Gödel Time Travel: New Highlights

Patrick Marquet
Calais, France. E-mail: patrick.marquet6@wanadoo.fr

The history of fascinating idea of time travel can be traced back to Kurt Gödel who found a solution of Einstein's field equations that contains closed time-like curves (CTCs). Those make it theoretically feasible to go on journey into one's own past. In what follows, we establish a realistic way to provide the required conditions to achieve this time displacement. After having given Gödel's model a physical meaning, we assign an object to move along a closed time-like curve using the warp drive technique. Provided the object bears circulating charges interacting with a surrounding electromagnetic field, it is possible to extract a negative energy necessary to sustain the warp drive without resorting to the hypothetical "exotic matter". In addition, this field/charge interaction has the virtue to drastically reduce the amount of required negative energy. Lastly, the entropy of the system is shown to be negative during the time journey into the past.

Notations

Space-time indices are: $\mu, \nu = 0, 1, 2, 3$.
Spatial indices are: $a, b = 1, 2, 3$.
The space signature is -2 (unless otherwise specified).
Newton's constant is G.

1 The generalized Gödel metric

The classical Gödel line element is generically given by the interval [1]

$$ds^2 = a^2\left(dx_0^2 - dx_1^2 + dx_2^2\,\frac{1}{2}\,e^{2x_1} - dx_3^2 + 2\,e^{x_1}dx_0 dx_2\right), \quad (1)$$

or equivalently

$$ds^2 = a^2\left[-dx_1^2 - dx_3^2 - dx_2^2\,\frac{1}{2}\,e^{2x_1} + (e^{x_1}dx_2 + dx_0)^2\right] \quad (2)$$

expliciting x_0

$$ds^2 = a^2\left[c^2dt^2 + \frac{1}{2}\,e^{2x}dy^2 - 2\,e^{x}c\,dt\,dy - dx^2 - dz^2\right], \quad (2\text{bis})$$

where $a > 0$.

In the cylindrical coordinates (t, r, ϕ) with the transformations

$$e^x = \cosh 2r + \cosh\phi\,\sinh 2r\,,$$

$$y\,e^x = \sqrt{2}\sinh\phi\,\sinh 2r\,,$$

$$\tan\frac{1}{2}\left[\phi + \left(ct - \frac{2t'}{2\sqrt{2}}\right)\right] = e^{-2r}\tan\frac{\phi}{2}\,,$$

the metric reads

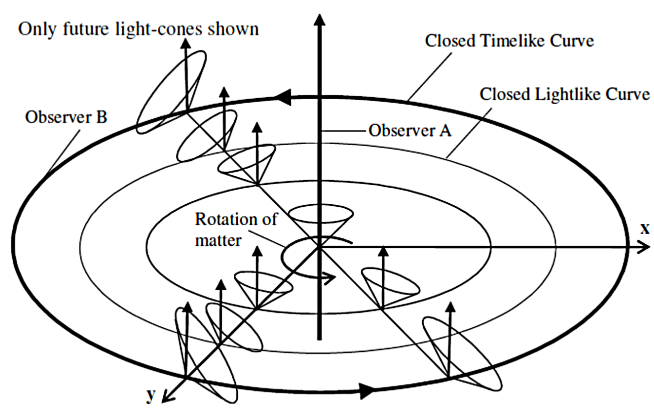
$$ds^2 = 4a^2\Big[(dt')^2 - dr^2 + \left(\sinh^4 r - \sinh^2 r\right)d\phi^2 + {} + 2\sqrt{2}\,\sinh^2 r\,d\phi\,dt'\Big] \quad (3)$$

(the inessential coordinate z is here suppressed).

In its original formulation, the Gödel universe describes a set of masses (such as galaxies, stars and planets) rotating about arbitrary axes.

The metric (3) exhibits a rotational symmetry about the axis $r = 0$ since we clearly see that the components of the metric tensor do not depend on ϕ.

For $r \geqslant 0$, we have $0 \leqslant \phi \leqslant 2\pi$ If a curve r_G is defined by $\sinh r = 1$ that is $r_G = \log\left(1 + \sqrt{2}\right)$, then for any curve $r > r_G$ we have $\sinh^4 r - \sinh^2 r > 0$. Such a curve which materializes in the "plane" $t = const$ is a closed time-like curve (CTC). The radius r_G referred to as the Gödel radius, induces a light-like curve or closed null curve where the light cones are tangential to the plane of constant t. With increasing $r > r_G$, the light cones continue to keel over and their opening angles widen until their future parts reach the negative numerical values of t.



As a consequence a spacecraft can move in such way that its chronological order with the positive cosmic time is reversed.

In order to make his metric compatible solution to Einstein's field equations, Gödel is led to introduce the cosmological constant Λ as

$$G_{\mu\beta} = \frac{8\pi G}{c^4}\,\rho c^2 u_\mu u_\beta + \Lambda g_{\mu\beta}\,. \tag{4}$$

To achieve this compatibility he then further sets

$$a^{-2} = \frac{8\pi G}{c^2}\,\rho\,,$$

$$\Lambda = -\frac{1}{2}R = -\frac{1}{2a^2} = -\frac{4\pi G}{c^2}\,\rho\,.$$

Finetuning the hypothetical cosmological constant with the (mean) density of the universe and the Ricci scalar R, appears as a rather dubious physical argument.

In our publication [2], we assumed that a is slightly space-time variable and we set

$$a^2 = \mathrm{e}^2. \tag{5}$$

As a result, the Gödel metric tensor components are conformal to the real Gödel metric tensor $g_{\mu\nu}$

$$(g_{\mu\nu})' = \mathrm{e}^{2U} g_{\mu\nu}\,, \quad (g^{\mu\nu})' = \mathrm{e}^{-2U} g^{\mu\nu}.$$

The exact Gödel metric reads now

$$ds^2 = \mathrm{e}^{2U}\left[c^2dt^2 + \frac{1}{2}\,\mathrm{e}^{2x}dy^2 - 2\,\mathrm{e}^x c\,dt\,dy - dx^2 - dz^2\right] \tag{6}$$

or

$$ds^2 = 4\,\mathrm{e}^{2U}\Big[(dt')^2 - dr^2 + \left(\sinh^4 r - \sinh^2 r\right)d\phi^2 + + 2\sqrt{2}\,\sinh^2 r\,d\phi\,dt'\Big]. \tag{7}$$

This implies that this metric is a straightforward solution of the field equations describing a peculiar perfect fluid [3–5]

$$G_{\mu\beta} = \frac{8\pi G}{c^4}\left[\left(\rho c^2 + P\right)u_\mu u_\beta - P g_{\mu\beta}\right]. \tag{8}$$

The model is now likened to a fluid in rotation with mass density ρ and pressure P with an equation of state $\rho = f(P)$.

The positive scalar U is shown to be

$$U(x^\mu) = \int \frac{dP}{\rho c^2 + P}\,. \tag{9}$$

From (7) one formally infers that the flow lines of matter of the fluid follow conformal geodesics given by

$$s' = \int \mathrm{e}^U ds\,. \tag{10}$$

The 4-vector $K_\nu = \partial_\nu U$ is regarded as the 4-acceleration of the flow lines [6]. The hallmark of the theory is the substitution (5). With this new definition,the Gödel space-time is no longer the representation of a cosmological model but it is relegated to the rank of an ordinary metric where its physical properties could allow for a possible replication.

Rotation of the model and closed curves now depend on the fluid characteristics.

To this effect consider the metric

$$ds^2 = c^2(dt'')^2 - \frac{(dr'')^2}{1 + (r''/2\,\mathrm{e}^U)^2} - - r''\left[1 - (r''2\,\mathrm{e}^U)^2\right]d\phi^2 + 2(r'')^2\frac{c}{\sqrt{2}\,\mathrm{e}^U}\,d\phi\,dt''. \tag{11}$$

As easily verified it is equivalent to the metric (7) if we set [7]

$$r'' = 2\,\mathrm{e}^U\sinh r\,, \quad t'' = \frac{2\,\mathrm{e}^U t'}{c}\,. \tag{12}$$

In this new representation, we see that when $r'' = 2\,\mathrm{e}^U$, the coefficient in front of $d\phi^2$ vanishes. If we choose the cosmic time t'' describing the evolution of our universe as the rotation-axis, then $r''_G = 2\,\mathrm{e}^U$ constitutes the Gödel radius for which the time lines close up and are tangential to the light cones (null curves). These curves are contained in the plane $t'' = const.$ in the same way as detailed above. Inspection shows that the fluid rotates with the angular velocity

$$\omega = \frac{c}{\sqrt{2}\,\mathrm{e}^U}\,. \tag{13}$$

Through the equation of state $\rho = f(P)$, the Gödel radius will be set by tuning the pressure parameter P of the considered fluid.

Referring to the work initiated in [8], we complete hereinafter our last publication [9] in which a spacecraft moves along a Gödel trajectory by using a warp drive propulsion. The required negative energy will be now given a physical meaning.

2 A short review on Alcubierre's theory

2.1 The ADM formalism

Arnowitt, Deser and Misner (ADM) suggested to a construct a space-time foliation of hypersurfaces parametrized by an arbitrarily chosen time coordinate value x^0 [10]. This foliation is caracterised by a proper time $d\tau$ between two nearby hypersurfaces

$$x^0 = const\,, \quad \text{and} \quad x^0 + dx^0 = const\,, \tag{14}$$

where $cd\tau$ is proportionnal to dx^0

$$cd\tau = N(x^a, x^0)\,dx^0, \tag{15}$$

and in the ADM terminology, N is called the *lapse function*.

Let us evaluate the 3-vector whose spatial coordinates x^a are lying in the hypersurface $x^0 = const$, and the vector is normal to the hypersurface on the second hypersurface $x^0 + dx^0 = const$, where those coordinates become

$$N^a dx^0,$$

and the vector N^a is called the *shift vector*.

From these definitions follows the derivation of the 3-tensor

$$K_{ab} = (2N)^{-1} \left(-N_{a;b} - N_{b;a} + \partial_0 g_{ab}\right), \qquad (16)$$

It represents the "extrinsic curvature", and as such describes the manner in which the hypersurface $x^0 = const$ is embedded in the surrounding space-time.

With this brief preparation we are now able to tackle our topic.

2.2 Alcubierre's function

In 1994, M. Alcubierre showed that a superluminal velocity can be achieved without violating the laws of Relativity. He considered a perturbed space-time region likened to bubble ("warp drive") which could transport a spacecraft in a surfing mode inside the bubble, the proper time $d\tau$ is the coordinate time element dt measured by an external observer called "Eulerian".

The motion is only achieved by the space wave, so that the occupants of the spacecraft are at rest and would not suffer any acceleration nor time dilation in the displacement [11]. This process requires a front contraction of the space while subject to a rear expansion. The spacecraft center distance located in the bubble

$$r_{\mathrm{s}}(t) = \sqrt{\left(x - x_{\mathrm{s}}(t)\right)^2 + y^2 + z^2} \qquad (17)$$

varies until $\mathrm{R_e}$, which is the external radius of the bubble.

With respect to the distant observer the apparent velocity of the spacecraft is

$$\mathrm{v_s}(t) = \frac{dx_{\mathrm{s}}(t)}{dt}, \qquad (18)$$

where $x_{\mathrm{s}}(t)$ is the coordinate of the bubble's trajectory along the x-direction.

Alcubierre then defined the step function $f(r_{\mathrm{s}}, t)$

$$f(r_{\mathrm{s}}, t) = \frac{\tanh\left[\sigma(r_{\mathrm{s}} + \mathrm{R_e})\right] - \tanh\left[\sigma(r_{\mathrm{s}} - \mathrm{R_e})\right]}{2\tanh(\sigma \mathrm{R_e})}, \qquad (19)$$

where $\mathrm{R_e} > 0$ is the external radius of the bubble, and σ is a "bump" parameter used to tune the wall thickness of the bubble. The larger the parameter σ, the greater the contained energy density; for its shell thickness decreases. Moreover, the absolute increase of σ means a faster approach of the condition

$$\left.\begin{array}{l} \lim\limits_{\sigma\to\infty} f(r_{\mathrm{s}}, t) = 1 \quad \text{for} \quad r_{\mathrm{s}} \in [-\mathrm{R_e}, \mathrm{R_e}] \\ \text{and is 0 everywhere else} \end{array}\right\}. \qquad (20)$$

The Alcubierre metric is

$$(ds^2)_{\mathrm{AL}} = -c^2dt^2 + \left[dx - \mathrm{v_s} f(r_{\mathrm{s}}, t)\, cdt\right]^2 + dy^2 + dz^2. \quad (21)$$

Inspection shows that

$$K_{ab} = -u_{a;b}, \qquad (22)$$

which is sometimes called the second fundamental form of the 3-space.

Within this formalism, the expansion scalar becomes

$$\theta = \partial_1 N^1 = -\operatorname{tr} K_{ab} \qquad (23)$$

that with (20) is

$$\theta = \mathrm{v_s} \frac{df}{dr_{\mathrm{s}}} \frac{x_{\mathrm{s}}}{r_{\mathrm{s}}}. \qquad (24)$$

Let us now write the Alcubierre metric in the following equivalent form

$$(ds^2)_{\mathrm{AL}} = -\left[(1 - \mathrm{v_s^2} f^2(r_{\mathrm{s}}, t)\right] c^2dt^2 - \\ - 2\,\mathrm{v_s} f\, cdt\, dx + dx^2 + dy^2 + dz^2. \qquad (25)$$

Taking account of (20) one finally find the energy density

$$(T^{00})_{\mathrm{AL}} = -\frac{c^4}{32\pi G}\mathrm{v_s^2}\left(\frac{df}{dr_{\mathrm{s}}}\right)^2 \frac{y^2 + z^2}{r_{\mathrm{s}}^2}. \qquad (26)$$

This expression is unfortunately negative as measured by the Eulerian observer and therefore it violates the weak energy conditions (WEC) [12]. Notwithstanding this violation, one is nevertheless forced to introduce a way to obtain a negative energy density. This possibility is examined below.

2.3 Nature of the negative energy

We consider a spacecraft having a shell whose thickness is $\mathrm{R_e} - \mathrm{R_i}$, where $\mathrm{R_e}$ is the external radius, while $\mathrm{R_i}$ is the inner radius. $\mathrm{R_e}$ coincides with the Alcubierre bubble which thus constitutes the whole spacecraft contour.

Consider now a charge μ circulating within the shell thus giving rise of a 4-current density

$$j^\alpha = \mu u^\alpha. \qquad (27)$$

This current is coupled to a co-moving electromagnetic field characterized with the 4-potential A^a which yields the interacting energy-momentum tensor

$$(T^{\alpha\beta})_{\mathrm{elec}} = \frac{1}{4\pi}\left(\frac{1}{4} g^{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta} + F^{\alpha\sigma} F^{\beta}_{\nu}\right) + \\ + g^{\alpha\beta} j_\nu A^\nu - j^\alpha A^\beta. \qquad (28)$$

The extracted energy density is

$$(T^{00})_{\mathrm{elec}} = \frac{1}{4\pi}\left(\frac{1}{4} F_{\gamma\delta} F^{\gamma\delta} + F^{0\nu} F^{0}_{\nu}\right) + j_\nu A^\nu - j^0 A^0. \quad (29)$$

Since we chose an orthonormal basis, we have

$$(T^{00})_{\mathrm{elec}} = \frac{1}{8\pi}\left(\boldsymbol{E}^2 + \boldsymbol{B}^2\right)\frac{1}{4\pi}\,\Delta(\Phi \boldsymbol{E}), \qquad (30)$$

where $\boldsymbol{E}$ and $\boldsymbol{B}$ are respectively the electric and magnetic field strengths derived from the Maxwell tensor

$$F_{\gamma\delta} = \partial_\gamma A_\delta - \partial_\delta A_\gamma. \qquad (31)$$

We assume that the field potential $A^{\alpha}(\Phi, \boldsymbol{A})$ is given in the Lorentz gauge.

The charge density is derived from

$$\Delta \boldsymbol{E} = 4\pi\mu\,, \tag{32}$$

which is just the time component of the 4-current density inferred from Maxwell's equations

$$\nabla_{\alpha} F^{\alpha\beta} = \frac{4\pi}{c}\, j^{\beta}. \tag{33}$$

Therefore negative energy density may be shown explicitly by the interaction tensor

$$(T^{00})_{\text{electitn}} = \frac{1}{4\pi}\, \boldsymbol{E}\,\Delta\Phi + \mu\Phi, \tag{34}$$

$$(T^{00})_{\textbf{elecint}} = \frac{1}{4\pi}\left(-\Delta\Phi - \frac{1}{c}\,\partial_t \boldsymbol{A}\right)\Delta\Phi + \mu\Phi \tag{35}$$

since $\boldsymbol{E} = -\Delta\Phi - \frac{1}{c}\,\partial_t \boldsymbol{A}$.

In (35) the first term in the brackets is always negative. As to the last term, it is made negative when the time varying charge density μ and the scalar potential Φ are 180° out of phase (method reached by the use of phasors).

We now suppose that the positive free radiative energy density

$$(T^{00})_{\text{elecrad}} = \frac{1}{8\pi}\left(\boldsymbol{E}^2 + \boldsymbol{B}^2\right)$$

is confined within the spacecraft, i.e., right to the inner side of the shell wall. The interacting tensor $(T^{00})_{\text{elecint}}$ is set so as to exhibit its energy density part on the *external side* of the shell. This is made consistent since the charges are circulating *inside* the surrounding shell of the spacecraft.

So we see that negative energy production can be achieved with such a configuration. The higher the charge density and the higher the scalar potential, then the most effective negative energy density.

The local field equations read

$$G_{\mu\beta} = \frac{8\pi G}{c^4}\left[(\rho c^2 + P)u_{\mu}u_{\beta} - P g_{\mu\beta} + (T_{\mu\beta})_{\text{elec}}\right]. \tag{36}$$

Remains now the energy density level $(T^{00})_{\text{elecint}}$ which is anticipated to be very huge. There is however a possible drastic reduction which adequately exploits the contribution of the electromagnetic field interacting with the charges.

2.4 Reducing the required negative energy

The spacecraft bubble is externally charged surrrounded by a comoving electromagentic field. As such it follows a *finslerian geodesic* [13] provided the ratio $\frac{\mu}{\rho}$ remains constant along the trajectory

$$(ds)_{\text{shell}} = ds + \frac{\mu}{\rho}\, A_{\alpha}\, dx^{\alpha}, \quad ds = \sqrt{\eta_{\alpha\beta}\, dx^{\alpha} dx^{\beta}}\,. \tag{37}$$

Neglecting the non-quadratic terms the metric reads

$$(ds^2)_{\text{shell}} = ds^2 + \left(\frac{\mu}{\rho}\, A_{\alpha} dx^{\alpha}\right)^2. \tag{38}$$

The interacting charge of the spacecraft must now be included in the metric (25).

Because we are considering only the energy density of the spacecraft-bubble as a whole, the spatial components of $\frac{\mu}{\rho} A_{\alpha} dx^{\alpha}$ in (38) can be neglected and the interaction term reduces to its time component

$$\frac{\mu}{\rho}\, A_0\, dx^0 = \frac{\Phi\mu}{\rho}\, c dt\,. \tag{39}$$

The metric (37) becomes now

$$ds^2 = -\left(1 + \frac{\Phi\mu}{\rho}\right)^2 c^2 dt^2 + dz^2 + dx^2 + dy^2. \tag{40}$$

Notice that the time component of the metric tensor

$$g_{00} = -\left(1 + \frac{\Phi\mu}{\rho}\right)^2 \tag{41}$$

can be expressed by the following formula

$$M = -(1 + N)\,, \tag{42}$$

where the lapse function is defined as

$$N = \Phi\,\frac{\mu}{\rho}\,. \tag{43}$$

The Alcubierre metric (25) reads now

$$ds^2 = -\left[M^2 - \text{v}_{\text{s}}^2 f^2(r_{\text{s}})\right] c^2 dt^2 - 2\,\text{v}_{\text{s}} f(r_{\text{s}})\, cdt\, dx + dz^2 + dx^2 + dy^2. \tag{44}$$

The interaction term should be only function of r_{s}, R_{e}, σ, and of the thickness $(\text{R}_{\text{e}} - \text{R}_{\text{i}})$, but not depending on the velocity v_{s}.

Here, our analysis is not too dissimilar to the approach detailed in [14, 15].

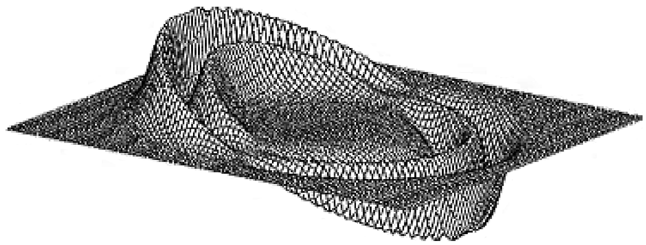
Finally, the negative energy density requirement is

$$\frac{c^4}{8G}\,\text{v}_{\text{s}}^2\left(\frac{df}{dr_{\text{s}}}\right)^2 \frac{y^2 + z^2}{r_{\text{s}}^2} = \left(\Delta\Phi + \frac{1}{c}\,\partial_t \boldsymbol{A}\right)\Delta\Phi + \mu\Phi\,. \tag{45}$$

The splitting shell/inner part of the spacecraft frame, is really the hallmark of the theory here it implies that the proper time τ of the inner part of the spacecraft is not affected by the term N.

The spacecraft-bubble follows the trajectory $x_{\text{s}}(t)$. Therefore For $\text{R} \leqslant \text{R}_{\text{e}}$, the bubble is assumed to be ruled by the new Alcubierre metric (44) expressed with the signature −2

$$ds^2 = (M^2 - \text{v}_{\text{s}}^2 f^2) c^2 dt^2 - 2\,\text{v}_{\text{s}} f(r_s)\, cdt\, dx - dz^2 - dx^2 - dy^2. \tag{46}$$



A 2D representation of the warped region according to (44). Propagation is from left (expansion) to right (contraction). The groove corresponds to the shell thickness determined by the function N.

This space-time is thus regarded as *globally hyperbolic* and the bubble will never know whether it moves along a CTC. As a result,the bubble is seen by a specific observer (see below) as being transported forward along the x-direction *tangential* to a CTC beyond the Gödel radius r_G.

We may now write down the *Gödel-Alcubierre metric*

$$ds^2 = e^{2U(1-f)} \left\{ \left[\left(1 + \frac{\Phi\mu}{\rho} \right)^2 - v_s^2 f^2 \right] c^2 dt^2 - \right.$$
$$- \left[f - \frac{1}{2} (1-f) e^{2x} \right] dy^2 -$$
$$\left. -2 \left[v_s f + (1-f) e^x \right] c dt dy - dx^2 - dz^2 \right\}. \quad (47)$$

In the absence of charge, beyond the distance R_e, we have $R > R_e \to \infty$ and $f = 0$ outside of the spacecraft-bubble and we retrieve Gödel's original modified metric (6).

3 Entropy along a Gödel trajectory

3.1 Relativistic thermodynamics

Consider a fluid that consists of n particles in motion within a given region. The primary variables are:

— The particle current

$$I^\mu = n u^\mu; \quad (48)$$

— The energy-momentum $T^{\mu\nu}$;

— The entropy flux S^μ,

where, obviously, $T^{\mu\nu}$ and I^μ are conserved

$$T^{\mu\nu}_{;\nu} = 0 \qquad I^\mu_{;\mu} = 0\,.$$

In a relativistic situation, the second law of thermodynamics requires

$$S^\mu_{;\mu} \geqslant 0\,. \quad (49)$$

For equilibrium states we have

$$S^\mu = n s u^\mu, \quad (50)$$

where s is the entropy per particle.

Denoting Q as the chemical potential and T the heat quantity (temperature) of the medium, the Euler relation reads

$$n s = \frac{\rho + P}{T} - \frac{Q n}{T}, \quad (51)$$

where ρ and P are respectively the density and pressure of the medium. We also have the Gibbs fundamental thermodynamic equation

$$T ds = ds \left(\frac{\rho}{n} \right) + P d \left(\frac{1}{n} \right) \quad (52)$$

or

$$T n ds = d\rho - \frac{\rho + P}{n} + dn\,. \quad (53)$$

From (51), we get

$$S^\mu = -\frac{Q I^\mu}{T} + \frac{(\rho + P)\, u^\mu}{T}\,. \quad (54)$$

Since in the rest system, the matter energy flux must vanish, we have

$$u_\lambda T^{\lambda\mu} = \rho u^\mu \quad (55)$$

and thus, we find the following expression for the entropy vector in equilibrium

$$S^\mu = -\frac{Q I^\mu}{T} + \frac{u_\lambda T^{\lambda\mu}}{T} + \frac{P u^\mu}{T}\,. \quad (56)$$

3.2 Applying to the time travel trajectory

Let us consider a spacecraft moving along a Gödel trajectory. We obviously neglect the chemical potential of the spacecraft's bodyframe as well as the pressure and the entropy vector reduces to

$$S^\mu = \frac{u_\lambda T^{\lambda\mu}}{T}\,. \quad (57)$$

This vector must be measured by the Eulerian observer which travels along the trajectory tangential to u^μ, where he "sees"

$$\frac{dt}{d\tau} = M^{-1}. \quad (58)$$

With this definition, it is easy to show that its velocity components are

$$(u^\mu)_E = \left[cM^{-1}, v_s f c M^{-1}, 0, 0 \right] \quad (59)$$

$$(u_\mu)_E = [\, cM, 0, 0, 0\,]\,. \quad (59\text{bis})$$

We are interested in the entropy scalar part

$$(S^0)_E = \frac{(u_0)_E\, (T^{00})_{AL}}{T} \quad (60)$$

with

$$(T^{00})_{AL} = -\frac{c^4}{32\pi G} \frac{v_s^2 \left(y^2 + z^2 \right)}{M^4 r_s^2} \left(\frac{df}{dr_s} \right)^2, \quad (61)$$

$$(u_0)_E = cM. \quad (62)$$

We clearly see that the entropy $(S^0)_E$ of the system is negative. Hence, the entropy S^0 attached to the spacecraft is seen negative with respect to the Eulerian observer which thus measures a "negentropy". While travelling to the past, the occupants of the spacecraft experience a positive entropy, i.e., they are ageing in their own proper time.

Conclusions

In the novel "The time machine" (1895) by H. G. Wells, an english scientist constructs a machine which allows him to travel back and forth in time.

Closed time-like curves were discovered in the 1920's, but it is really in 1988 that time travel possibility was seriously considered by physicists in the stunning article [16]. The Gödel solution was mainly regarded as a mathematical curiosity and thus it was almost forgotten. We have succeeded in reviving his work by using some transformations which give Gödel's mathematical derivation a full physical significance. In this view, the major contribution of J. Natàrio's work [17, 18] introduces now a complementary perspective.

So far, Gödel's model only depicts a travel into the past. What about the journey home? If advanced civilizations harness the time travel technology, they must be able to return to their own present, meaning a reversed time orientation. In the light of the aforementioned derivations we conclude that they should take another path. A possibility arises by considering our recent publication [19]. In this article, we recalled that the current Einstein's field equations are inferred from the second Bianchi's identity which is verified by the Riemann tensor. The latter tensor can be particularized to the *Landau-Lifchitz superpotential* [20], which is shown to yield two opposite field equations (not necessarily symmetrical) coupled with a common index.

Identifying the time coordinate chosen as the cosmic time-axis, to this common index, the solution of the second field equation would ten display a reversed time orientation. In this case an advanced civilization could adequately exploit this circumstance to return to its epoch.

Much remains to be worked out on the subject, but we trust that Gödel's legacy will continue to stimulate my fundamental researches in this field.

Submitted on May 31, 2024

References

1. Gödel K. An example of a new type of cosmological solutions of Einstein's field equations of gravitation. *Review of Modern Physics*, 1949, v. 21, no. 3, 447–450.
2. Marquet P. The exact Gödel solution. *Progress in Physics*, 2021, v. 17, no. 2, 133–138.
3. Eisenhart L.P. Space-time continua of perfect fluids in General Relativity. *Trans. Amer. Math. Soc.*, 1924, v. 26, 205–220.
4. Synge J.L. Relativistic hydrodynamics. *Proc. Lond. Math. Soc.*, 1937, v. 43, 376–416.
5. Lichnerowicz A. Les Théories Relativistes de la Gravitation et de l'Electromagnétisme. Masson et Cie, Paris, 1955.
6. Hawking S.W., Ellis G.F.R. The Large Scale Structure of Space time. Cambridge University Press, 1987.
7. Kajari E., Walser R., Schleich P., Delgado A. Sagnac effect of Gödel's universe. arXiv: 0404032 (2004).
8. Tippett B.J., Tsang D. Traversable achronal retrograde domain in space-time. arXiv: 1310.7985 (2013).
9. Marquet P. Gödel time travel with warp drive propulsion. *Progress in Physics*, 2022, v. 18, no.1, 82–88.
10. Arnowitt R., Deser S., Misner C. Dynamical structure and definition of energy in General Relativity. *Physical Review*, 1959, v. 116, no. 5, 1322–1330.
11. Alcubierre M. The warp drive: hyper fast travel within General Relativity. *Classical and Quantum Gravity*, 1994, v. 11, L73–L77.
12. Marquet P. The generalized warp drive concept in the EGR theory. *The Abraham Zelmanov Journal*, 2009, v. 2, 261–287.
13. Marquet P. Geodesics and Finslerian equations in the EGR theory. *The Abraham Zelmanov Journal*, 2010, v. 3, 90–100.
14. Loup F., Waite D., Held R., Halerewicz E.Jr., Stabno M., Kuntzman M., Sims R. A causally connected superluminal Warp Drive spacetime. arXiv: 020.2021 (2002).
15. Loup F., Waite D., Halerewicz E.Jr. Reduced total energy requirements for a modified Alcubierre warp drive spacetime. arXiv: 010.70975 (2001).
16. Morris M.S., Thorne K.S., Yurtsever U. Wormholes, time machine and the weak energy condition. *Phys. Rev. Let.*, 1988, v. 61, no. 13, 1446–1449.
17. Natàrio J. Optimal time travel in the Gödel universe. arXiv: 1105.619 (2011).
18. Natario J. Warp Drive with zero expansion. arXiv: 01.10086 (2002).
19. Marquet P. Twin universes confirmed by General Relativity. *Progress in Physics*, 2022, v. 18, no. 1, 89–94.
20. Landau L., Lifshitz E. The Classical Theory of Fields. Addison-Wesley, Reading, 1962.