Graph Theory Entropy Values for Lepton and Quark Discrete Symmetry Quantum States and Their Decay Channels

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We assume that each lepton family and each quark family represents its own unique discrete symmetry modular group, one which is also a binary subgroup of SU(2). Equivalently, we have a different regular 3-D polyhedral group for each lepton family and a different regular 4-D polytope group for each quark family. Being discrete symmetry subgroups representing 3-D and 4-D geometric objects that are known also as complete graphs, they each possess a different graph theory entropy based upon the number of connected paired vertices. We examine the various decay channels of the leptons and the quarks that obey all the conservation laws as well as the special theory of relativity to look for a violation of a fundamental graph theory entropy inequality constraint. Such a violation and its experimental verification would confirm the importance of graph theory entropy in particle physics.

1 Introduction

The identification of the symmetry group or groups for the lepton families and for the quark families of the Standard Model (SM) has been an interesting challenge for decades. In recent years there has been an emphasis on the discrete symmetry modular groups [1] such as $\Gamma_3 = A_4$, $\Gamma_4 = S_4$, $\Gamma_5 = A_5$, as well as their double groups Γ'_3, Γ'_4, and Γ'_5, where the A_4, S_4, and A_5 refer to equivalent permutation groups. These discrete symmetry modular groups not only connect directly to a top-down approach using superstring concepts (see e.g. [2, 3] for recent reviews) but also represent the discrete symmetries of the regular polyhedrons in R^3. However, no discrete symmetry group or set of discrete symmetry groups has been accepted yet, even though neutrino mass values are predicted, because the true mass values for the neutrinos are not known for direct comparison [4].

In a series of articles and conference presentations since 1987 we have proposed [5–7] that each lepton family represents a unique discrete symmetry binary subgroup of the continuous group SU(2), or equivalently, of the quaternion group Q and the modular group. That is, they represent the only finite quaternion subgroups that enclose a 3-D volume. Specifically, the electron family (ν_e, e^-) represents 2T = (3,3,2) = Γ'_3, the muon family (ν_μ, μ^-) represents 2O = (3,4,2) = Γ'_4, and the tau family (ν_τ, τ^-) represents 2I = (3,5,2) = Γ'_5, where the first and second group notations are also the familiar geometrical names for the 3-D regular polyhedron groups, i.e. the Platonic solids in R^3. As subgroups of SU(2), there is an upper and lower quantum state in each family.

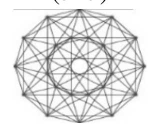
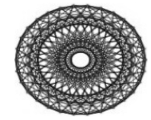
We proposed also [5, 6] that quark families represent the related discrete symmetry groups for the 4-D regular polytopes that enclose a volume, (3,3,3) for the (u, d) family, (3,3,4) for the (c, s) family, (3,4,3) for the (t, b) family, and (3,3,5) for the predicted 4th quark family, i.e. a top/bottom family (t', b') or (T, B). Of course, the predicted 4th quark family (T, B) has not been discovered yet, but its existence might resolve several problems within the SM and would increase the value of the Jarlskog constant for the baryon asymmetry of the Universe (BAU) by a factor of about 10^{13} [8].

How do we know that the lepton families and the quark families represent these particular discrete symmetry groups? By imposing the conservation of total lepton family number as the rationale for lepton family mixing and the conservation of total bayon number as the rationale for quark family mixing, we derived the lepton mixing matrix and the quark mixing matrix from first principles without any free parameters [6]. That is, with these discrete symmetry groups, by having a linear superposition of their quaternion group generators for the lepton families and separately for the quark families, we could mimic the continuous symmetry group SU(2) for each and therefore meet the continuous symmetry requirement of Noether's theorem [9] for a conservation law.

All 9 predicted elements of the lepton 3x3 Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix match the experimentally determined value ranges, while 8 of 9 elements in the quark 3x3 Cabibbo-Kobayashi-Maskawa (CKM) agree with experimentally determined value ranges, with only the V_{ub} element disagreeing. Of course, with 4 quark families predicted, one has a 4x4 quark mixing matrix CKM4, and we predict reasonable values for the 4th column and the 4th row. Our values also agree with recent concerns that the first row of the normal 3x3 CKM matrix does not sum to unity (for a review see [10, 11]).

However, this mismatch of family numbers, 4 to 3, might raise concerns for triangle anomaly cancellations, which normally cancel with 3 lepton families matching 3 quark families 1-to-1. But we have the cancellation because the lepton families and quark families separately form linear superpositions to each collectively mimic SU(2), so the anomaly cancellation still occurs via quark SU(2) negative contribution against lepton SU(2) positive contribution.

Table 1: Fermion Group and Graph Entropy Assignments

Family	ν_e, e^-	ν_μ, μ^-	ν_τ, τ^-	u, d	c, s	t, b	T, B
Group	(332)	(342)	(352)	(333)	(334)	(343)	(335
Graph							
n	4	6	12	5	8	24	120
H	2.0	2.585	3.585	2.322	3.0	4.585	6.907

In addition to providing the rationale for the lepton family mixing and for the quark family mixing, we predicted [7] a normal neutrino mass hierarchy (NH) to the neutrino mass values with m_1 = 0.3 meV, m_2 = 8.9 meV, and m_3 = 50.7 meV, reasonable mass values just within the proposed cosmological constraint limit of 60 meV [12]. We await further experiments that will determine the actual neutrino mass values in the near future.

In the following, we utilize the mathematical graph theory property that each discrete symmetry group represents a 3-D or 4-D complete graph and that graph theory identifies an entropy for each complete graph that is determined by its number of vertices, or nodes. This graph theory entropy is not the entropy normally considered in the decay of particles but is an additional entropy to be considered.

Why do we investigate graph theory entropy for these lepton family and quark family discrete symmetries? Because if space happens to be discrete at the Planck scale of about 10^{-35} meters, then this graph entropy could be important. The hope is that we might encounter a graph theory entropy forbidden decay that is allowed by all the known conservation laws and the special theory of relativity (STR). This constraint placed by these graph entropy values for the decays of the leptons and the quarks could either provide further support for the possible existence of a 4th quark family or possibly eliminate a 4th quark family. Therefore, an investigation into the properties and predictions of graph theory entropy seems justified.

2 Graph entropy

Each lepton family and each quark family is represented by a complete undirected graph as illustrated in Table 1, meaning that every pair of distinct vertices is connected by a unique edge [13]. A complete graph G with n vertices has a graph theory entropy

$$H(G) = \log_2 n\,, \tag{1}$$

and given two complete graphs G_1 and G_2 their union entropy

$$H(G_1 \cup G_2) \leq H(G_1) + H(G_2)\,. \tag{2}$$

Therefore the two resulting graphs must have at least the entropy of the original graph. So when a lepton or a quark decays, the total graph entropy of the particle products of the decay must be at least equal to the graph entropy of the decaying particle or else the decay cannot occur.

As an example, consider the weak interaction decay of the muon μ^- first to the W^- plus the muon neutrino, and then the W decays:

$$\mu^- \to W^- + \nu_\mu \to e^- + \bar{\nu}_e + \nu_\mu\,. \tag{3}$$

In Table 1 are given the entropy values for the lepton families and the quark families. The entropy $H(\mu^-) = 2.585$, and the final products of the decay sum to a total entropy 6.585, so the entropy inequality condition is met,

$$H(e^- + \bar{\nu}_e + \nu_\mu) > H(\mu^-) \tag{4}$$

as expected for this prevalent decay channel for the muon.

Tables 2 and 3 contain the entropy values for spontaneous decay channels for the leptons and for spontaneous semileptonic decay channels for the quarks that obey the conservation laws and the special theory of relativity (STR). I have included decay channels for a predicted 4th quark family (T,B).

The up quark in the 1st quark family has a smaller mass than its down quark partner, so a spontaneous decay channel is not available to the down quark, and therefore no decay channel is shown even though given enough energy in a proton-proton collision, for example, the up quark in a proton can change into a down quark to produce a neutron plus a positron and electron neutrino.

Table 4 provides the entropy values for spontaneous quark decays to a different quark plus a meson. With no evidence for the predicted 4th quark family and for the mass values of the T and B quarks, the last two columns contain entropy values for both reasonable decay channels as well as possibly some forbidden decay channels. The predicted mass values for the B and T quarks are estimated by using a four family Koide formula [14].

Intermediate stages in each decay process involve a weak interaction boson, a W or a Z, which have the extremely short lifetime [4] of about 3 x 10^{-25} seconds. Alternative quantum states for the W's and Z can be expressed in terms of the discrete symmetry of the 2I group as the direct product 2I x 2I', where 2I' is the group 2I with one of its quaternion generators modified to provide "reciprocal" operations to ensure that

Table 2: Final Graph Entropy Values for Lepton Decays

Particle	Mass (MeV)	Group	H	Decay Channel Total H	
				$e^- + \bar{\nu}_e + \nu_\mu$	$\mu^- + \bar{\nu}_\mu + \nu_\tau$
e^-	0.511	(332)	2.0		
μ^-	105.66	(342)	2.585	6.585	
τ^-	1776.84	(352)	3.585	7.585	8.755

all 7 families properly experience the weak interaction [15]. Therefore,

$$W^+ = |\nu_\tau\rangle|\tau^+\rangle \quad (5)$$

$$Z^0 = (|\nu_\tau\rangle|\bar{\nu_\tau}\rangle + |\tau^-\rangle|\tau^+\rangle)/\sqrt{2} \quad (6)$$

$$W^- = |\tau^-\rangle|\bar{\nu_\tau}\rangle \quad (7)$$

$$\gamma = (|\nu_\tau\rangle|\bar{\nu_\tau}\rangle - |\tau^-\rangle|\tau^+\rangle)/\sqrt{2}\,. \quad (8)$$

In each expression for an electroweak (EW) boson there exits the product of two identical complete graphs for the tau family representing the discrete symmetry groups 2I and 2I'. Each group in the product uses the same 12 vertices as the single tau family graph for 2I and therefore this product has the same graph theory entropy value as the tau family entropy $H = 3.585$. The W and Z bosons, having the extremely short lifetimes, immediately decay to the various long-lived final states which have final graph theory entropies that are expected to obey the graph entropy inequality in (2).

3 Discussion

We have applied graph theory entropy to lepton and quark decay channels. We had hoped to encounter some forbidden decays as a result of additional graph theory entropy restrictions for decays that are allowed by the normal conservation laws and STR. That is, we looked for forbidden decays that would violate the final state graph entropy inequality in (2):

$$H(G_1 \cup G_2) \leq H(G_1) + H(G_2)$$

where $G_1 \cup G_2$ represents the initial decaying particle state graph and G_1 and G_2 are the final state particle graphs.

We did find two decays that are in violation of this graph entropy inequality in two separate channels, both of them for 4th quark family T and B semileptonic decays to the (u, d) quark family plus the electron family as shown by the two underlined and bold entries in Table 3. These decays:

$$T^{+2/3} \rightarrow d^{-1/3} + W^+ \rightarrow d^{-1/3} + e^+ + \nu_e \quad (9)$$

$$B^{-1/3} \rightarrow u^{+2/3} + W^- \rightarrow u^{+2/3} + e^- + \bar{\nu}_e \quad (10)$$

satisfy all normal constraints but would be prohibited in graph theory because the initial graph entropy of 6.907 would result in a final graph entropy of 6.322, a violation of the graph entropy inequality rule. No other violations were found in the decay channels.

Unfortunately, we have no evidence that the 4th quark family actually exists, so we cannot check Nature for this violation yet. Therefore, we must wait for the opportunity in the future should the T and B quarks make their existence known.

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Table 3: Final Graph Entropy Values for Quark Semileptonic Decays

Quark	$u^{+2/3}$	$d^{-1/3}$	$c^{+2/3}$	$s^{-1/3}$	$t^{+2/3}$	$b^{-1/3}$	$T^{+2/3}$	$B^{-1/3}$
Mass	2.3 MeV	4.8 MeV	127.5 MeV	95 MeV	173.2 GeV	4.8 GeV	3.4 TeV	95 GeV
Group	(333)		(334)		(343)		(335)	
H	2.322		3.0		4.585		6.907	
Decay Channel								
$d^{-1/3} + e^{+} + \nu_e$			6.322		6.322		**<u>6.322</u>**	
$u^{+2/3} + e^{-} + \bar{\nu}_e$		6.322		6.322		6.322		**<u>6.322</u>**
$s^{-1/3} + e^{+} + \nu_e$			7.0		7.0		7.0	
$c^{+2/3} + e^{-} + \bar{\nu}_e$						7.0		7.0
$b^{-1/3} + e^{+} + \nu_e$					8.585		8.585	
$t^{+2/3} + e^{-} + \bar{\nu}_e$								
$B^{-1/3} + e^{+} + \nu_e$					10.907		10.907	
$T^{+2/3} + e^{-} + \bar{\nu}_e$								
$d^{-1/3} + \mu^{+} + \nu_\mu$			7.492		7.492		7.492	
$u^{+2/3} + \mu^{-} + \bar{\nu}_\mu$						7.492		7.492
$s^{-1/3} + \mu^{+} + \nu_\mu$					8.17		8.17	
$c^{+2/3} + \mu^{-} + \bar{\nu}_\mu$						8.17		8.17
$b^{-1/3} + \mu^{+} + \nu_\mu$					9.755		9.755	
$t^{+2/3} + \mu^{-} + \bar{\nu}_\mu$								
$B^{-1/3} + \mu^{+} + \nu_\mu$					12.077		12.077	
$T^{+2/3} + \mu^{-} + \bar{\nu}_\mu$								
$d^{-1/3} + \tau^{+} + \nu_\tau$					9.492		9.492	
$u^{+2/3} + \tau^{-} + \bar{\nu}_\tau$						9.492		9.492
$s^{-1/3} + \tau^{+} + \nu_\tau$					10.17		10.17	
$c^{+2/3} + \tau^{-} + \bar{\nu}_\tau$						10.17		10.17
$b^{-1/3} + \tau^{+} + \nu_\tau$					11.755		11.755	
$t^{+2/3} + \tau^{-} + \bar{\nu}_\tau$								
$B^{-1/3} + \tau^{+} + \nu_\tau$					14.077		14.077	
$T^{+2/3} + \tau^{-} + \bar{\nu}_\tau$								

Table 4: Final Graph Entropy Values for Quark Decay Channels to Mesons

Quark	$u^{+2/3}$	$d^{-1/3}$	$c^{+2/3}$	$s^{-1/3}$	$t^{+2/3}$	$b^{-1/3}$	$T^{+2/3}$	$B^{-1/3}$
Mass	2.3 MeV	4.8 MeV	127.5 MeV	95 MeV	173.2 GeV	4.8 GeV	3.4 TeV	95 GeV
Group	(333)		(334)		(343)		(335)	
H	2.322		3.0		4.585		6.907	
Decay Channel								
$u + d\bar{u}$				6.966		6.966		6.966
$u + s\bar{c}$						8.322		8.322
$c + d\bar{u}$						7.644		7.644
$c + s\bar{c}$						9.0		9.0
$d + \bar{d}u$			6.966		6.966		6.966	
$d + \bar{s}c$			8.322		8.322		8.322	
$d + \bar{b}t$							11.492	
$s + \bar{d}u$					7.644		7.644	
$s + \bar{s}c$					9.0		9.0	
$s + \bar{b}t$							12.17	
$b + \bar{d}u$					9.229		9.229	
$b + \bar{s}c$					10.585		10.585	
$b + \bar{b}t$							13.755	
$B + \bar{d}u$							11.551	
$B + \bar{s}c$							12.907	
$B + \bar{b}t$							16.077	