs of orbital periods and distances that fulfill Kepler's laws. Einstein's field equations do not reduce the theoretical variety of possible orbits, but increases it even more.

However, if we express the orbital periods in electron units, we can realize that they approximate integer powers and roots of Euler's number, and in this way, they avoid destabilizing parametric resonance. This requirement reduces dramatically the number of possible orbits.

For instance, if we express Jupiter's orbital period in years (11.862), in days (4332.59) or in seconds ($3.74343 \cdot 10^8$), there is no way to verify whether this value is special or not. If we express Jupiter's orbital period in oscillation periods of the electron, we can realize that it is indeed very special, because it approximates the 66^{th} power of Euler's number:

$$\ln\left(\frac{T_O(Jupiter)}{2\pi \cdot \tau_e}\right) = \ln\left(\frac{3.74343 \cdot 10^8 \text{ s}}{2\pi \cdot 1.28809 \cdot 10^{-21} \text{ s}}\right) = 66.00$$

The same is valid for the orbital period 686.98 days ($5.93551 \cdot 10^7$ seconds) of the planet Mars that equals the 66^{th} power of Euler's number multiplied by the *angular* oscillation period of the electron:

$$\ln\left(\frac{T_O(Mars)}{\tau_e}\right) = \ln\left(\frac{5.93551 \cdot 10^7 \text{ s}}{1.28809 \cdot 10^{-21} \text{ s}}\right) = 66.00$$

Consequently, the Jupiter-to-Mars orbital period ratio is 2π:

$$T_O(Jupiter) = 2\pi \cdot T_O(Mars)$$

This transcendental ratio allows Mars to avoid parametric orbital resonance with Jupiter. Approaching an integer power of Euler's number relative to the electron's natural period of oscillation prevents both Jupiter's and Mars' periodic orbital motion from provoking electron based eigenresonance. Since the proton-to-electron ratio approximates an integer power of Euler's number and its square root, both planets avoid also proton based eigenresonance.

In [16] we have shown that integer and rational powers of $e = 2.71828\ldots$ and $\pi = 3.14159\ldots$ form two complementary fractal scalar fields of transcendental attractors – the *Euler field* and the *Archimedes field*.

The rotational periods of planets and planetoids of the solar system approximate integer powers of Euler's number and its square root relative to the angular oscillation period of the electron. Since the proton-to-electron ratio approximates the 7^{th} power of Euler's number and its square root, the rotational periods approximate integer powers of Euler's number relative to the angular oscillation period of the proton, as we have shown in [16].

For instance, the current sidereal rotational period of the Earth equals 23 h, 56 min and 4.1 s, or 86164.1 s. In general, the duration of the sidereal day should increase, because it is believed that the rotation of the Earth is slowing down. Indeed, if we express the sidereal rotational period of the Earth in electron units, we can realize that it must increase in order to reach the 59^{th} power of Euler's number and its square root:

$$\ln\left(\frac{T_R(Earth)}{\tau_e}\right) = \ln\left(\frac{86164.1 \text{ s}}{1.28809 \cdot 10^{-21} \text{ s}}\right) = 59.47$$

However, our numeric physical approach suggests that the rotation of the Earth will slow down only until the sidereal day reaches a duration of 24 hours, 47 minutes and 1 second, or 89221 s that corresponds with the Euler-attractor:

$$\tau_e \cdot e^{59} \cdot \sqrt{e} = 89221 \text{ s}$$

When the sidereal period of rotation has reached that Euler-attractor, the rotation of the Earth should be stabilized, and should not slow down more. By the way, the sidereal rotational period of the planet Mars 24 hours, 37 minutes and 22.7 seconds, or 88642.7 s is much closer to that attractor:

$$\ln\left(\frac{T_R(Mars)}{\tau_e}\right) = \ln\left(\frac{88642.7 \text{ s}}{1.28809 \cdot 10^{-21} \text{ s}}\right) = 59.49$$

Probably, smaller bodies with faster rotation can reach numerical attractors faster than larger bodies. The sidereal rotational period 9.07417 h = 32667 s of the planetoid Ceres, for example, has already reached an Euler-attractor:

$$\ln\left(\frac{T_R(Ceres)}{\tau_e}\right) = \ln\left(\frac{32667 \text{ s}}{1.28809 \cdot 10^{-21} \text{ s}}\right) = 58.50$$

In general, every prime, irrational or transcendental number generates a unique fundamental fractal field of its own integer and rational powers that causes physical effects which are typical for that number.

For instance, integer and rational powers of 2 and 3 generate two different fractal scalar fields – the fundamental binary and the fundamental ternary fields, which are the strongest providers of parametric resonance.

On the contrary, the golden ratio $\phi = (\sqrt{5} + 1)/2 = 1.618\ldots$ makes difficult its rational approximation, since its continued fraction does not contain large denominators. So, the fundamental field of its integer and rational powers should be a perfect inhibitor of resonance amplification. This is why the Venus-to-Earth orbital period ratio approximates $1/\phi$, as already shown by Butusov [17] in 1978.

In [16] we have proposed to name this field after Hippasus of Metapontum who was an ancient Greek philosopher and early follower of Pythagoras, and is widely credited with the discovery of the existence of irrational numbers, and the first proof of the irrationality of the golden ratio.

Although the golden ratio is irrational, it is a Pisot number, so its powers are getting closer and closer to whole numbers, for example, $\phi^{10} = 122.99\ldots$ This is why the Hippasus field can inhibit resonance within small frequency ranges only. Hence, in systems with many coupled periodic processes, the Hippasus field can produce two opposing effects: over small frequency ranges, the Hippasus field can inhibit parametric resonance, but over large frequency ranges, it provides the long-period appearance of resonance amplification. Euler's number is not a Pisot number, so that the Euler field permits coupled periodic processes to avoid parametric resonance also over very large frequency ranges. As we have shown in [6, 7], typical examples are the orbital and rotational periods of planets and planetoids.

Conclusion

The use of natural units of measure gives physical meaning to the numerical properties of the readings and allows the study of physical effects caused by their numerical relations.

In the case of frequency ratios, the readings are real numbers that can approximate integer, rational, irrational algebraic or transcendental values.

In application to real systems of coupled periodic processes, transcendental numerical relations can avoid destabilizing parametric resonance and provide lasting stability.

In units of the electron metrology (table 1), the orbital and rotational periods of large bodies of the solar system approximate integer powers of Euler's number and its roots multiplied by the natural oscillation period of the electron. This we have verified [6] also for 1430 exoplanets.

The perihelion and aphelion of a planetary orbit, if expressed in units of the electron metrology, give the lower and upper approximations of integer powers of Euler's number, as we have shown in [7]. As a consequence, the gravitational parameters of the Sun and its planets, if expressed in electron units, approximate integer powers of Euler's number.

The maxima in the frequency distribution of the number of stars as function of the distance between them, expressed in electron units, correspond with integer powers of Euler's number and its roots. In [18] we have shown this for 18336 interstellar distances in the solar neighborhood.

All these findings allow us to interpret the approximation of integer powers of Euler's number and its roots as general evolutional trend.

In this context, also the current temperature 2.726 K of the cosmic microwave background radiation (CMBR) does not appear as to be accidental. In [8] we have shown that this temperature, if expressed in electron units, approximates an integer power of Euler's number. Consequently, it is very unlikely that the temperature of the CMBR will still decrease. This conclusion contradicts the big bang model of a cooling down universe. However, a resonating with protons and electrons fulfilling the entire cosmic space microwave radiation could probably impede the formation of molecules essential for life. By obeying the Euler field, the CMBR allows life to arise. From this point of view, the Euler field can be seen as a promoter of life on a cosmic scale.

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References

1. The International System of Units. International Bureau of Weights and Measures, 2019, ISBN 978-92-822-2272-0
2. Barrow J. D. Natural Units Before Planck. *Quarterly Journal of the Royal Astronomical Society*, vol. 24, pp. 24–26.
3. Planck M. Über Irreversible Strahlungsvorgänge. *Sitzungsbericht der Königlich Preußischen Akademie der Wissenschaften*, 1899, v.1, 479–480.
4. Müller H. Scale-Invariant Models of Natural Oscillations in Chain Systems and their Cosmological Significance. *Progress in Physics*, 2017, v. 13, 187–197.
5. Müller H. Fractal Scaling Models of Natural Oscillations in Chain Systems and the Mass Distribution of Particles. *Progress in Physics*, 2010, v. 6, 61–66.
6. Müller H. Physics of Transcendental Numbers meets Gravitation. *Progress in Physics*, 2021, vol. 17, 83–92.
7. Müller H. Physics of Transcendental Numbers as Forming Factor of the Solar System. *Progress in Physics*, 2022, v. 18, 56–61.
8. Müller H. On the Cosmological Significance of Euler's Number. *Progress in Physics*, 2019, v. 15, 17–21.
9. Dombrowski K. Rational Numbers Distribution and Resonance. *Progress in Physics*, 2005, v. 1, no. 1, 65–67.
10. Panchelyuga V.A., Panchelyuga M. S. Resonance and Fractals on the Real Numbers Set. *Progress in Physics*, 2012, v. 8, no. 4, 48–53.
11. Minton D. A., Malhotra R. A record of planet migration in the main asteroid belt. *Nature*, Vol. 457, 1109–1111, (2009).
12. Workman R. L. et al. (Particle Data Group), *Prog. Theor. Exp. Phys.*, 083C01 (2022), www.pdg.lbl.gov
13. Müller H. The Physics of Transcendental Numbers. *Progress in Physics*, 2019, vol. 15, 148–155.
14. Hilbert D. Über die Transcendenz der Zahlen e und π. *Mathematische Annalen*, 43, 216–219, (1893).
15. Müller H. Global Scaling of Planetary Systems. *Progress in Physics*, 2018, v. 14, 99–105.
16. Müller H. Physics of Irrational Numbers. *Progress in Physics*, 2022, vol. 18, 103–109.
17. Butusov K. P. The Golden Ratio in the Solar system. *Problems of Cosmological Research*, vol. 7, Moscow–Leningrad, 1978.
18. Müller H. Physics of Transcendental Numbers Determines Star Distribution. *Progress in Physics*, 2021, vol. 17, 164–167.